

# High-Frequency Buncher and $\phi$ - $\delta E$ Rotation for the $\mu^+$ - $\mu^-$ Source

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## Abstract

An alternative scenario for bunching and rf rotation of  $\mu^+$ 's from the proton source is presented. It consists of a drift section, a variable frequency  $\sim 300 \rightarrow 180$  MHz bunching section, and a fixed frequency ( $\sim 180$  MHz)  $\phi$ - $\delta E$  rotation section. In initial 1-D simulations, the overall capture performance of the system is similar to that of induction linac + buncher scenarios developed for the neutrino factory. The total rf required for the system is quite modest.

## Introduction

In various scenarios for a  $\mu^+$ - $\mu^-$  Collider or a  $\nu$ -Factory a phase-energy ( $\phi$ - $\delta E$ ) rotation is performed on the beam exiting the decay channel.[1, 2, 3] In that rotation the beam is allowed to lengthen and an rf system (which decelerates the high-energy "head" of the bunch and accelerates the low energy "tail") is used to reduce the energy spread. The resulting beam, which is a long bunch with smaller energy spread, has an energy spread reduced to a level where the bulk of the  $\mu$ -beam is captured by a downstream bunching and/or cooling system.

The difficulty with previously proposed  $\phi$ -E rotation systems is that they require either very-low-frequency rf, or an induction linac, matched to the elongated bunch length of the  $\phi$ -E rotated system. This long-wavelength (or long rise-time) acceleration system would require new technology development and considerable expense.

We are thus interested in alternatives which avoid such new acceleration systems, and use established high-frequency rf systems. In this paper we present an approach which uses high frequency rf systems for bunching the beam, and then reducing the bunch to bunch energy difference, obtaining a beam distribution similar to that obtained from the neutrino source  $\phi$ -E rotation system + initial buncher system. An example with these properties is presented. In this example, the rf systems are chosen with frequencies in the  $\sim 200$ – $300$  MHz frequency regime, which is a frequency range where rf systems have been readily developed. The properties of this example are explored and guidelines for optimization and implementation are developed.

## Proposed Alternative: Initial Example

The first example was adapted from the Fermilab neutrino factory scenario parameters [1]. An initial beam with a small phase spread, but large energy spread (similar to beam from a  $\pi \rightarrow \mu$  production target) was generated and drifted for 100m.

This is followed by an "adiabatic buncher" section, in which an rf system is gradually increased in gradient, "adiabatically" capturing the beam into a string of bunches. In a usual rf system the central energy of each bunch would be the same, corresponding to the synchronous energy of the fixed frequency rf system. In this case the rf frequency would decrease along the length of the buncher, following the constraint that the phase difference between two reference energies

remains a fixed number  $N_w$  of wavelengths, as the beam propagates down the buncher. Thus the reference energies remain at zero phase in their respective bunches, and stable phases and energies are obtained for  $N_w - 1$  evenly spaced intermediate points, and at locations before and after the reference energies. For the first example the central energy was at 125 MeV kinetic energy and the reference energies at +50 MeV and -50 MeV from that value, with  $N_w$  set at 15. With these numbers the matched rf frequency at the beginning of the buncher section is ~300 MHz, and at the end of the 60m buncher it is reduced to ~180 MHz.

In the bunching system the rf bunching gradient is gradually increased from zero to a value of 4.8 MV/m over the length of the buncher (with a quadratic ramp in this first example). The goal here is to attempt an “adiabatic capture”, in which the beam within each bunch is compressed in phase so as to be concentrated near the zero phase for each bunch. Note that, since each of the bunches is centered at different energies, they all have different longitudinal oscillation frequencies, and a simultaneously matched compression for all bunches is not possible. Instead a quasi-adiabatic capture obtaining an approximate bunch length minimization in each bunch is attempted.

Following the buncher the rf frequency is fixed to the matched value at the end of the buncher and the rf gradient is increased to as large a value as possible. In the initial example this matched frequency is ~183 MHz, while the rf gradient is 10 MV/m. In this system, the centers of the low-energy bunches increase in energy, while the centers of the high-energy bunches decrease in energy, similar to the particle motion in a fixed frequency system with a large energy spread and zero initial phase spread. After  $\frac{1}{4}$ -synchrotron oscillation (~8.4 m), the energy spread is ~ minimized. At that point the beam is now in a string of similar-energy bunches, and would be captured into a ~180-MHz ionization cooling system matched to the central energy of the beam.

In our initial simulations we have approximated the following cooling system as a longitudinal buncher, with a bunching voltage corresponding to the net bunching voltage available in prototype cooling designs (~4 MV/m), and have tracked beam through a bunching transport of ~100 m, to obtain an initial estimate of the beam density that might be longitudinally captured in the cooling system.

Some simulation results of this process have been developed and are displayed in figs. 1A-D, with a complementary view in figures 2A-C.

### **Scenario Discussion**

The key components of the scenario are : 1. a drift section, 2. a buncher section, which separates the beam into a sequence of bunches of different energies, and 3. A  $\frac{1}{4}$ -wave debuncher, which effects an approximate  $\phi$ - $\delta E$  rotation of the bunch centers. This would then be followed by an rf system for cooling.

This can be compared with the Fermilab scenario of Study 1, which had a 50m drift section, a 100m induction linac for  $\phi$ - $\delta E$  rotation, and a ~30m buncher for capturing the beam into the 200 MHz cooling section. The major change is replacement of the induction linac by a short fixed-frequency buncher, obtainable by changing the order of bunching and  $\phi$ - $\delta E$  rotation.

Even if the basic structure of the capture, buncher and  $\phi$ - $\delta E$  rotation is maintained, the system has a large number of interrelated parameters, and it seems somewhat unlikely that our initial attempt

was fully optimized. In order to systematize future optimizations, we list the key parameters of each component, and discuss our initial parameter choices.

The key parameters are:

1. The Drift section: The key parameter here is the length of the section, which was arbitrarily set to 100 m for similarity to previous  $\mu$ -collection systems. Future studies which include transverse motion must consider the apertures and focusing fields; previous studies have used fixed-field solenoids for transverse focusing. These focusing parameters are also critical system parameters.
2. The Buncher section: The length of the section, the bunching voltage, and the voltage increase program can all be explored. In the initial example, the length of the buncher was 60 m, the final bunching voltage was 4.8 MV/m and a parabolic increase in voltage was used ( $V_{rf}' = V_{final}' (z/L_{buncher})^2$ ); a linear ramp appeared to give slightly inferior bunching. Also the bunching rf voltage (and phase) could be varied from the fixed wave number spacing between reference energies. In the initial example, the distance between the reference energy particles at kinetic energy  $E = 175$  MeV and  $E = 75$  MeV was set to a constant number of wavelengths ( $N_w = 15$ ). The length of the buncher was 60m and the final bunching voltage was 4.8 MV/m. This obtains an rf wavelength which increases from 1.036 m to 1.637 m over the length of the buncher.
3. The  $\phi$ - $\delta E$  rotation: The length and rf voltage of the phase rotation section ( $L_{RFR} = 8.4$  m and  $V' = 10$  MV/m in our initial example) are the key parameters. In general, more gradient would be better and the optimum rf rotation section length should be adjusted to provide  $\sim 1/4$  synchrotron oscillation for the beam. The rf frequency is constant and set to the matched value at the end of the buncher, with 15 rf wavelengths between the reference particles, and with zero initial phase for the reference particles. The rf wavelength and phase could be perturbed from this to optimize performance. Also the central reference energy (125 MeV in our case) could be perturbed for optimization.
4. The cooling system parameters: On entering the cooling section the rf wavelength was readjusted to match the spacing between the reference particles to 15 rf wavelengths, obtaining  $\lambda_{COOL} = 1.678$  m, so that the bunches remain centered in the capture buckets. The total number of captured and cooled particles will depend on the capture bucket of the cooling system. The values used here (4 MV/m) approximate the rf bucket area of the cooling systems used in feasibility study 1 ( $V'=10$  MV/m, with stable phase at  $30^\circ$ , where reacceleration matches energy loss in the absorbers). In future studies our approximate system should be replaced by an actual cooling + rf system, with the problems of maintaining beam capture with the energy straggling in cooling.

In addition we must consider the initial beam parameters, where we chose a full bunch length of  $L_B = 1.2$ m and a kinetic energy of 125 MeV with reference energies of  $\pm 50$ MeV, which are used in setting the matched frequencies of the rf modules. The system captures a large fraction of the beam up to  $\sim \pm 100$  MeV. Future studies should vary the parameters, particularly the design energy and energy spread, and match these more closely to optimum production values. We have investigated increasing initial bunch length to 4m and obtained little capture degradation. (In the simulations discussed below,  $L_B = 1.2$ m (full width) was used for Figs. 1A-D, and  $L_B = 4$ m was used for Fig. 2A-C.)

## Discussion of simulations

To illustrate the method we display an example with 1-D simulations of the longitudinal motion within a buncher-phase rotation system. Simulation results are displayed in Figs. 1 and 2.

In this example an initial muon beam is generated within a uniform kinetic energy spread from 45 MeV to 225 MeV and an initial gaussian longitudinal distribution (in  $c\tau$ ) from  $-0.6$  m to  $0.6$  m. ( $2\sigma$  cutoff with  $\sigma(\text{gaussian}) = 0.3$  m) The reference energies used to define the reference rf wavelengths are  $E=75$  MeV and  $175$  MeV. Fig. 1A shows the beam after a  $100$  m drift, where the beam has extended itself to  $\sim 40$  m length, with the lower energy beam trailing behind. Reference particles at  $75$ ,  $125$  and  $175$  MeV are shown by crosses.

Following this the beam is put into a buncher, where an rf voltage bunches the beam. The distance between the reference particles is fixed at  $15$  rf wavelengths; this means the rf wavelength increases as these particle drift apart. The beam forms into a string of bunches at different energies, as shown in Figure 1B, which displays the beam at the end of a  $60$  m buncher, where the rf wavelength is  $183$  MHz.

This is followed by a high-gradient fixed-frequency rf system for  $\phi$ -E rotation. After a  $8.4$  m,  $10$  MV/m system the bunches are approximately rotated to similar central energies, as shown in Figure 1C.

Following this the beam would be inserted into a cooling system. In the 1-D simulation we have approximated the longitudinal acceptance of the cooling system by a  $4$  MV/m bunching system, with the rf wavelength slightly lengthened to  $1.678$  m to match the bunch center spacings after the end of the  $\phi$ - $\delta E$  rotation rf. Fig. 1D shows beam after  $100$  m of this buncher; most of the initial beam ( $\sim 80\%$ ) has been captured within these approximate cooling bunches.

To clarify some of the issues involved in bunching system, we present a somewhat different view of the same system in figs 2A-C. Here the beam longitudinal motion is graphed with particle phase within its bunch as the longitudinal coordinate; this phase is constrained to lie between  $-\pi$  and  $+\pi$ , so all of the subbunches of the beam are shown overlapping in phase. In this system the distribution is nearly unchanged over the  $100$  m drift. However the  $60$  m buncher concentrates the beam near zero phase in all bunches, as shown in figure 2A. (In this simulation we used an initial longitudinal distribution from  $-2$  to  $+2$  m ( $2\sigma$  gaussian cut-off), this obtains smoother distributions than the narrower  $\pm 0.6$  m case of Figs. 1.)

After the buncher the beam passes through the fixed frequency rf rotation system. This fixed frequency rf rotation is then seen as an approximate  $1/4$ -synchrotron oscillation, taking the large energy spread, short phase-width seen in Fig 2A to a reduced energy-spread, full phase-width beam seen in Fig. 2B.

Note that in Fig. 2B the beam appears to fill the entire phase length from  $-\pi$  to  $\pi$ . This is an illusion, since the beam from the bunches at the high energy head of the beam is concentrated at larger phase and the initially lower energy bunches are concentrated at smaller phase; the overlap of all bunches covers all phases. To minimize the phase-width of the cooling bunches, the

wavelength of the cooling is made slightly greater than the  $\phi$ - $\delta E$  rotation wavelength, matching to the distance between the centers of the beam within the reference bunches.

After this rotation the beam is inserted into a fixed frequency cooling section. The longitudinal distribution ( $\phi$ - $\delta E$ ) at the end of the 100m section is shown in figure 2C, where it can be seen that most of the beam (~80%) is captured into the center of a cooling rf bucket.

To clarify the mechanism for  $\phi$ - $\delta E$  rotation we show in Fig. 3 the beam that results from a large energy spread, zero phase width beam passing through the 8.4 m 10 MV/m rf system; this system obtains a long bunch, reduced energy spread beam, with the  $\sim 1/4$ -synchrotron oscillation visible in the simulation results. The beam shape approximates beam shape features seen in Figs. 1C and 2B, indicating that the single bunch  $\phi$ -rotation has been spread over the many bunch centers, with the beam in each bunchlet approximating a section of the rotation.

We note that, as shown in Fig.3, we obtain only an approximately linear  $\phi$ - $\delta E$  rotation, and a nonlinear  $\phi$ - $\delta E$  relationship is seen, resulting from the sinusoidal rf system and the energy-velocity relationship. (The figure suggests that adding a higher harmonic rf system could reduce the energy spread in our case; this will be investigated in future studies.)

### Summary and future directions

The bunching and  $\phi$ - $\delta E$  rotation system shown here (see fig. 4) is much more modest than the induction linac + buncher of the feasibility study, with the 60-m buncher requiring a total of 96 MV of low-gradient rf, and the single-frequency 8.4 m  $\phi$ - $\delta E$  rotator requiring 84MV of high-gradient rf. These rf systems are at  $\sim 200$  MHz frequencies where the required cavities and gradients are within the capabilities of present technology and should be relatively affordable. Note that we require a number of different frequency cavities, ranging from  $\sim 300$  MHz to  $\sim 180$  MHz; explicit development of these similar but different systems could add some complication.

A clear deficiency in the present analysis is that only longitudinal motion has been included in these initial simulations. Future studies must include transverse motion effects. Also the initial beams were unrealistic flat-distribution beams; realistic initial beams should also be used and the scenario parameters reoptimized in these more realistic simulations.

As discussed above, there are a large number of interrelated parameters in our buncher +  $\phi$ - $\delta E$  rotation scenario, and they have not been systematically optimized. It is likely the buncher could be improved considerably with a more optimized solution which would shape the bunchlets to smaller widths than this initial attempt. The bunching wavelengths could be changed from our initial values. In general, we expect longer wavelengths (lower frequency) systems would have better acceptance, if high gradients and moderate costs can be maintained. Transverse motion difficulties are likely to be less at lower frequencies. Also, a larger energy width could be captured if the central capture energy were increased; however, this must be weighted by the  $\mu$ -production density, which would shift the optimum to lower energies. Variations of features and details of the scenario will be attempted in order to improve the method.

This method should be compared with the baseline methods which combine low-frequency acceleration with the buncher. Our initial simulation here indicates that its performance should be similar to that of the induction linac + buncher section of ref. 1. In addition, it has the significant advantage that the same system would obtain strings of both  $\mu^+$  and  $\mu^-$  bunches, with

the bunches interleaved at  $180^\circ$  intervals. And this method would avoid the relatively large cost of the induction linac acceleration system, or any other low-frequency rf system.

### **Acknowledgment**

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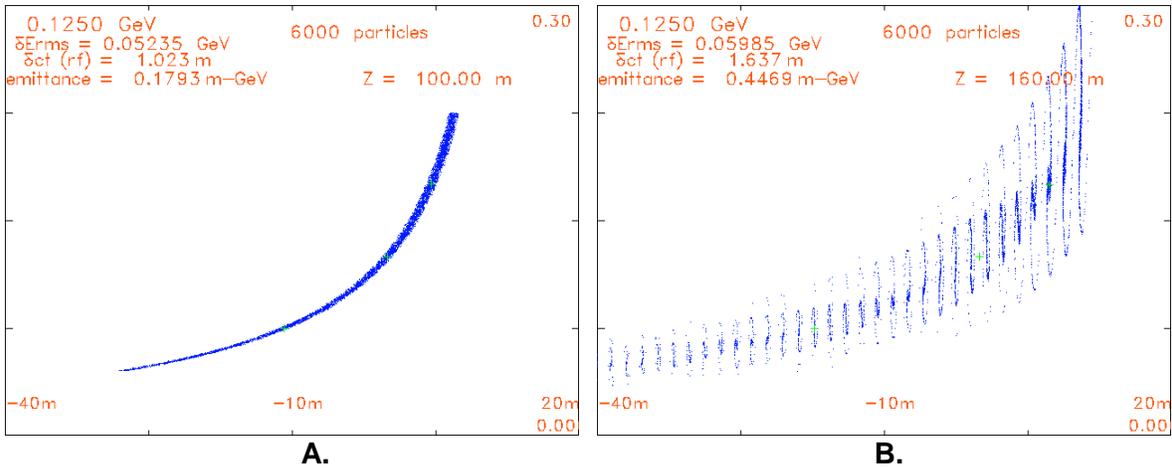
### **References**

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- [2]  $\mu^+\mu^-$  Collider - A Feasibility Study, BNL-52503, Fermi-Lab-Conf.-96-092, LBNL-38946 (1996), presented at the Snowmass 96 workshop (1997).
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Fig. 1A-D – Beam distributions in z-E space (position  $c\tau$  and kinetic energy E) in a simulation of longitudinal motion in a high-frequency buncher- rf rotation system. The initial beam extends from 45MeV to 225MeV kinetic energy, with a short phase width of  $\pm 0.6\text{m}$  (gaussian in  $c\tau$ , with  $\sigma=0.3$ , and a  $2\sigma$  cutoff).

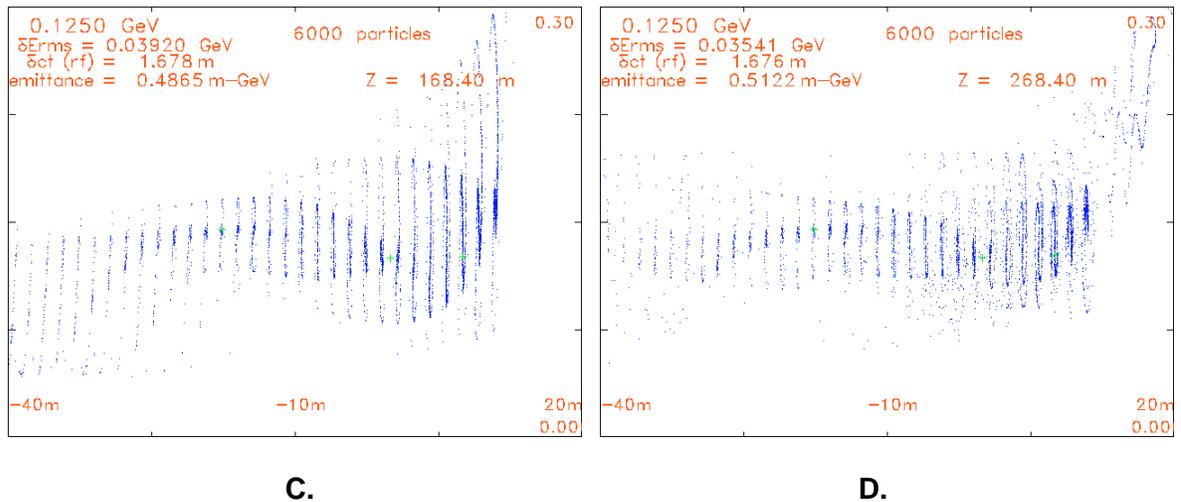
A- After a 100m drift, the distribution extends from  $\sim -30$  to  $+10\text{m}$ .

B- After a 60m buncher section, in which the frequency ramps from 300 to 183MHz, with rf gradient increasing parabolically to 4.8 MV/m, the beam has developed a bunch structure.



C- After a 8.4m single-frequency 10MV/m  $\phi$ -E rotation, the bunchlet energies are more equal.

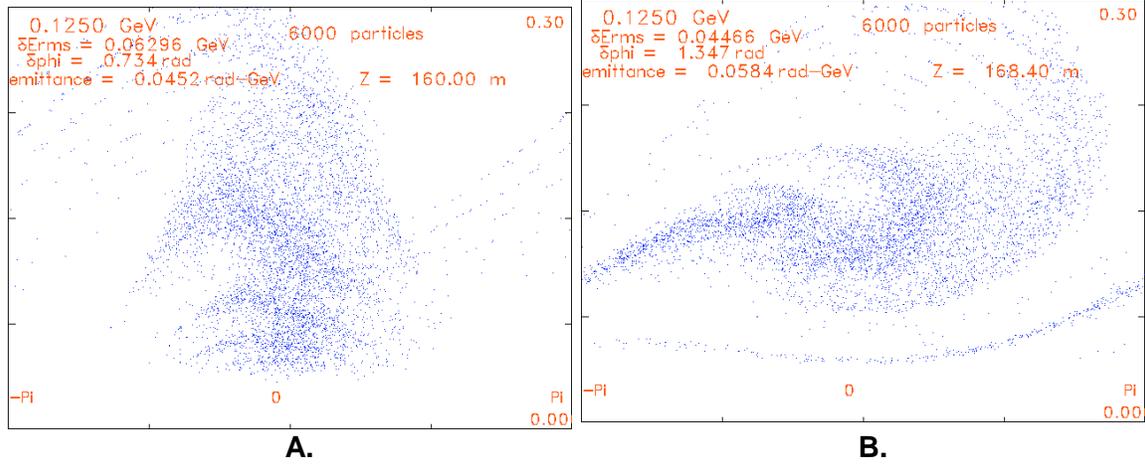
D- After an additional 100m with a buncher approximating the bunching in a cooling system,  $\sim 80\%$  of the initial distribution is within cooling rf buckets.



Figs 2: The same drift, buncher,  $\phi$ -E rotation, and cooler as figs. 1A-D.  $\phi$ -E distributions are displayed, where  $\phi$  is the rf phase within each bunch (so that the distributions of different bunches are superimposed). (The initial  $c\tau$  width was  $\pm 2m$ , larger than that of figs. 1A-D.)

A: Beam at the end of the drift and 60m buncher,

B: Beam at the end of the  $\phi$ -E rotation section,



C: Beam after 100m cooling buncher; ~80% of the initial beam is captured in the cooling rf bucket.

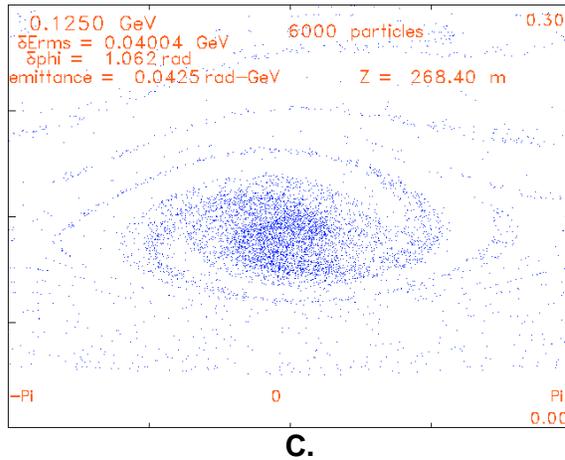
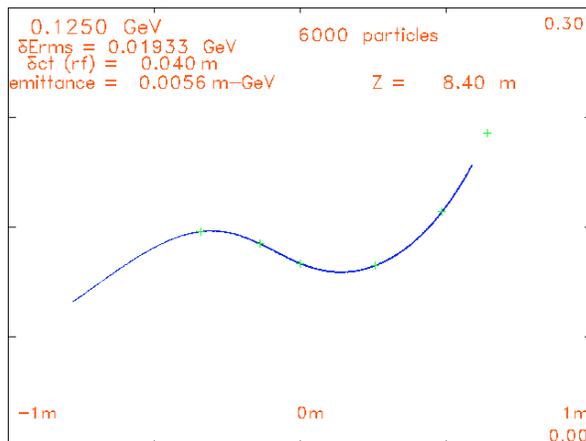


Fig. 3. Beam of large energy spread and zero phase width after a single  $\phi$ - $\delta E$  rotation ( $L = 8.4m$ ,  $V' = 10MV/m$ )



## Overview of transport



Fig.4 Overview of the transport system used above, which is a 100m drift, a 60m buncher, a 8.4m.  $\phi$ - $\delta E$  rotator, and a 100m cooling system.