DO WE NEED A "COOLING" EXPERIMENT?

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Introduction

The question "Do we need a cooling experiment ?" is being debated. I would like to try and define what our goals should be. Obviously the answer is "yes" we do need to demonstrate cooling, but what does this mean? I believe that it requires a test channel that uses hardware that has been studied using our simulation programs, and which also has had prototypes built and measured in order to verify that it will perform as required by the simulation studies. This is a complimentary process. The simulation will define the parameters that must be achieved. The R&D will verify by laboratory studies that the required field accuracy can be obtained, that the component can operate with the required lifetime, that the field geometry can be tied to survey markers with an acceptable accuracy, that the radiation hardness is satisfactory, that it is possible to assemble and service complete units, etc.. These processes are not independent of each other, but interact and we have seen many cases where field configurations used in the simulation could not be realized in an engineering design.

The question of length is complicated. Shown below is a typical simulation of a FOFO cooling channel.

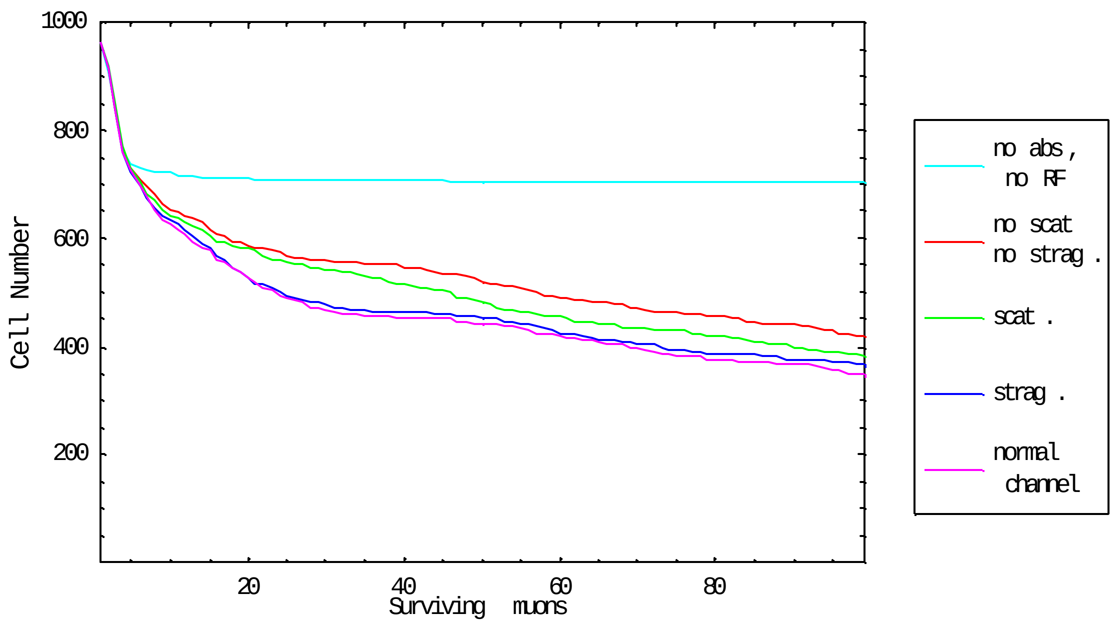

Fig. 1 Attenuation of a FOFO cooling channel with 100sections. 1000 muons are injected at the left-hand side. The initial loss is because the beam emittance is too large for the channel. The five curves are: (a) Empty channel, no absorber, no RF, (b) RF is on but there is no scattering or straggling, (c) RF on, scattering on, straggling off, (d) RF on, scattering off, straggling on, (e) Normal FOFO cooling channel.

In the example shown, the simulation started with an emittance larger than the channel acceptance and the excess particles are rejected. Much of this loss is due to betatron motion and occurs in a few meters. The long sloping portion after this initial loss is due to various stochastic processes operating in the synchrotron phase space and controlling this is crucial for a successful cooling channel.

The above discussion illustrates why a cooling demonstration is complicated to implement and much more work is necessary in this area in order to define a real experiment.

It should be pointed out that the problems that we have faced in the simulation studies so far involve the whole system. There is never a case where only one small unit such as a cavity, an absorber, or a solenoid causes trouble. The problems have required understanding particle loss mechanisms and understanding how second order correlations in the beam phase space can affect the cooling. In the case of a real channel the R&D will have to include development of suitable instrumentation to measure the beam properties in an intense bunched beam environment.

In addition to the problems alluded to above, there may be fundamental physics processes that we need to understand better. It is this question that I would like to address in this note. There are three processes that are involved: dE/dz, scattering, and straggling. The first controls the cooling, and the latter two control the heating. In addition to these processes, there can be random errors in the fields that can contribute to the heating of the beam. The thesis of this note is that proper use of our existing simulation programs can define how accurately we need to know all of these quantities. After we learn how sensitive we are to the above effects, then we can decide whether a scattering and/or straggling experiment makes sense, and the accuracy of the experiment defined. Such studies will also define the accuracy of magnetic field configurations, and the beam survey accuracy that is required for the successful operation of a cooling channel..

At the moment, I think that the experiment to “demonstrate cooling using a large solenoid and particle tracking” does not meet the above requirements. It requires precision tracking in a field which is far from uniform and for particles in the 200 MeV/c region where multiple scattering is a major problem for precision measurements. It develops and uses none of the hardware that is specific to a cooling channel. It does not answer any of the relevant questions that have to do with length. The length scale is of the order of a betatron wavelength (2 meters or so) in the transverse and a synchrotron wavelength (8 meters or so) in the longitudinal direction. It is an experiment that only measures things we know already and if it gets the wrong answer will be criticized as having been done poorly.

What is known about cooling?

The model we have for cooling is formulated in a continuous medium where there is momentum loss in the direction that particle is moving, and in which there is continuous replacement of the z component of the lost momentum by a RF system. This is similar to what happens in the case of synchrotron radiation in an electron synchrotron. This is a dramatic and well-understood demonstration of cooling that has been around for the last 40 years. The equation for the normalized emittance is given below:

$$\frac{d\epsilon_n}{dz} = -\frac{1}{P}\,\frac{dP}{dz} + \beta\,\gamma\,\frac{\beta_t}{2}\,\frac{d}{dz}\,\langle\theta^2\rangle$$

The first term is the momentum loss per unit length in the absorber and it cools (in analogy to the synchrotron radiation energy loss) and the second term is the stochastic process of multiple scattering that heats the beam (which corresponds to the quantum fluctuations in a synchrotron). Both terms are controlled by material the beam travels through. As mentioned above, it assumes that a RF system is replacing the momentum loss in the z direction. The above equation applies to a continuum, but it can be easily generalized to the case of a focusing system using discrete components. The focusing length in the channel is proportional to β_τ , the β function, which needs to be small. A solution of the above equation looks like:

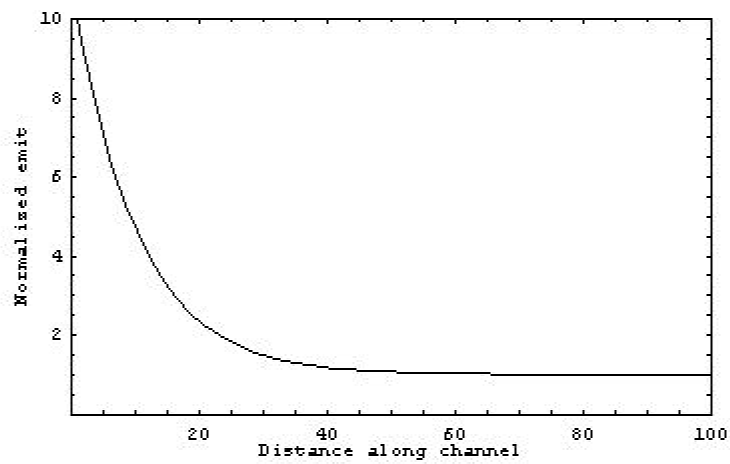

Fig. 2. Schematic behavior of a cooling channel showing the normalized emittance as a function of distance along the channel.

The initial slope is given by 1/P dP/dz and the final value is given by:

$$\epsilon_{min} = \frac{\beta_t}{2}\,E_s^2\,\frac{1}{\beta\, m\, c^2\, X_0\, dE/dz}$$

The Gaussian multiple scattering formula is buried in this equation as well as the energy loss per unit distance. Hydrogen is chosen for two reasons: its large radiation length and the fact that dE/dz is twice as large for hydrogen as for any other element. We can see from this model where the physics of scattering enters the problem. As long as the emittance is larger than the minimum value, the major effect is from the 1/P dP/dz

Term. As the emittance approaches the equilibrium value, the cooling term comes into balance with the heating due to multiple scattering.

There are three effects that need investigation at the single particle level:

1. How well do we know the scattering distribution in hydrogen and in the elements that form the windows in the system?
2. How well do we know dE/dz?
3. How well do we understand straggling?

And then there are questions of how we incorporate this information into the simulation:

1. How do we incorporate the plural and single scattering tails into the theory?
2. How does straggling get incorporated?

These last two questions can be answered by comparing the analytical results given by theory with the simulation results. Paul Lebrun has started on this in MUCOOL note #30, but more work needs to be done. I give the theory in MUCOOL notes #16 & #20. In any case, this question is merely one of computer power and although important does not need to concern us here.

The Particle Data Group quotes that the Bethe-Bloch equation describes the "energy loss of pions in a material such as copper to about 1% accuracy for energies between about 6 MeV and 6 GeV."   Radiation and density effects are not important at our energies, which are about 200 MeV/c and are not far from minimum.

What do we know about scattering?

There are three regions that re important for scattering: the central Gaussian core, plural scattering where several single scattering in series produce a combined p_t with a probability that is larger the single scattering cross section at the same p_t, and finally the single particle scattering. The cooling channel covers all three regions as the max P_t in the channel is of the order of 50 MeV/c.
We now discuss what is known.

First, for the single scattering we note that the proton scatters like a point modified by a form factor

$$G\left(q^2\right) = \frac{1}{\left(1 + \frac{q^2}{q_0^2}\right)^2} \qquad \text{where } q_0 = .71\,\text{GeV/c}$$

Since the scattering cross section is proportional to the square of the form factor. The scattering at P_t = 50 MeV/c is attenuated by about 2% and hence is not important, but is easy to incorporate into the scattering model. We note in passing that mu – e scattering cannot take place at angles beyond m_e / m_{mu} = 5 mr or a P_t of about 1 MeV/c for a typical beam of 200 MeV/c. See MUCOOL note #16.

A number of experiments have been performed to measure Coulomb scattering. There is a list of them in Rick Fernow's MUCOOL note #124. We will comment on two experiments here. The first is by Shen et al in PRD 20, 1584, 1979 and was done in the course of measuring the Coulomb intereference term in hadron proton scattering. A summary of their results is given in the following table.

Target	**Z**	**Measured / Moliere**
Hydrogen	1	0.993 +/- .008
Beryllium	4	1.018 +/- .037
Carbon	6	1.026 +/- .040
Aluminum	13	1.008 +/- .027
Copper	29	0.991 +/- .027
Tin	50	1.010 +/- .021
Lead	82	0.981 +/- .022

The experiment used a number of different beam momenta between 50 and 200 GeV/c and various projectile particles. We consider how it applies to cooling.

The equation for Rutherford Scattering is:

$$\sigma(p_t)\, p_t\, dp_t = \frac{\text{constant}\, F(p_t)\, p_t\, dp_t}{p_t^4}$$

Where p_t is the momentum transfer and $F(p_t)$ is the atomic form factor. It is important to note that except for small a beta dependence in the constant (beta is of order .85) the scattering only depends on the momentum transfer, p_t , and not the momentum, P_0, of the beam. Shen's experiment explored an RMS p_t of about 3.8 MeV/c which is similar to that for a 30 cm long liquid H_2 absorber in a muon beam with momentum 200 MeV/c. This is comparable to the values used in some of the cooling channels that have been studied.

The experiment was analyzed using the Thomas Fermi model of the atom, which is not so good for hydrogen, but satisfactory for the other elements listed in table above. The wave function of hydrogen is known and the elastic and inelastic form factors for the other elements are known from x-ray crystallographic tables, and so there is no need to use this model (see reference to Hubbell et al). The two graphs below show the elastic and inelastic form factors for hydrogen and beryllium:

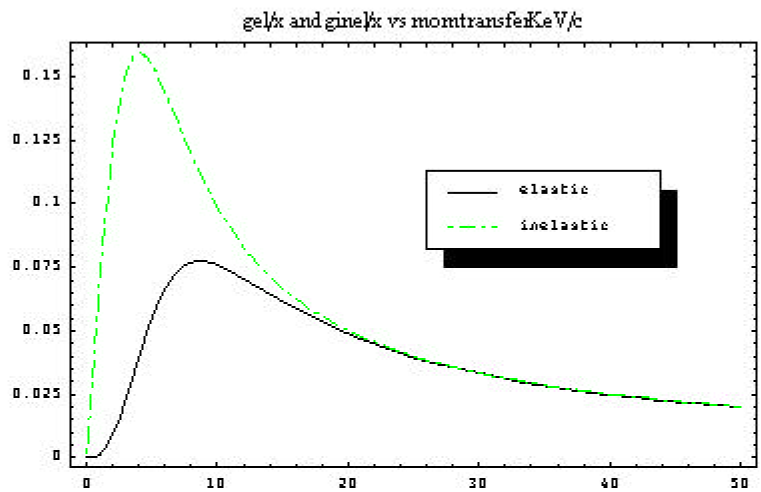

Fig.3 The inelastic and inelastic form factors for hydrogen divided by the momentum transfer. The horizontal scale is in KeV/c. Both form factors approach 1 at a p_t of about 20 KeV/c.

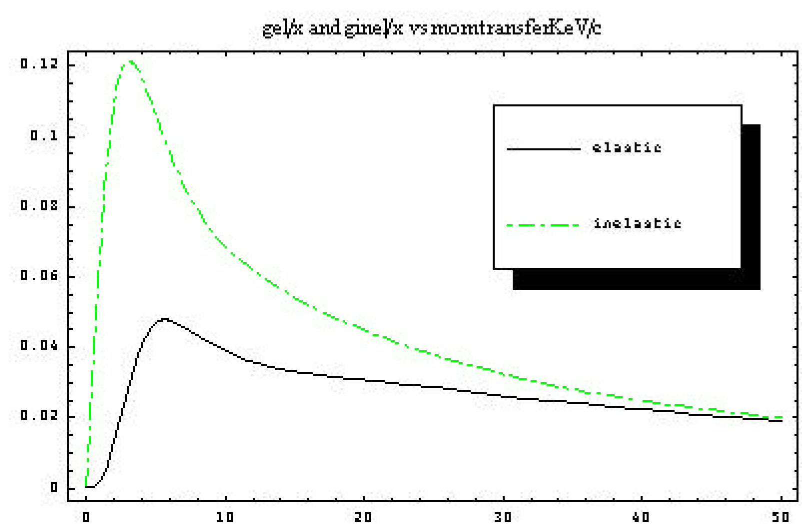

Fig.4. Elastic and inelastic atomic form factors for Beryllium dived by the momentum transfer. The horizontal scale is in KeV/c. The form factors are from Hubbell et al.

The following table compares the Thomas-Fermi calculation of the RMS p_t in MeV/c for the four lightest elements with that obtained using a correct model for the atomic form factors. The x-ray elastic and inelastic form factors are used for He, Li, and Be and for hydrogen its atomic wave function. The target thickness is chosen so that all have a radiation length that is equal to that of 32 cm of liquid H_2.

	Hydrogen	Helium	Lithium	Beryllium
Thomas-Fermi	3.441	3.663	3.644	3.635
Correct atom	3.343	3.500	3.547	3.611

The biggest difference is for hydrogen, 3%, and the calculation with correct form factors approaches that of the TF model as Z increases as it should. Neither of the values for hydrogen includes molecular effects and hence should be increased by 2.88% according to the Particle Data Group.

Bernstein and Panofsky in PR 102, 522, 1956 have measured the radiation length of liquid hydrogen and compared it with theory. This experiment is sensitive to the same Coulomb integrals that are involved in scattering. In addition, the molecular effects are calculated and agree with the 2.9% effect quoted in the PDG tables. They quote an experimental value that is 2.4%+/-2.8% below the best-calculated value. I take this as a direct measurement of the form factor for liquid hydrogen, and it is within 3% of the calculated value.

How about straggling? I haven't completed a study of this effect. However, there is an interesting observation that straggling may not be the most important effect causing the longitudinal losses. The multiple scattering causes the length of the particle path to change in a stochastic manner, and this in turn shows up in the longitudinal phase space. We are in the process of understanding the relative importance of the two effects. One of the beauties of the simulation program is that the different stochastic processes can be turned on one at a time…unlike nature!

To summarize, I believe we know by, direct measurement, dE/dx and multiple scattering to a few percent, and these determine the cooling curve completely in the transverse phase space. When one questions how well we need to know these numbers, it is important to fold these uncertainties into all the rest of the unknowns like: timing of the individual RF cavities, their voltage, the length of the Hydrogen absorber cells (the windows bulge, there may be bubbles, density is a function of temp and pressure), the momentum of the incoming beam, and many other things. The effect of variations in the channel parameters can be studied and assigned limits by the use of our simulation programs. I think that an important ameliorating effect will come from the fact that the channel, by its very nature, will damp out the effect of short term errors and will respond adiabatically to long term ones.

It is my belief that there is a lot of quantitative work that needs to be done with the tools we now have in hand before we can propose a definitive "cooling demonstration".

Some references that contain pertinent work are given below. They are not meant to be complete. They can all be found via the web on the MUCOOL page:

http://www-mucool.fnal.gov/notes/notes.htm

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J.H.Hubbell, Wm. J. Veigele, E.A. Briggs, R.T. Brown, D.T. Cramer, and R.J. Howerton J. Phys.Chem. Ref. Data, Vol.4, No. 3,1975.

Mucool 16 **Kinematics and cross section for mu - e scattering** A. V. Tollestrup

Mucool 20 **Moliere Scattering for 186 MeV/c muons in Hydrogen** Alvin. V. Tollestrup

Mucool 30 **Modeling of Multiple Scattering Effects in the Presence of a Strong Magnetic Field** Paul Lebrun

Mucool 37 **Errors in a discrete solenoid** Juan C. Gallardo

Mucool 41 **Errors in a continuous solenoid** Juan C. Gallardo

Mucool 73 **Sensitivity of the 15 T. Alternate Solenoid Cooling Channel to Dipole Errors caused by misalign...** Paul Lebrun

Mucool 89 **R&D for MUCOOL** Alvin Tollestrup

Mucool 91 **Effect of solenoid errors on the performance of the cooling channel** Juan C. Gallardo

Mucool 123 **Suitability of Moliere scattering theory for ionization cooling simulations** R.C. Fernow

Mucool 124 **Comparison of Moliere scatttering theory with Mott scattering in liquid hydrogen** R.C. Fernow

Appendix

I would like comment on the proposal for a cooling experiment. The information that I am addressing is found at:

http://puhep1.princeton.edu/mumu/cool1.ps

The experiment starts with a muon beam that has a relatively small emittance and runs it through a U diffuser to increase the angular spread. Then the emittance is measured with chambers before it enters a hydrogen absorber cell. The final measurement is made again in a TPC.

Comments:

1. A serious technical problem that must be addressed is the precision measurement of the four-vectors before and after the hydrogen cell. This will lead to a data set that in principle describes scattering and energy loss in hydrogen. However these measurements must be carried out at 200 MeV/c where scattering is a major problem, and in a magnet whose field is not particularly uniform. A question: if this is just a scattering and energy loss measurement, why not setup a good geometry experiment and make the measurement directly as is being done in the experiment presently being set up at TRIUMF? Why blow up the beam with a diffuser and make a very difficult measurement in a non-uniform magnet field?

2. This is not a cooling demonstration, as it does not replace the z component of momentum. It relies on the conservation of phase space in a conservative system to calculate what the invariant emittance would be if a RF cavity were present after the absorber. The "cooling for a single particle" will be determined by its before and after energy and thus is not a demonstration of cooling, but a calculation of cooling. The cooling of a real bunch averages in a complicated manner over all of the particles in the bunch. It is affected by both straggling and scattering which can result in different transit times for different particles and consequently a variation in the amount of pz that is replaced.   The resulting synchrotron oscillations cause particle loss and is a major problem as shown by the simulation.

3. None of the apparatus used is that which must be developed to instrument and cool a real bunched beam.