Muon Cooling Rings for the ν Factory and the $\mu^+\mu^-$ collider

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Abstract. We designed two storage ring models for 6 dimensional phase space cooling of intense muon beam for the neutrino factory and the muon collider, by using the SYNCH code. We demonstrate the 6 dimensional muon phase space cooling with the first 8 cell cooling ring model by using a tracking simulation code, ICOOL, with distributed Lithium lenses with the β at 30 cm, wedge absorbers, and RF cavities, with the muon momentum at 500 MeV/c. Phase space cooling is done by the transverse ionization cooling, and the wedge absorbers contribute to the emittance exchange from the longitudinal phase space to the transverse phase space.

The second muon cooling ring has a 1.25 m long Lithium lens with the β at 1 cm in a straight section of a race track ring. Tracking simulation study is in progress to demonstrate the 6 dimensional phase space cooling of muon beam.

1. Introduction

In order to reduce the 6 dimensional phase space of muons within their lifetime, the ionization cooling is considered to be one of the most promising method, where both transverse and longitudinal momenta are reduced due to the energy loss in absorbers, and the only longitudinal components of the muon momenta are restored through the accelerating fields of RF cavities. The multiple Coulomb scattering contributes to heat the transverse phase space. And the normalized transverse equilibrium depends on material kinds and the transverse beta function where an absorber is located. Because of the difficulty of extraction and injection, muon cooling rings have to be designed without using solenoid as focusing elements. Initially, we designed muon cooling rings with quadrupole magnets, RF cavities, wedge absorbers of liq. H_2. [1] In order to increase the acceptance and to reduce the circumference, we then designed muon cooling rings with zero-gradient dipole magnets with edge focusing, RF cavities, and liq. H_2 wedge absorbers. Work is still in progress in improving the cooling performance in muon cooling rings.

Lithium lens is an active focusing element with energy absorber function at the same time. With β at 1 cm with high current density Lithium lenses, the normalized transverse emittance can be at 100-200 mm $\cdot$ mrad, which is low enough for a $\mu^+\mu^-$ collider.

By using the muon cooling rings and repeat the phase space cooling multiple times, the cost of the muon cooling channel can be reduced for the Neutrino Factory designs, and by using Lithium Lens with small beta in a storage ring, the 6 dimensional phase space cooling can be achieved for the muon collider designs.

2. Phase space cooling in a muon cooling ring

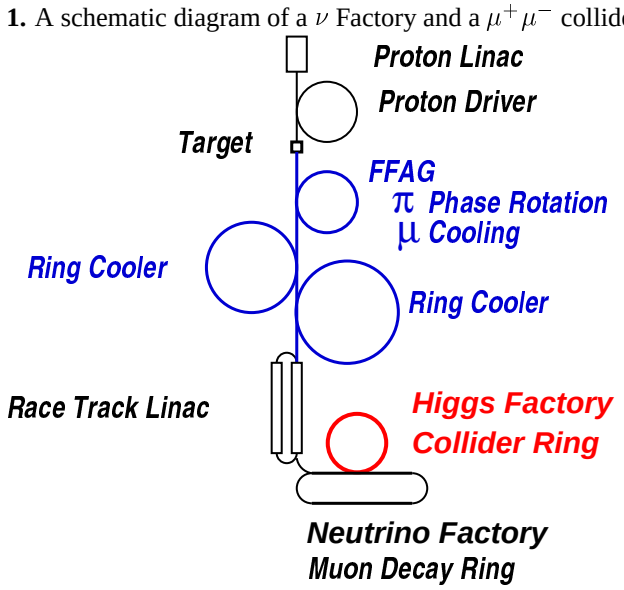
Figure 1 shows a schematic diagram of a ν Factory and a $\mu^+\mu^-$ collider. Two stages of muon phase space cooling rings are used in the figure.

In the longitudinal phase space, a wedge absorber in a dispersive region can reduce the energy spread of muons, where the straggling of the dE/dx and the slope of the dE/dx

as a function of the muon momenta contributes to heat the longitudinal phase space by widening the energy spread. The wedge absorbers perform the emittance exchange from the longitudinal phase space to the transverse(horizontal) phase space, due to the change of the muon trajectories in a storage ring when relative muon momenta were changed through absorbers in dispersive regions. Table 1 lists a comparison of an electron damping ring and a muon cooling ring on elements of damping, excitation, and the partition numbers in the transverse phase space and in the longitudinal phase space. Partition numbers in the Robinson's theorem are also listed. The muon cooling ring with wedge absorbers is similar to the well known electron damping rings in the damping and excitation terms.

Figure 1 shows a schematic diagram of a ν Factory and a $\mu^+\mu^-$ collider. Two stages of muon phase space cooling rings are used in the figure.

Figure 1. A schematic diagram of a ν Factory and a $\mu^+\mu^-$ collider



We designed an eight cell ring, with eight 45 degree bending cells, with a storage ring design code, SYNCH [3] by using two sets of zero-gradient dipole magnets with edge focusing. Figure 2 shows a schematic diagram of the eight cell muon cooling ring, and a blow-up of a half of a 45 degree bending lattice. A circumference is 28.8 m and a radius is 4.6 m. 2.5 cm long liq.H_2 wedge absorber and 7 cm long Lithium lens with field gradient of 3.3 Tesla/m, a set of two 0 gradient dipole magnets with bending angles of 44 degree and -22 degree with edge focusing, and a half of a 201 MHz RF cavity is shown in a half of a 45 degree cell. The cell length is 3.6 m. Figure 3 shows β_x, β_y and η in a 45 degree bending cell in the SYNCH modeling. The maximum β_x is 1.8 m, and the maximum β_x is 2.7 m at the outside dipoles. At the center of the 45 degree cell where the liq.H_2 wedge absorbers and Lithium lenses are placed, the minimum β_x, β_x are 30 cm each, and the maximum η, the dispersion, is 38 cm.

SYNCH is a linear matrix program to design a storage ring. It does not have any of the following, acceleration through RF cavities, dE/dx energy loss and straggling, multiple Coulomb scattering in the absorbers or in the Lithium lenses, and the effect of nonlinear field on particle tracking. We use the fitted values of the ring parameters of SYNCH as input parameters of the ICOOL [4], a tracking code with nonlinear field configurations.

Table 2 lists parameters of the eight cell muon cooling ring. Figure 4 shows evolution of the normalized transverse emittances, ϵ_{nx}, ϵ_{ny}, and the normalized longitudinal emittance, ϵ_{nz} and a merit factor as a function of the path length along the central trajectory. Here, x

Figure 2. A top view of a muon cooling ring and a schematic diagram of a half of a 45 degree bending cell

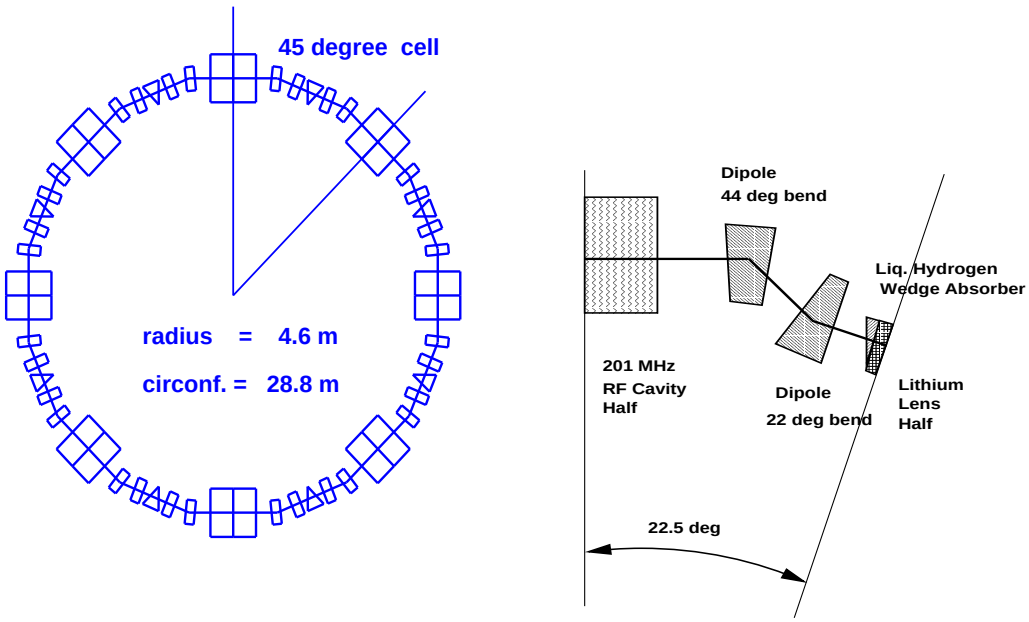
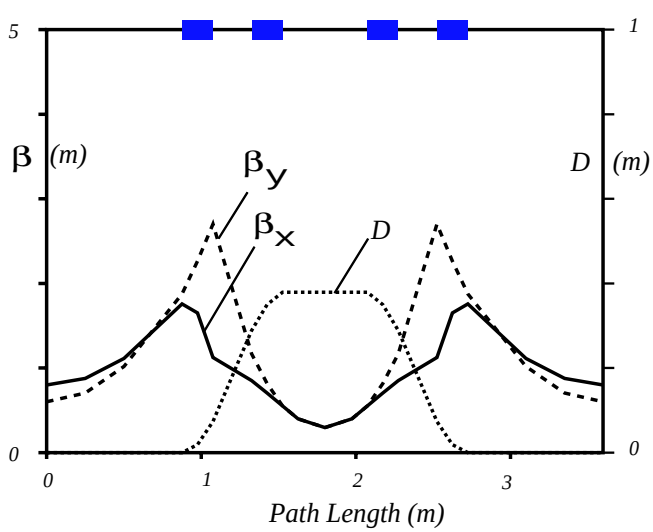


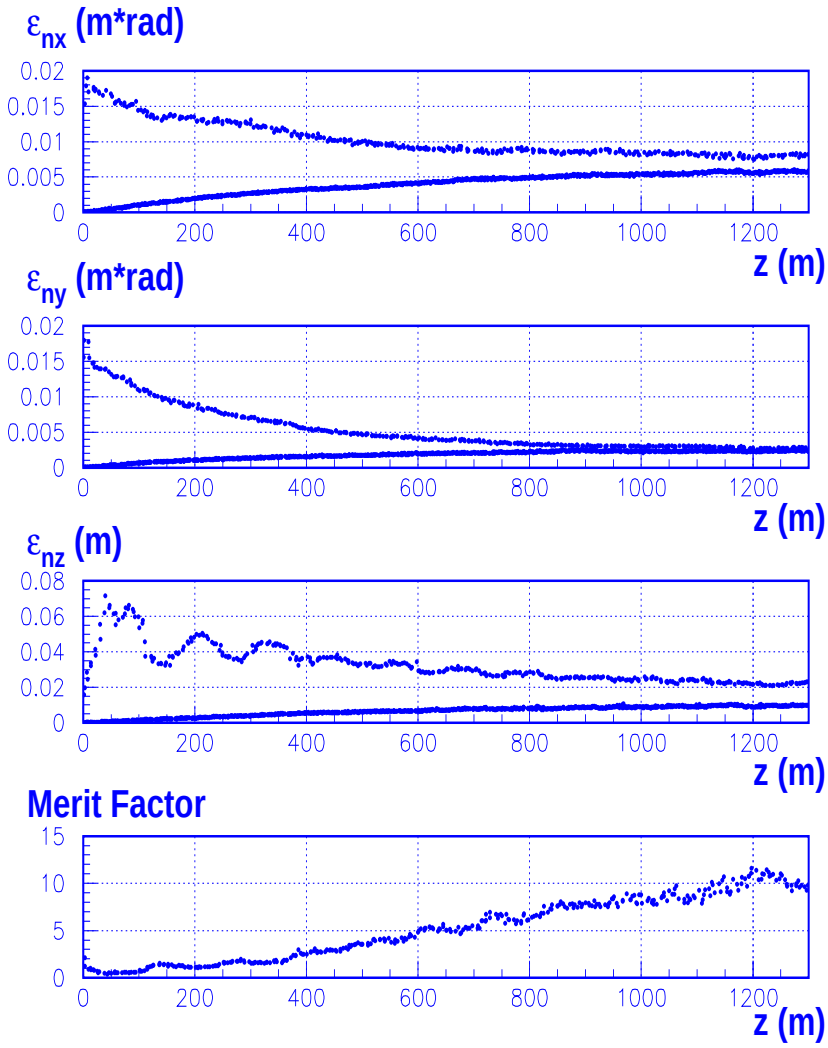
Figure 3. β_x, β_y and η as a function of z in a 45 degree bending cell



is the horizontal coordinate and its positive direction goes outside of the muon cooling ring, y is the vertical coordinate, and the z goes along the central trajectory with beam. With the minimum β at 30 cm at the wedge absorbers and the Lithium lenses, expected normalized vertical equilibrium emittance $\epsilon_{ny,qui}$ is 1.9 mm·rad, and we obtained 2.3 mm·rad in the ICOOL simulation which is close enough to the expected number.

A merit factor is defined as a ratio of the initial normalized 6 dimensional emittance to the final normalized 6 dimensional emittance, multiplied by the muon transmission, without including the muon decay factor. With the average muon momentum at 500 MeV/c, the average survival factor of muons due to decay at z at 1200 m is 69 %. The merit factor in this

Figure 4. x, y, z normalized emittances and a merit factor as a function of z in the ICOOL tracking simulation on the 8 cell muon cooling ring



simulation shows a merit factor of around 10 at z at 1200 m, which corresponds to 42 turns.

3. A muon cooling ring with a Lithium lens with the β at 1 cm

We designed a race track ring model where a 1.08 m Lithium lens with the β at 1 cm, sandwiched by two 9 cm long matching Lithium lens with the β at 5 cm, is installed in a straight section. Table 2 lists parameters of the race track muon cooling ring. Figure 5 shows the schematic diagram of the race track ring. The circumference is 64.8 m, the straight sections are 18.0 m each. Figure 6 shows β_x, β_y inside two matching Lithium lenses and a central Lithium lens. Matching of the β function through a matching Lithium lens is done by:

$\beta_{match} = \sqrt{\beta_{in} \cdot \beta_{out}}, \qquad \lambda_{osci}/2 = \pi/2 \cdot \beta_{match}$

where $\beta_{match}, \beta_{in}, \beta_{out}$, and λ_{osci} are an equilibrium beta function of the matching Lithium lens, beta functions outside the matching Lithium lens, and the wave length of the beta oscillation in the matching lens.

The set of Lithium lenses are connected to special matching lattice which is shown in Figure 7. Maximum β_x and β_y are 10.0 m and 12.9 m respectively in a quadrupole magnet close to the Lithium lens set. Figure 8 shows $\beta_x and \beta_y$ in four 4.50 m long straight cells on the other side of the straight section. No Lithium lens is installed here. Figure 9 shows β_x, β_y, and η in the whole race track muon cooling ring.

Figure 5. A schematic diagram of a race track muon cooling ring with a Lithium lens with the β at 1 cm

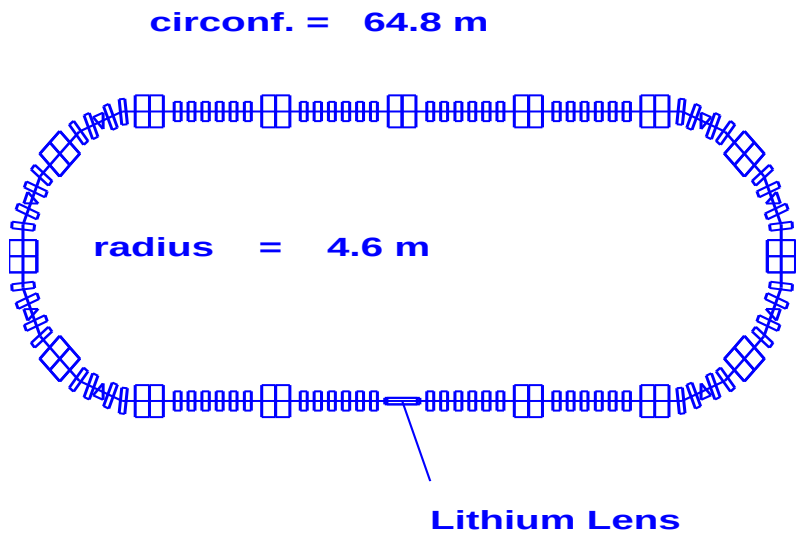


Figure 6. β_x, β_y as a function of z in two matching Lithium lenses and in a central Lithium lens where the minimum β is 1 cm.

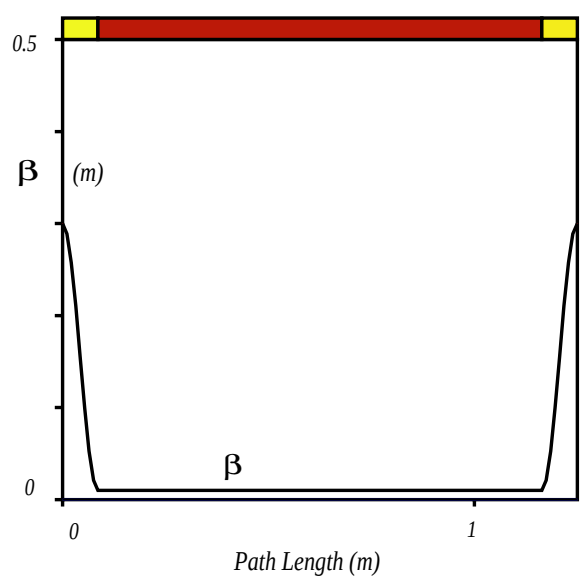


Figure 7. β_x, β_y as a function of z in a straight section with a Lithium lens at the β at 1 cm

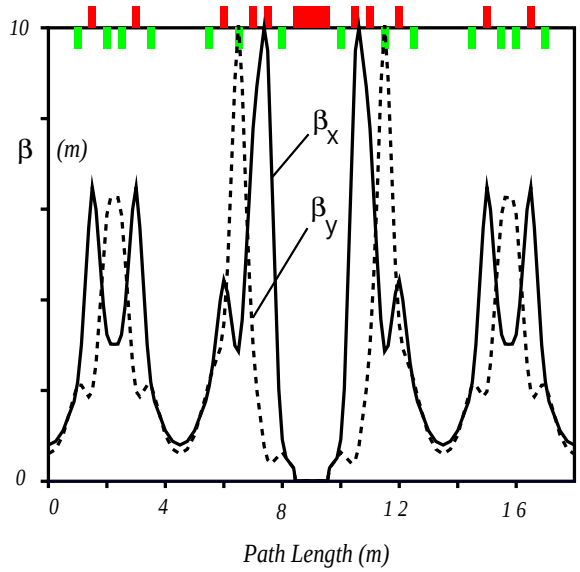
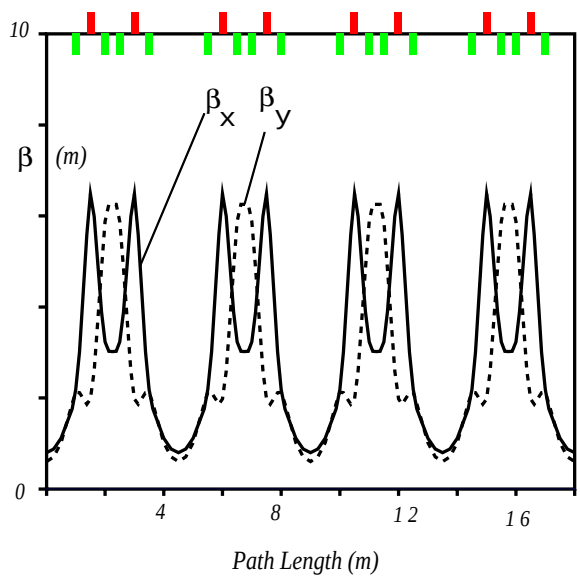


Figure 8. β_x, β_y as a function of z in a straight section without a Lithium lens



4. Conclusion

We designed an eight cell muon cooling ring by using SYNCH, a storage designing code, with zero-gradient dipole magnets with edge focusing. We demonstrated the 6 dimensional muon phase space cooling in the eight cell ring with a tracking simulation code, ICOOL, with liq. H_2 wedge absorbers, Lithium lenses with β at 30 cm, RF cavities to compensate the z component of the muon momentum.

We designed a race track muon cooling ring with 1 m long Lithium lens with β at 1 cm, with the SYNCH code. Study is in progress to obtain the 6 dimensional muon cooling in this cooling ring.

Figure 9. β_x, β_y and η as a function of z in a race track ring

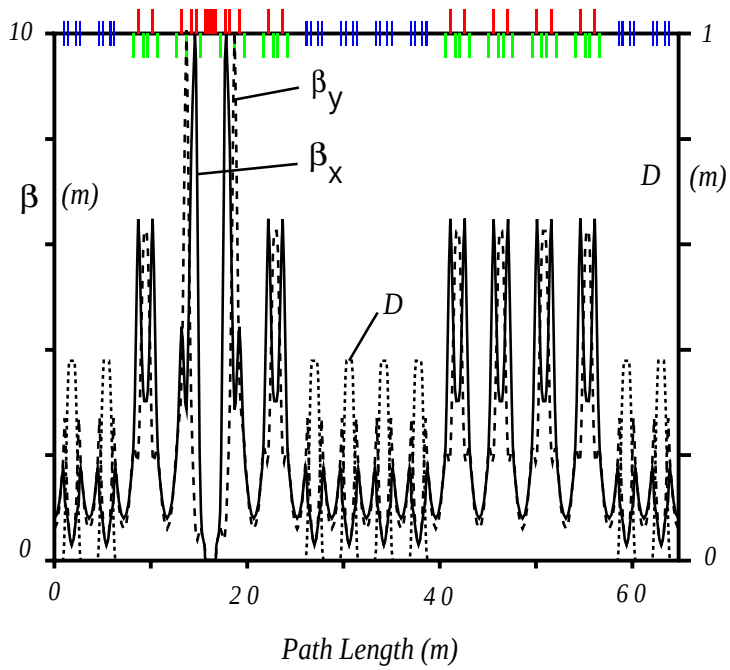


Table 1. Comparison of an electron damping ring and the Muon Cooling ring

	e **Damping Ring**		
phase space	x	y	z
Damping	x' synch.rad. +RF	y' synch.rad. +RF	synch.rad. $\Delta E \propto E^4$
Excitation	x-x' orbit change		quantum fluct. $\propto E^{3.5}$
Partition #	$(1-\mathcal{D})$	1	$2+\mathcal{D}$

	μ **Cooling Ring with Wedge Absobers**		
phase space	x	y	z
Damping	x' Ion.Cooling	y' Ion.Cooling	$\Delta E \propto$ E in Wedge
Excitation	x-x' orbit change mult.scat.	mult.scat.	$\frac{dE}{dx}$ struggling $\propto E^2$
Partition #	2-d	2	d

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Table 2. Parameters of an 8 cell muon cooling ring and a racetrack muon cooling ring

	8 Cell Ring	Racetrack Ring	
muon momentum	500	500	MeV/c
Circumference	28.8	64.8	m
straight section length		36.0	m
Structure of half cell	2 dipoles with edges	2 dipoles with edges	
Number of bending cells	8	8	
Number of straight cells	0	8	
bend cell length	3.6	3.6	m
straight cell length		4.5	m
max. β bending cell	1.8/2.7	1.8/2.7	m
min. β bending cell	0.30	0.30	m
max. η bending cell	0.38	0.38	m
max. β straight cell		10.0/12.9, 6.5/6.3	m
min. β straight cell		0.30	m
max. Quad grad		22.7	Tesla/m
length of Lithium lens	59	125	cm/turn
equilib. β in Lithium lens	0.30	0.01/0.05	m
dB_ϕ/dr of Lithium lens	0.034	167/5.5	Tesla/cm
length of liq.H_2 wedge absorber	0.40	0.40	m/turn
energy loss	57	115	MeV/turn
dipole length	0.2	0.2	m
dipole bend angles	44.2, -21.7	44.2, -21.7	degree
dipole edge angles	30/-3, -11/-11	30/-3, -11/-11	degree
dipole magnetic field	6.5, -3.2	6.5, -3.2	tesla
RF cavity length	1.0	1.0	m
number of RF cavities	8	15	
RF frequency	201	201	MHz
total drift space	21.4	44.8	m/turn
Cell tunes bend cell	0.72/0.70	0.72/0.70	
Cell tunes straight cell		0.39/0.47, 0.77/1.22	
Ring tunes	3.09/3.75	2.87/2.80	
chromaticities bend cell	-0.86/-0.69	-0.86/-0.69	
chromaticities straight cell		-2.75/-2.49, -9.29/0	
momentum compaction	-0.062	-0.062	
transition gamma	4.02	4.02	

References

[1] H. G. Kirk, D. B. Cline, Y. Fukui, A. A. Garren, *Progress Toward A Muon Ring Cooler*, SNOWMASS-2001-M101, Jun 2001. 5pp., Prepared for APS / DPF / DPB Summer Study on the Future of Particle Physics (Snowmass 2001), Snowmass, Colorado, 30 Jun - 21 Jul 2001.

[2] M. M. Alsharo'a, et. al., the NuMu Collaboration, *Status of Neutrino Factory and Muon Collider Reasearch and Developmemt and Future Plans*, FERMILAB-PUB-02-149-E, Jul 2002. 103pp, submitted to Phys. Rev. ST Accel. Beams

[3] A. A. Garren, A. S. Kenney, E. D. Courant, A. D. Russel, and M. J. Syphers. *SYNCH - A Compuetr System for Synchrotron Design and Orbit Analysis, User's Guide*, SSCL-MAN-0030, 1993

[4] R. Fernow, *A Simulation Code for Ionization Cooling of Muon Beams*, Part. Accel. Conf., Edts., A. Luccio and W. MacKay, Proc. 1999, p. 3020.