Cooling Channel with Lithium Lenses for a Muon Collider .

V.Balbekov
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

June 20, 2003

Abstract

Production of muon bunch for a $\mu^+\mu^-$collider is considered. Front-end simulation begun in Ref. [1] and [2] is continued in this paper by means of a linear ionization cooling channel with lithium lenses, solenoids, and 201 MHz RF cavities. Special lattice is designed to mitigate chromatic effects and provide maximal energy acceptance of the channel. Two versions are investigated which differ in number and length of cooling cells and Li lenses. It is shown that normalized transverse emittance 0.5-0.7 mm is achievable at the channel length 70-90 m. A possibility to improve the channel performances are discussed.

1 Introduction

It is common knowledge that, at given number of particles, luminosity of any collider is inversely proportional to the number of bunches. For traditional colliders, applicability of this statement can be restricted because of beam-beam effects which limit number of particles in the bunch. However, for the discussed $\mu^+\mu^-$collider these effects are probably neglected [3], and a collection of all muons in a single bunch is the best option, indeed.

A system for production of a single high intensity muon bunch was proposed and simulated in Refs. [1] and [2]. It includes proton driver, target station, phase rotation channel, bunch compressor, and RFOFO ring cooler. Produced muon bunch contains about 0.054 muons per incident proton and has normalized r.m.s. emittance about 2.5 mm in transverse direction and 3.2 mm in longitudinal one.

Because the transverse emittance is still large for a collider, a further cooling is required. The most prospective device for s final cooling is probably lithium lens that can create and support very small beta-function. However, design of matching sections between the lenses is very difficult and challenging problem. Unavoidable strong modulation of the beta-function causes chromatic effects what considerably restricts energy acceptance of the channel. Method proposed in Ref. [4] is used in this note to resolve this problem. Two ideas constitute its basis:

1. A usage of phase advance per cell as small as possible to provide maximal energy interval between linear betatron resonances.

2. A usage of special lattice to suppress 2 betatron resonances and to get an additional increase of the energy acceptance by factor about 3.

A shortcoming of this cooling channel is a big number of short Li lenses required. Therefore, a modification is considered in 4th section with less number of longer lenses that maybe is preferable from engineering point of view.

2 Short Period Cooler: Lattice

A periodical cooling channel composed of cells shown in Fig.1 is considered in this section. The cell includes Li lens, small radius-high field solenoid, and big radius-low field solenoid with 201.25 MHz cavities inside. The realistic magnetic field plotted in Fig.3 is used at the simulation; however, we apply also hard edge approximation for comparison and easy explanations.

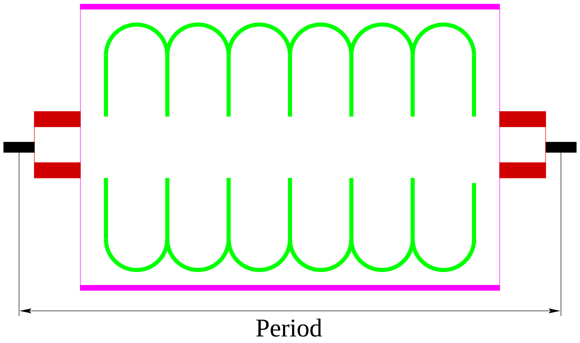

Figure 1: Schematic of the cell. Black (zone 1) - Li lens, red (zone 2) - high field solenoid, magenta (zone 3) - low field solenoid, green - RF cavities.

First of all note that transverse motion becomes unstable at several momenta because of linear parametric resonances. Spacing between corresponding stop-bands can be estimated by the formula:

$$\Delta p \simeq \pi p_0/\mu_0,$$

where μ_0 is betatron phase advance per cell at central momentum p_0. For example, at $p_0 =$ 164 MeV/c and $\mu_0 = 3\pi$ we have $\Delta p \simeq 55$ MeV/c corresponding to the energy interval $\Delta E \simeq 39$ MeV. Taking into account a width of the stop-bands and strong modulation of the beta-function near them, a permissible energy spread of cooled beam can be estimated as 30-35 MeV, what is considerably less of typical energy spread of muon beams.

There is a simple way to double one stable region. For this purpose, the following conditions must be satisfied at the momentum p_0:

1. Phase advance per cell μ_0 is a multiple of π.
2. Phase advance in high field solenoid $\phi_2 = \pi/2$.
3. Own beta-functions of separate regions in Fig.1 are: $\beta_2 = \sqrt{\beta_1\beta_3}$.

For above example, resonance $\mu_0 = 3\pi$ will be suppressed at these conditions, increasing available space between the neighboring 2π and 4π resonances up to 80-90 MeV.

An additional improvement of the system was proposed in Ref. [4]. It was shown that small change of phase advances in Li lens and low field solenoid allows to suppress the 2π resonance also. Following this, we take for the further consideration the lattice parameters listed in Table 1 (hard edge approximation). Field of the solenoids and gradient of Li lens are given at $E_0 = 195$ MeV.

Table 1: Parameters of the cell (hard edge approximation)

Region	Own beta-function (cm)	Phase advance/π	Length (cm)	Field, Gradient (T, T/cm)
Li lens	4.25	0.9505	12.69	3.027
High-field solenoid	$4.25*3.33 \simeq 14.15$	0.5	22.23	7.726
Low-field solenoid	$4.25*3.33^2 \simeq 47.13$	1.0495	155.39	2.320

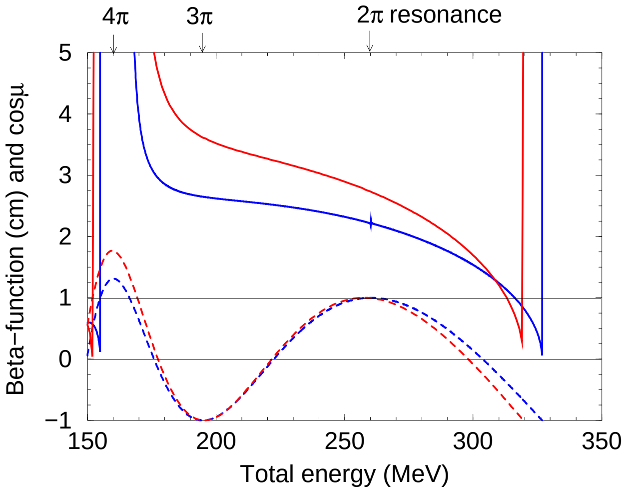

Figure 2: Beta-function at the center of Li lens (solid lines) and $\cos\mu$ (dashed) *vs* total energy. Blue – hard edge approximation, red – realistic field.

Dependence of beta-function in the center of Li lens on total energy is plotted in Fig.2 by solid blue line, and $\cos\mu$ is plotted also by dashed line. It is seen that 2π and 3π resonances are really suppressed, and region of stability stretches about from 170 to 320 MeV. Because of strong modulation of beta-function π and 4π-resonances, energy acceptance is less – about from 185 to 285 MeV; however, it is quite enough to accept a beam from the RFOFO ring cooler.

It is important to emphasize that similar result can be obtained also at the appropriately tuning realistic magnetic field. The following method of the tuning is used in this paper:

- Coils of inner-outer radii 6-14 cm (zone 2) and 69-70 (zone 3) are used as a basis. Their initial current density is 76.85 A/mm^2 and 92.31 A/mm^2 correspondingly. Such long coils would create fields listed in Table 1. However, their actual initial length is restricted and taken from the same table, so the real field is more complicated, of course (see Fig.3 as an example).

- On the first stage of the tuning, realistic field is applied only in zone 3 – inner part of the long solenoid, whereas the hard edge approximation is used in other parts. Length and current density of the long coil are tuned to suppress 2π and 3π resonances.

- Then a penetration of the solenoid field into Li lens is taken into account. It leads to some detuning of the cell, i.e. 2π and 3π resonances appear again. The length and gradient of Li lens are tuned to suppress them.

- Finally, the realistic field is applied everywhere, and length and current density of the short coil are tuned as before.

- As a result, we obtain a resonance free area between π and 4π resonances. However, it can be shifted relatively initial (hard-edge) position on the axis of energy. The last step is simple scaling of the field and gradient at the same geometry to move the region of stability in a desirable position. In our case, the scaling coefficient 0.9414 is applied.

Obtained parameters of the equipment are given in Table.2, axial magnetic field is plotted in Fig.3, and dependence of beta-function in the center of Li lens on total energy is shown by red lines in Fig.2. It is seen that 2π and 3π resonances are suppressed as before, though beta-function is

Table 2: Parameters of Li lens and solenoid coils

Element	Length (cm)	Inner radius (cm)	Outer radius (cm)	Current density (A/mm^2)
Li lens	13.45	-	3	355.0
High-field coil	22.45	6	14	84.62
Low-field coil	206.66	69	71	81.69

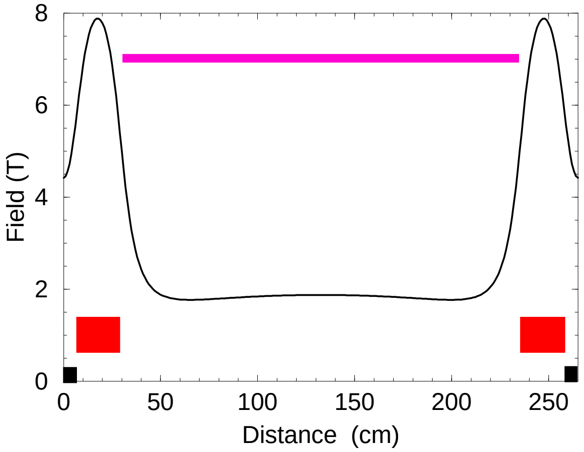

Figure 3: Axial magnetic field of the cell.

slightly more and region of stability is narrower then in hard edge approximation. Note also that the solenoids field penetrates into Li lens being about 5.8 T at its ends and 4.4 T at the center. It considerably increases focusing strength so that the lens gradient required now is only 2.23 T/cm instead of 3.03 T/cm in hard edge approximation.

An important issue is a modulation of beta-function along the cell, especially in the Li lens. It is plotted for realistic field in Fig.4, where total energy of muon is used as a parameter. According left picture, maximal beta-function at 220 MeV (center of used energy interval) is about 1 m what

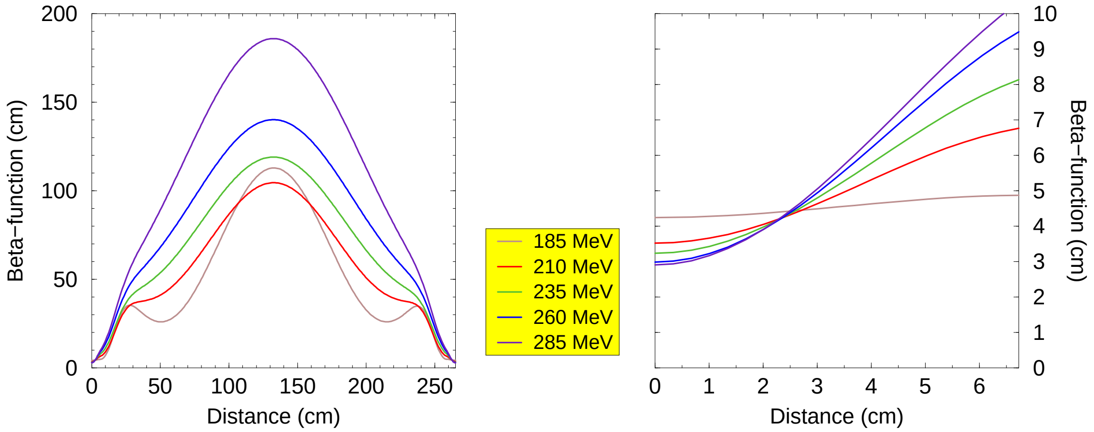

Figure 4: Beta-function *vs* distance at different total energy (realistic field). Left – cell, right – a half of Li lens.

is quite admissible. Beta-function inside of Li lens is plotted in right Fig.4. In average, it is not very different from initial ('hard edge') value 4.25 cm but has rather strong modulation. Therefore the lens radius is taken as large as 3 cm to accept muon beam without significant particle loss. Own surface field of the lens is only 6.7 T but total field is about 8.9 T on the ends because of contribution of the solenoids.

3 Short Period Cooler: Cooling Simulations

Both hard edge approximation and realistic field simulation are considered in this section. The same input is used in both cases obtained in Ref. [2] where cooling in the RFOFO ring was simulated. Central energy of this beam is about 220 MeV that is taken as the reference energy of the Li lens cooler. Note that this energy is shifted to left edge of the region of stability (see Fig.2). The point is that, according Fig.4, average beta-function in Li lens, as well as its modulation, fast increases when energy approaches π resonance (though beta-function at the center tends to 0).

A realistic matching section between RFOFO and Li lens coolers is not considered in this paper because its design strongly depends on extraction system. A simple scaling in transverse phase space is performed instead to accommodate it to the acceptance of the channel. A transformation in longitudinal space is not required because almost the same accelerating system is used both if RFOFO ring and Li lens cooler. Radio frequency and synchronous phase are 201.25 MHz and 30° in these simulations, whereas accelerating gradient is 15.5 MV/m at hard edge approximation and 11.9 MV/m at realistic magnetic field. The difference appears to even energy loss in the Li lens and energy gain in the cavities which in both cases fully occupy the long solenoid.

Evolution of the beam parameters at the cooling is shown in Fig.5 where left plot concerns hard edge approximation, and right one – realistic field. The curves are rather like thought the cooling at 'hard-edge' channel is slightly faster, and final emittance is less. There are several reasons for this including more beta-function and narrower stable zone (see Fig.2), and less accelerating gradient of the realistic channel. Probably, it means that some improvement of the system is still possible.

After 35-40 periods transverse cooling becomes slower then longitudinal heating which arises mostly because energy loss dE/dz is a descending function of energy. The beam parameters after the cooling are listed in columns A and B of Table 3 (columns C and D are commented in the next section). The cooling factors are defined as a ratio of final emittances to initial ones, 6D merit factor is the beam density in 6D phase space related to the initial density.

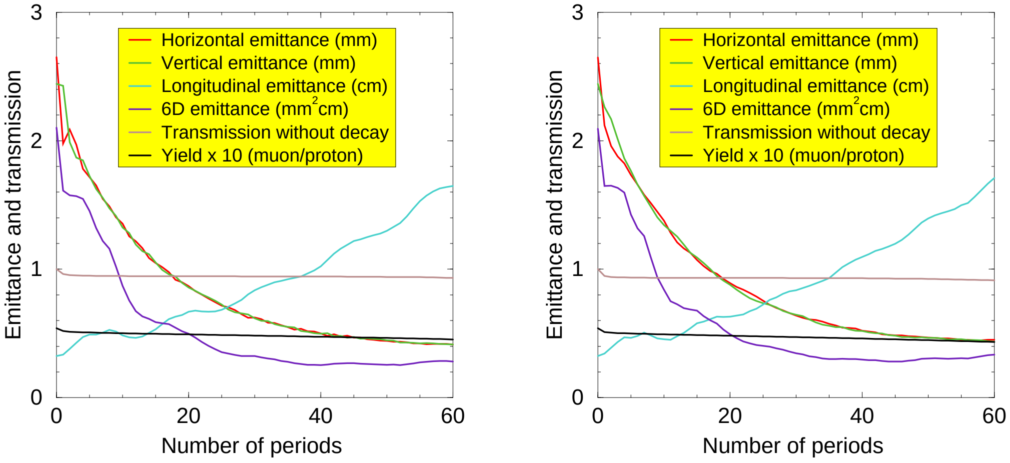

Figure 5: Evolution of the beam parameters at the cooling. Left – hard edge approximation, right – realistic field.

Table 3: Beam parameters after the cooling

Parameter	A: short period, hard edge apx.	B: short period, realistic field	C: long period, hard edge apx.	D: long period, realistic field
No. of periods	40	35	14	12
Length (m)	85.0	92.8	75.7	72.0
Trans. emittance (mm)	.498	.567	.645	.893
Trans. cooling factor	5.11	4.48	3.95	2.85
Long. emittance (mm)	10.2	9.31	8.86	8.29
Long. cooling factor	.317	.348	.366	.391
6D emittance (mm^3)	2.54	3.00	3.69	6.62
6D cooling factor	8.28	6.98	5.70	3.16
Transmission w/o decay	.943	.931	.937	.897
Yield (muon/proton)	.0474	.0466	.0476	.0457
6D merit factor	7.27	6.02	5.34	2.84

4 Longer Period Cooler

A drawback of considered cooler is a large amount (35-40) of very short (about 13 cm) Li lenses. It is a result of an aspiration to get as large energy acceptance as possible by using of cells with minimal phase advance. Obtained energy acceptance is at least 100 MeV what is probably even surplus because r.m.s energy spread of injected beam is only about 8.4 MeV. Therefore, it looks reasonably to consider a system with more phase advance and less energy acceptance, but with longer cell and correspondingly with less amount of the lenses. It is really possible because it turns out that all the above ideas, including a suppression of 2 resonances, retain their validity at phase advances 3π in Li lens and low field solenoid (see Fig.1).

The lattice parameters obtained by this way are listed in Table 4 (hard edge approximation) and Table 5 (realistic field). Axial field of the solenoid is plotted in Fig.6. Dependence of beta-function at the center of Li lens on total energy is shown in Fig.7. It is seen again that results of hard

Table 4: Parameters of the long cell (hard edge approximation)

Region	Own beta-function (cm)	Phase advance/π	Length (cm)	Field, Gradient (T, T/cm)
Li lens	4.25	2.91093	38.87	3.352
High-field solenoid	$4.25*3.33 \simeq 14.15$	0.5	22.23	8.555
Low-field solenoid	$4.25*3.33^2 \simeq 47.13$	3.08907	457.36	2.569

Table 5: Parameters of Li lens and solenoid coils (realistic field)

Element	Length (cm)	Inner radius (cm)	Outer radius (cm)	Current density (A/mm^2)
Li lens	38.48	-	3	512.9
High-field coil	21.81	6	14	99.84
Low-field coil	518.19	69	71	97.58

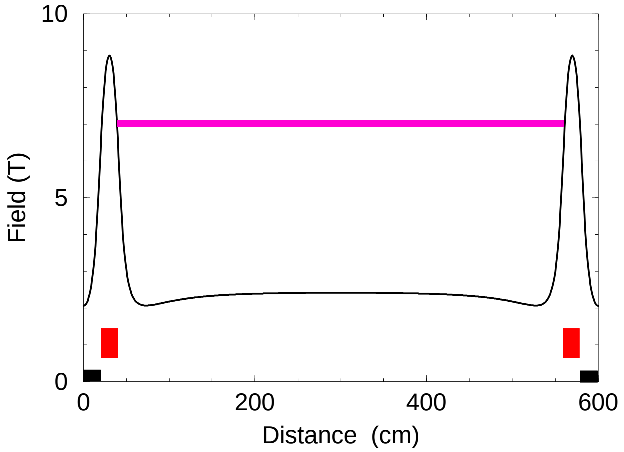

Figure 6: Axial magnetic field of the long cell.

edge approximation (blue lines) are very close to the realistic field calculations (red). As expected, region of stability is about 3 times narrower then for the short period channel. Dependence of beta-function of real cell on longitudinal coordinate is shown in Fig.8 for the cell (left) and for Li lens only (right). Its modulation is considerably more, especially near 5π resonance, what probably means that performances of this channel worse then of short periodic one.

Results of the cooling simulation are presented in Fig.9 and in Table 6, columns C and D. Note that accelerating gradient of realistic cell is 14.1 MV/m now, but all other parameters of accelerating system are the same as before. As was expected, number of cells and Li lenses is decreased by factor 3, and total length of the channels is less also. Transverse cooling of the beam is somewhat worse in comparison with previous, however longitudinal emittance growths slightly slower. Probably, it testifies that, because of betatron resonances, energy acceptance of the channel is not enough large and many particles reach the stop-bands and perish at relatively small amplitude of synchrotron

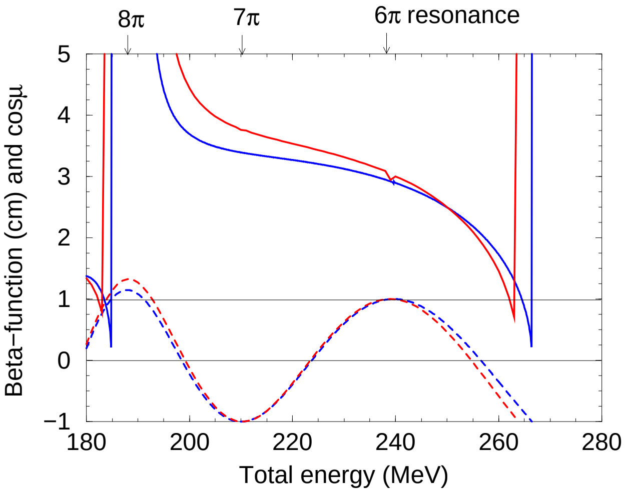

Figure 7: Beta-function at the center of Li lens (solid lines) and $\cos\mu$ (dashed) *vs* total energy. Blue – hard edge approximation, red – realistic field.

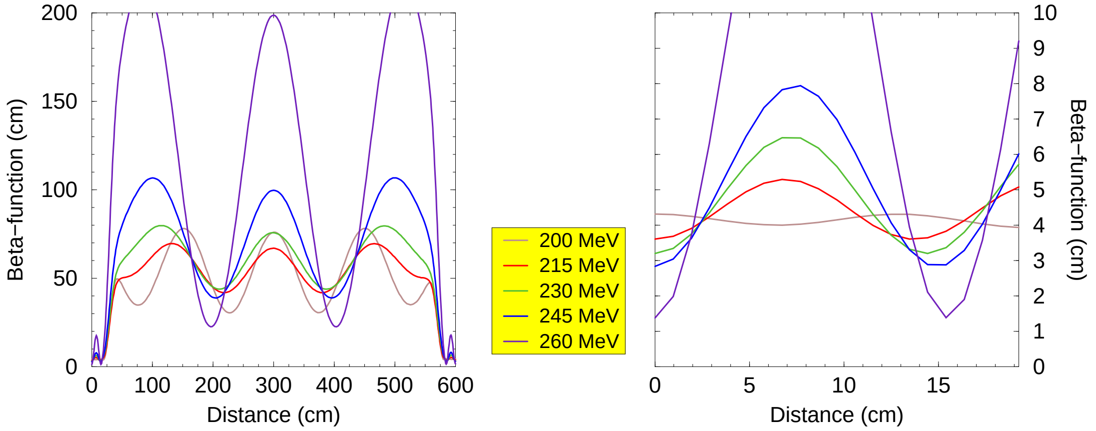

Figure 8: Beta-function *vs* distance at different total energy. Left – cell, right – a half of Li lens.

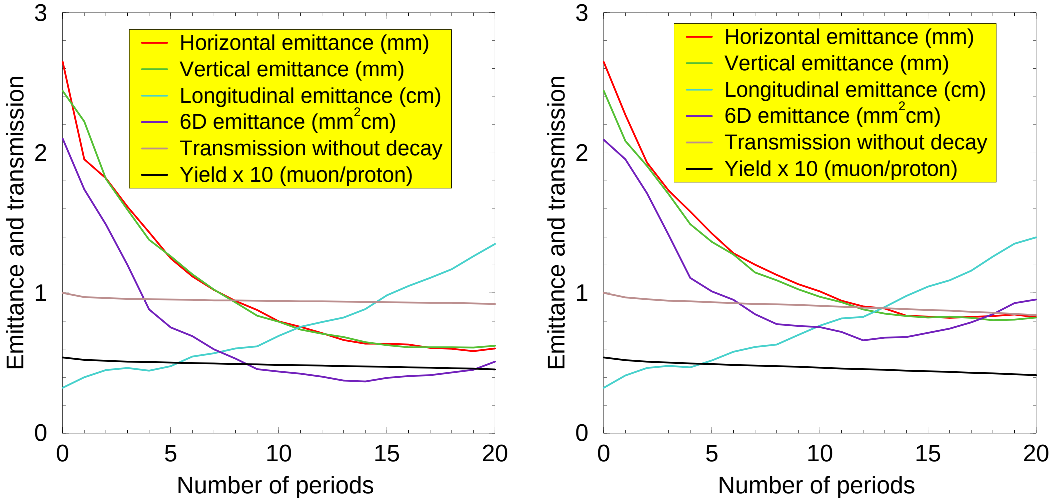

Figure 9: Evolution of the beam parameters at the cooling. Left – hard edge approximation, right – realistic field.

oscillations. Note also that the difference of realistic performances and hard edge approximation is more then in short period channel, what probably means that the tuning of the lattice is not enough perfect and can be improved.

5 Conclusion

It is shown that normalized transverse emittance about 0.5 mm is achievable by means of a short period Li lens cooling channel of length about 90 m. The cooling rate is restricted first of all by longitudinal heating, because ionization energy loss dE/dz is a descending function of energy in considered energy interval. Therefore an integration of emittance exchange in the channel looks as the most important task. Another problem is a small length of Li lenses which can be only ~ 3 or ~ 10 times more of beta-function to provide enough large energy acceptance. More advances methods of a suppression of linear betatron resonances are required to apply longer lenses. Serious limiting factor is also large field of a matching solenoid which reaches 10 T in considered design. A usage for the matching of ends of the Li lenses with less current density is impossible in this case, because at short central part the ends will give prevalent contribution to multiple Coulomb scattering. A decrease of solenoid field is possible, in principle, however it is not clear now how to combine it with a suppression of betatron resonances.

References

[1] V.I.Balbekov, N.V.Mokhov, *Bunch Compressor for a Muon Collider*, Fermilab MCNote-272 (2003).

[2] V.Balbekov, *Cooling of a Compressed Bunch in the RFOFO Ring*, Fermilab MCNote-276 (2003).

[3] C.M. Ankenbrandt et al. *Phys. Rev. ST. Accel. Beams*, **2**, 081001 (1999).

[4] V.Balbekov, *Achromatic cooling hcannel with Li lenses*, Fermilab MCNote-245 (2002).