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Modeling solenoids using coil, sheet and block conductors

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A new current block routine has been written that offers a simple and accurate method of describing solenoid fields in ICOOL. Blocks are convenient because (1) there is a one-to-one correspondence between these elements and physical magnet coils, (2) the current density can be entered directly in amps/mm^2, and (3) one can avoid infinities that arise from asking for field points at arbitrary locations in the conductor. We have found that the new routine generates fields in the beam aperture with relative field accuracy ~10^{-3} at the worst spots and considerably better over most of the aperture. The accuracy is also fairly good in the conductor region, except for the area in the immediate vicinity of the conductor edges.

1. Introduction

The most realistic solenoid modeling in ICOOL gets the magnetic field from sets of individual conductors. Three types of azimuthally symmetric conductor are available. The “coil” is a wire with zero thickness. The “sheet” has a finite length, but zero radial thickness. The “block” has both finite length and thickness. It is most convenient to describe a solenoid using blocks because (1) there is a one-to-one correspondence between these elements and physical magnet coils, (2) the current density can be entered directly in amps/mm^2, and (3) one does not have to worry about infinities from asking for a field point at certain locations in the conductor, as one does with the coil and sheet elements. However, most previous modeling has been done using sheets because the block routine currently used in ICOOL gave less accuracy. In this note we quantitatively compare the accuracy of modeling a solenoid using coils, sheets and blocks. We introduce a new algorithm for computing the field of a block conductor. This new model gives similar accuracy to the sheet model and thus should become the standard technique for accurate solenoid modeling in ICOOL.

2. Magnetic field equations

In this section we present the equations for finding the magnetic field from coil, sheet and block conductors. Simple equations are given for the field at the center of a solenoid in each case in order to check for proper normalization. We also describe the new algorithm for finding block fields.

2.1 Coil conductor

Circular current loops have been used as the default solenoidal conductor element in other programs, for example INTMAG [1]. Consider a circular coil with radius *a* carrying a current *I*. If we locate the origin of a cylindrical coordinate system at the center of the coil, the axial field B_o at the origin is

$$B_o = B_Z(0,0) = \frac{\mu_o I}{2a}$$

where $\mu_o = 4\pi\ 10^{-7}$ is the permeability constant. For (r,z) points off the symmetry axis the field can be written [2]

$$B_r = \frac{\mu_o I}{2\pi}\frac{z}{r\zeta}\left[-K(k) + \frac{a^2+r^2+z^2}{(a-r)^2+z^2}E(k)\right]$$

$$B_z = \frac{\mu_o I}{2\pi}\frac{1}{\zeta}\left[K(k) + \frac{a^2-r^2-z^2}{(a-r)^2+z^2}E(k)\right]$$

where *K(k)* and *E(k)* are complete elliptic integrals with argument

$$k = \sqrt{\frac{4ar}{(a+r)^2+z^2}}$$

and the parameter ζ is

$$\zeta = \sqrt{(a+r)^2+z^2}$$

2.2 Sheet conductor

Solenoids are described by current sheets in other programs, for example RAYTRACE [3]. Consider a circular sheet with radius *a* and length 2*L* carrying a current per unit length *I'*. If we locate the origin of a cylindrical coordinate system at the center of the sheet, the axial field at the origin is

$$B_o = \mu_o I' \frac{L}{\sqrt{L^2 + a^2}}$$

For (r,z) points off the symmetry axis the field can be written [3,4] in terms of elliptic integrals. Let us define the functions

$$b_z(r,z) = \frac{\mu_o I'}{\pi} \frac{za}{\zeta(a+r)} \left[K(k) + \frac{a-r}{2a} \left(\Pi(k,c) - K(k) \right) \right]$$
$$b_r(r,z) = \frac{\mu_o I'}{\pi} \frac{\zeta}{4r} \left[2\left(K(k) - E(k) \right) - k^2 K(k) \right]$$
$$c = -\frac{4ar}{(a+r)^2}$$

where $\Pi(k,c)$ is a complete elliptic integral of the third kind. The magnetic field from the sheet is given in terms of these functions by

$$B_Z(r,z) = -b_z(r,z-L) + b_z(r,z+L)$$
$$B_r(r,z) = b_r(r,z-L) - b_r(r,z+L)$$

2.3 Block conductor

If we locate the origin of a cylindrical coordinate system at the center of the block, the axial field at the origin is

$$B_o = \mu_o JL \ln\left[\frac{a_2 + \sqrt{L^2 + a_2^2}}{a_1 + \sqrt{L^2 + a_1^2}} \right]$$

where $2L$ is the length of the block, a_1 (a_2) is the inner (outer) radius and J is the current density.

An analytic solution for the field from a solenoidal block conductor does not appear to exist. The present block model [5] in ICOOL makes use of the vector potential for a planar current loop. In order to obtain an expression that can integrated axially and radially to make a block, the vector potential is elegantly approximated in a series that can be integrated term by term. The field components can then also be written as series of functions found from this approximate vector potential. We will refer to this algorithm as BLOCK-S (for series) in the following discussion.

The new block algorithm discussed here takes the current sheet solution as its starting point. The radial integration is done numerically using a 10-point Gaussian quadrature procedure. This is an "open" method that does not require evaluation of the function at the end points of the integration. When the field point is inside the conductor the radial integration is broken into two or more regions with a boundary at the radial position of the field point. This guarantees that the field point never coincides with the location of any of the sheets used in the integration. We will refer to this algorithm as BLOCK-I (for integration) in the following discussion.

3. Solenoid model

We now set up a test example so that we can test the accuracy of the various routines for calculating the field. Consider a solenoid 1 m long and 1 m in diameter. The conductor is located in the axial region ±50 cm and the radial region 50<r<60 cm. Fig. 1 shows the layout.

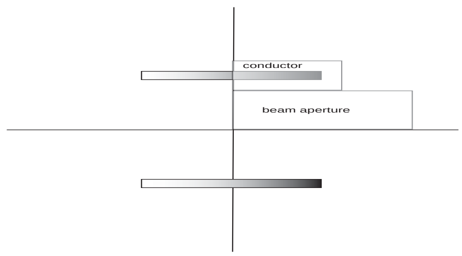

Figure 1. Layout of the solenoid test example.

We also identify two regions in Fig. 1 that will be used in the subsequent analysis. The “beam aperture” region is the region where the field quality needs to be good enough for particle tracking. We define this region to extend from the axis to 80% of the inner radius of the conductor. In practice some of the excluded region must anyway be taken up by insulation, support tubes, cryogenics and beam tubes. The second “conductor” region will be used to study the field behavior nearby and inside the conductor itself.

For the coil model the conductor region is approximated by 100 x 10 coils with 1 cm spacing. The nearest coil is 0.5 cm from the actual boundary of the conductor region. For the sheet model the conductor region is approximated by 10 sheets with 1 cm spacing. The sheets were 99 cm long, so again the nearest portion of a sheet is 0.5 cm from the actual boundary of the conductor region. The block models used blocks the same size as the conductor region. The current in each of the models was adjusted to give the same 1 T field at the center of the solenoid.

The field quality for the models was tested by examining the appropriate 2-dimensional Maxwell equations for div B

$$\partial_r B_r + \frac{B_r}{r} + \partial_z B_z = 0$$

and curl B

$$\partial_z B_r - \partial_r B_z = \mu_o J$$

3.1 Field quality in the beam aperture

For each of the models an (r,z) field grid was made with 1 cm spacing. Fig. 2 shows the field distribution on the grid. The field is mostly axial on this grid, except for the upper right corner nearest the end of the conductor.

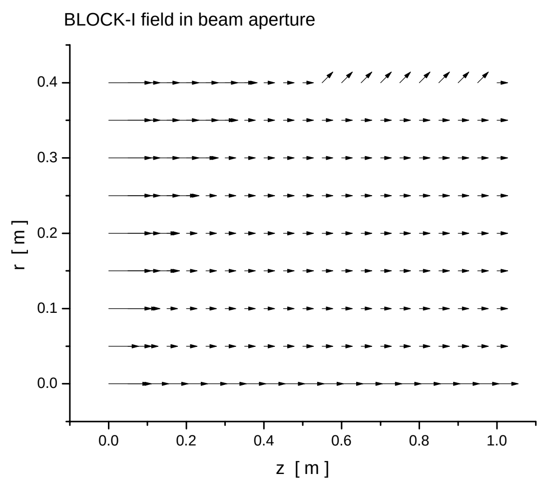

Figure 2. Magnetic field distribution in the beam aperture.

In the beam aperture we have $J = 0$ in the curl relation. Table 1 gives a summary of the field quality results.

Table 1: Field quality in beam aperture region

model	div B [G/m]	r1 [cm]	z1 [cm]	curl B [G/m]	r2 [cm]	z2 [cm]	time [s]
coil	53	39	50	36	39	56	23.33
sheet	52	39	49	36	39	55	0.94
block-S	3471	39	50	4313	39	40	0.15
block-I	53	39	50	36	39	56	1.88

The second column lists the maximum error in div *B* on the grid. The maximum error occured at the location (r1,z1). Likewise the fifth column gives the maximum error in curl *B* which occurred at the location (r2,z2). The last column gives the execution time required to compute field at the 41 x 101 grid points. All the models give similar accuracy, except for the BLOCK-S model, which is substantially worse. We see that the maximum error occurs at the maximum radius used in the Maxwell equation grid, closest to the conductor. The peak error in div *B* is directly below the end of the conductor, while the peak error in curl *B* is more distributed longitudinally. This can also be seen in Figs. 3 and 4, which show contour plots of the error distributions.

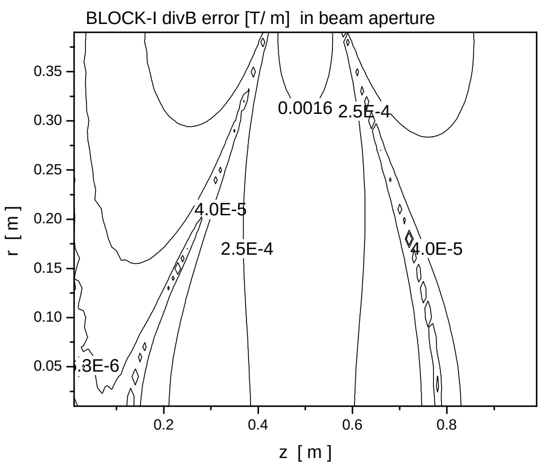

Figure 3. Error contours for div *B* in the beam aperture.

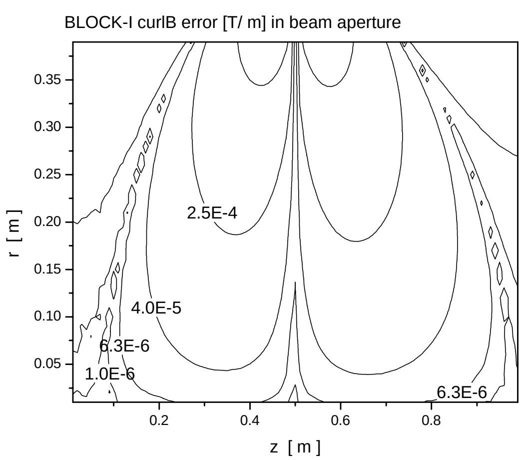

Figure 4. Error contours for curl *B* in the beam aperture.

The errors in div *B* and curl *B* are essentially gradient errors δg in the field. Using a maximum error δg ~50 G/m, the maximum relative field error on the grid is then

$$\frac{\Delta B}{B} \approx \delta g \frac{r_{\max}}{B_o} \approx 2 \times 10^{-3}$$

The accuracy is much better near the axis, where δg ~1 G/m near the end of the conductor, and better still near the center of the solenoid, where δg ~0.01 G/m .

3.2 Field quality in the conductor region

For each of the models another (r,z) field grid was made for the conductor region with 1 cm spacing. Fig. 5 shows the field distribution on the grid. The field reversal takes place in the upper part of the conductor.

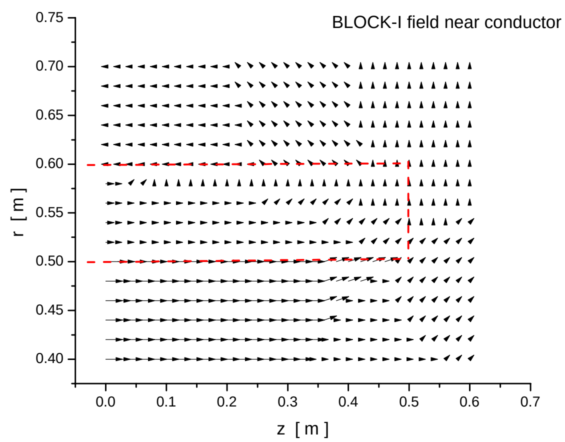

Figure 5. Magnetic field distribution in the conductor region.

Table 2 gives a summary of the field quality results.

Table 2: Field quality in the conductor region

model	div B [T/m]	r1 [cm]	z1 [cm]
coil	1.85	60	49
sheet	1.11	60	49
block-S	3.87	51	50
block-I	1.15	51	50

We only list the div *B* errors here because of problems calculating curl *B* on the conductor edge, which we discuss below. Note that the units on the div *B* error are in T/m here. We see that the maximum error occurs at either the upper or lower corner at the end of the conductor. None of the models can give an accurate value for the field at these locations. This can also be seen in Figs. 6 and 7, which show contour plots of the error distributions.

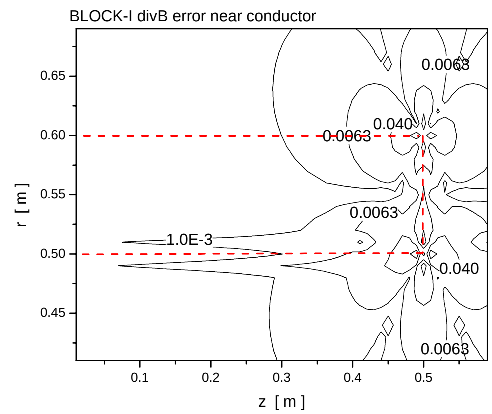

Figure 6. Error contours for div *B* in [T/m] in the conductor region.

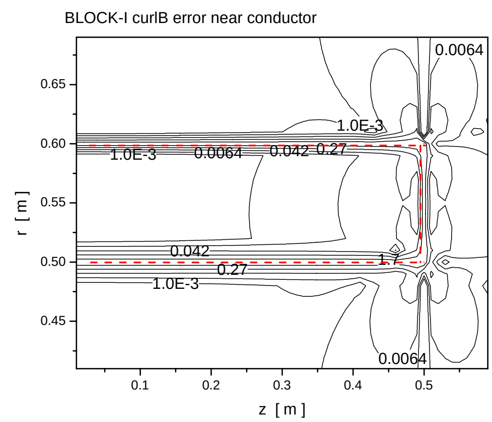

Figure 7. Error contours for curl *B* in [T/m] in the conductor region.

Note in Fig. 7 that the error in curl *B* is small both outside and inside the conductor, but that it spikes along the immediate edge of the conductor. It is likely that this comes from not properly accounting for the magnitude of *J* when calculating the curl *B* relation in this case. Further improvements in the algorithm would be necessary if it is important to accurately determine *B* for this case.

4. Conclusions

The new BLOCK-I routine offers a simple and accurate method of describing solenoid fields in ICOOL. The method has essentially taken the past practice of summing sets of radial sheets and incorporated it into an easier-to-use package. We have found that these routines generate fields in the beam aperture with relative field accuracy ~10^{-3} at the worst spots and considerably better over most of the aperture. The accuracy is also fairly good in the conductor region, except for the area in the immediate vicinity of the conductor edges.

The BLOCK-I routine gives much better results than the BLOCK-S routine presently used in ICOOL. This is probably because only 9 terms are used in the series expansion and the convergence is fairly slow.

Acknowledgements

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References

[1] R. Becker, INTMAG: a program for the calculation of magnetic fields by integration, Nuc. Instr. Meth. B42:303-306, 1989.

[2] W.R. Smythe, Static and Dynamic Electricity, McGraw-Hill, New York, 1950, p. 270-271. You must divide Smythe's expressions by 4π to obtain the S.I. units that we use here.

[3] S. Kowalski & H.A. Enge, RAYTRACE Users Manual, 1986, p. 23-4. The normalization factor BF in the RAYTRACE equations correspond to the factor $\mu_o I'$ here.

[4] M.W. Garrett, Calculation of fields, forces, and mutual inductances of current systems by elliptic integrals, J. Appl. Phys. 34:2567-2573, 1963.

[5] J.C. Gallardo, Note on vector potential and magnetic field for a solenoid, BNL Muon Tech. Note MU-040, 2000.