MUC-NOTE-ACCELERATION-297

ICOOL Acceptance Studies of PRISM

R.B.Palmer, J.S.Berg, R.Fernow

9/28/04

ABSTRACT

Simulations of the scaling FFAG PRISM[1] lattice, including end effects and overlapping fields, have been performed using ICOOL[2]. It is found that the transverse acceptance is somewhat greater for a field assumption that does not contain too rapid field variation between focus and de-focus magnets. The acceptance is also increased if the field index scaling magnet designs are replaced with linear non-scaling magnets with approximately the same axial fields and gradients.

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1 introduction

PRISM[?] (see figure 1) is a 10 cell scaling Fixed Fieald Alternating Gradient (FFAG) storage ring to be used at JPARC to phase rotate muons (reducing their energy spread while inccreasing their pulse duration). The source, from a target, will have a very large transverse momentum phase space and huge energy spread. The requirements on the ring are, besides the inevitably large momentum compaction, are for maximum energy and transverse momentum acceptance. The physical aperture is large: approximately 34 cm vertically and 80 cm horizontally. This study is aimed at determining and studying the dynamic acceptances.

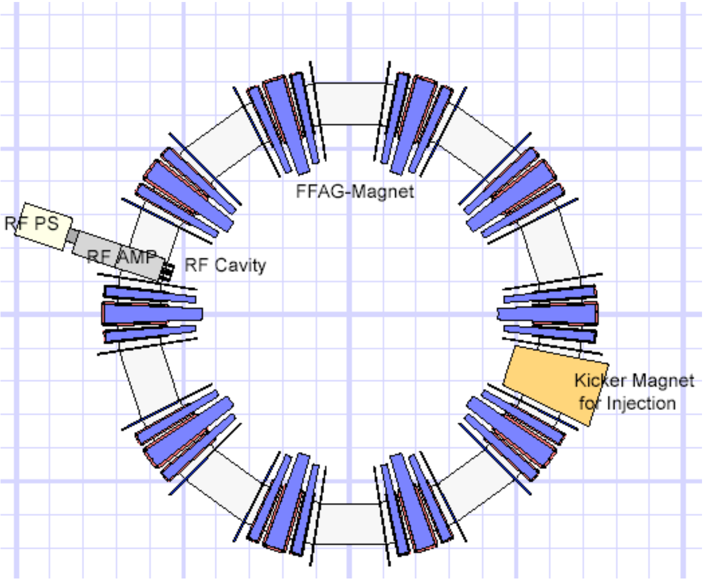

Figure 1: PRISM Layout

While ICOOL allows hard edged bending magnets with specified radial field index k (as used in scaling FFAG's), it does not now specifically allow such indices in systems with soft field ends and inter penetrating neighboring magnet fields as are significant in PRISM and other low momentum FFAGs. This study solves the problem by approximating the field index fields by a sum of five multipoles, and uses ICOOL's facility to allow fields so specified to have variations in length defined by Fourier sums of terms specifying periodic variations around a circular reference orbit.

The radial field variations are assumed to have an ideal scaling FFAG dependence with a given value of the field index k. Such fields are then approximated by a sum of multipoles up to the 5th order (dipole, sextupole, octupole decapole and dodecapole). The strength of each multipole is given by the Taylor series:

$$B(x) \;=\; B_o\left(1+\frac{x}{R}\right)^k \;\approx\; B_o\left\{1 \;+\; k\left(\frac{x}{R}\right) \;+\; \frac{k(k-1)}{2!}\left(\frac{x}{R}\right)^2 \;+\; \text{etc}\right\}$$

$$=\; B_o \;+\; \sum_{n=1}^{n=5} M_n \;\left(\frac{r}{R}\right)^n$$

where the M_n's are multipoles given by

$$M_n = B_o\,\frac{\prod_{i=0}^{i=n-1}(k-i)}{n!}$$

Given specified magnets' nominal lengths and nominal fields on the nominal radius, the simulated magnets are assumed to have hyperbolic tangent fall offs at each end i with a slope parameters Γ_i. i.e. at distances z_i from end i: $dz_i = z_i/\Gamma$ and nominal field B_o:

$$B \;=\; \frac{B_o}{2}\;\left\{\frac{(e^{dz_1}-e^{-dz_1})}{(e^{dz_1}+e^{-dz_1})} \;-\; \frac{(e^{dz_2}-e^{-dz_2})}{(e^{dz_2}+e^{-dz_2})}\right\}$$

Fields, so calculated, are Fourier transformed into 50 Fourier components with periods of 1 cell, 1/2 cell, 1/4 cell, etc. Each Fourier component could have differing multipole components, but in this study, the multipole components for all Fourier components are taken to be identical. This assumption corresponds to field profiles that are independent of radial position, as is approxiamtely the PRISM case as shown in Arimoto's calculated fields[3]. The multipoles for all Fourier components and their differentials up to a specified order, are used by ICOOL to calculate the fields at points off the reference circle. Fourier components, as apposed to tables, are prefered because they guarantee smoothness from cell to cell and thus avoid large improper higher differential components that could result in errors in field determinations further from the circle.

The following plot shows the observed vertical dynamic aperture vs. the calculation's order, for two of the cases that will be discussed in this paper. The plot suggests that errors from use of third order calculkations are of the order of 5% in amplitude (10% in acceptance), while the use of 5th order reduces such errors to the order of 1% in amplitude (2% in acceptance). The qualitative results shown in figures 4 to 10 used 3rd order calculations, but the final acceptances shown in figure 13 and table 5 were done to 5th order.

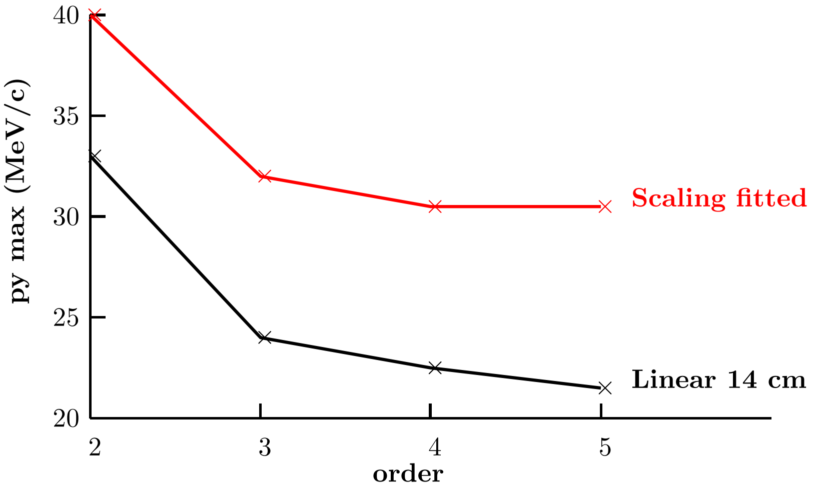

Figure 2: Acceptance vs. the order of ICOOL's off axis field calculations

In this study, particles were introduced at a plane in the center of a long straight, and their positions and angles were examined after each of 100 identical cells. The number 100 corresponds to 10 turns for lattices with 10 cells, as in Prism.

Such positions and angles, plotted in phase diagrams, for particles injected at small angles, allow the derivation of closed orbits and tunes. Injection at larger angles allow studies of acceptance, x-y coupling and other non-linear effects.

We have looked at two different assumptions for the magnets nominal lengths and fields. In both cases parameters were adjusted to achieve specific tunes in x (in the machines radial direction) and y (vertical). In the first case, we used the magnet angles as given in Arimoto's Nufac04 talk[3] and picked the shape parameters Γ=15 cm based on a typical value for a stand alone dipole with a gap of ± 17 cm. In the second case, we fit the shape and magnet parameters to approximate the field vs. angle plot shown by Arimoto. The first case generated fields that are clearly different from those shown by Arimoto, and might not be worthy of mention but for its observed better acceptance, and the lesson that this may teach us.

Tracking one of 7 single particles through 100 cells takes approximately 6 seconds on a 2.4 GHz Pentium laptop. Much of the time is taken by the calculations of the off axis fields.

2 Case 1: With Fixed End Shapes

Parameters were taken from Sato's talk at Nufac04[1], as shown in table 1, and spaces between magnets taken from the figure 2 of Arimoto's talk[3].

The shape parameter Γ (see above in the introduction) was taken at 15 cm for all ends - a value consistent with the half gap dimension of 17 cm in a stand alone dipole.

When run with the field magnitudes given in table 1, ICOOL gave significantly higher tunes than those quoted by Sato. The strengths of the nominal fields were then adjusted to obtain approximately Sato's tunes. The results were found to be insensitive to differences in the tunes at the level of the differences from Sato's exact values. The resulting parameters are shown in table 2.

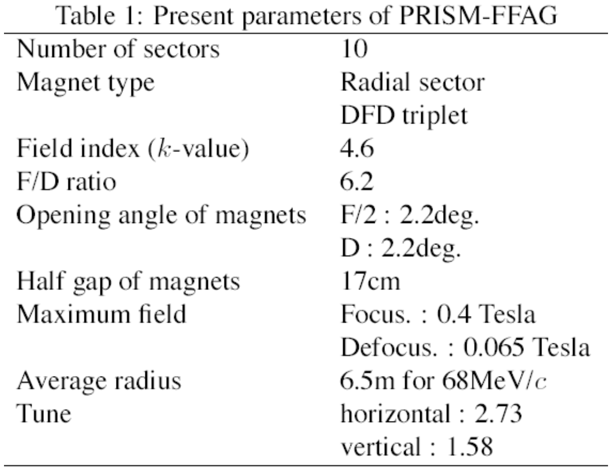

Table 1: Present parameters of PRISM-FFAG

Number of sectors	10
Magnet type	Radial sector DFD triplet
Field index (k-value)	4.6
F/D ratio	6.2
Opening angle of magnets	F/2 : 2.2deg. D : 2.2deg.
Half gap of magnets	17cm
Maximum field	Focus. : 0.4 Tesla Defocus. : 0.065 Tesla
Average radius	6.5m for 68MeV/c
Tune	horizontal : 2.73 vertical : 1.58

Table 1: Parameters from Sato

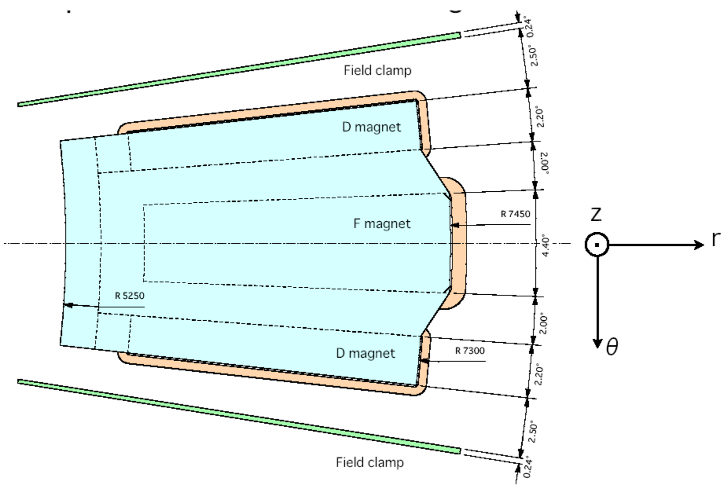

Figure 3: Magnet Dimensions from Arimoto

exponent k		4.6
cell	m	4.085
number of cells		10
nominal radius	m	6.5
nominal momentum	MeV	68
Shape parameter Γ	m	0.15

	Len m	B T	Grad T/m	Sext T/m^2	Oct T/m^3	Dec T/m^4	Dodec T/m^5
gap	1.316						
1	.25	-.0553995	-.2548377	-.4587078	-.3975467	-.1590187	-1.908224E-02
gap	.227						
2	.499	.34092	1.568232	2.822818	2.446442	.9785766	.1174292
gap	.227						
3	.25	-.0553995	-.2548377	-.4587078	-.3975467	-.1590187	-1.908224E-02
gap	1.316						

Tables 2: Parameters used in case 1

2.1 Small amplitude results

Plots from a run with small amplitude particles follow. The x and y positions plotted are at the center of the long drift. The x is with respect to a circle with a 6.5 m radius. As expected, the nominal momentum closed orbit at that location is negative because the trajectory there is straight. Fig.4a shows the fields on the nominal radius vs length along the circumference. Fig4b shows the closed orbit vs momentum.

The Fig. 5a and 5b show the x and y tunes which are, as expected, essentially independent of momentum.

Table 3 gives some properties obtained.

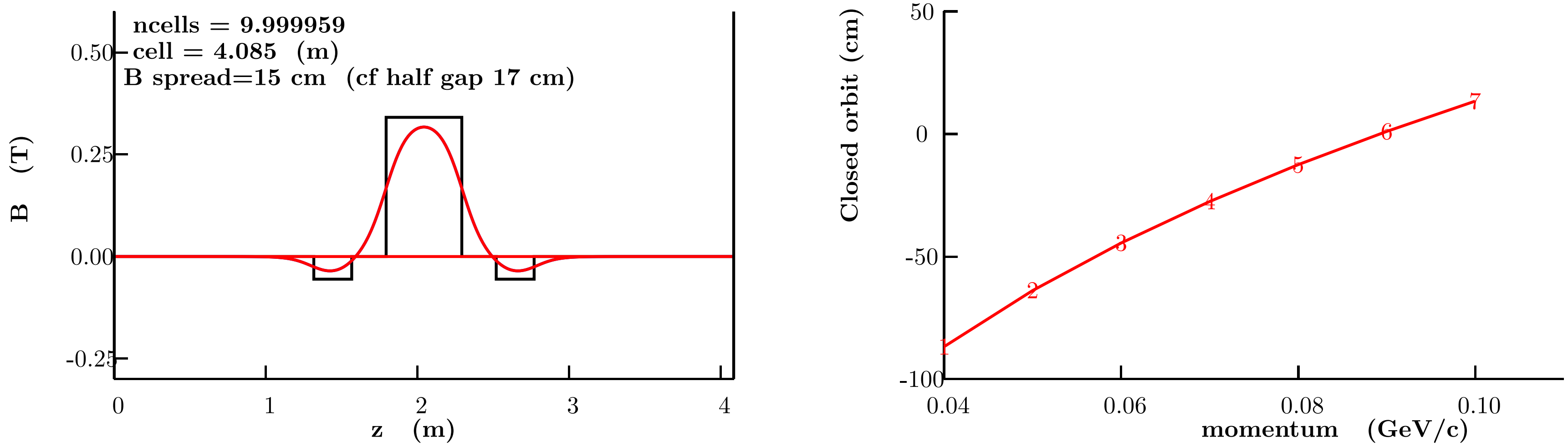

Figure 4: a) Vertical field vs azimuthal position b) Closed orbit position at center of long gap

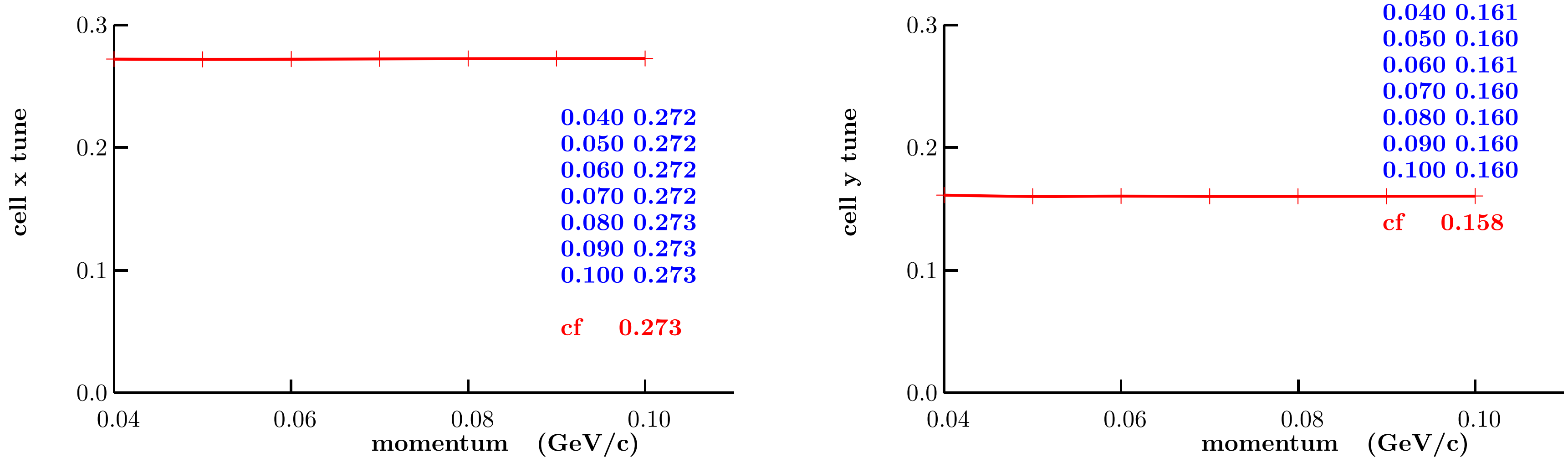

Figure 5: Tunes vs. momentum in the a) x and b) y directions

beta x	m	1.75
beta y	m	3.8
momentum range for ± 40 cm		× 2.11
x tune at 68 MeV/c		.272 x 10 = 2.72
y tune at 68 MeV/c		0.160 x 10 = 1.60

Table 3: Resulting small amplitude parameters for case 1

2.2 Dynamic aperture

We now fix the initial momenta, but vary the initial amplitudes in x, y, or both, to determine the dynamic acceptance of the lattice. First we look at x motion with truly zero initial y amplitude (no assymetry whatever in up/down). The phase plot and tune vs amplitude is shown in figure 6a and b:

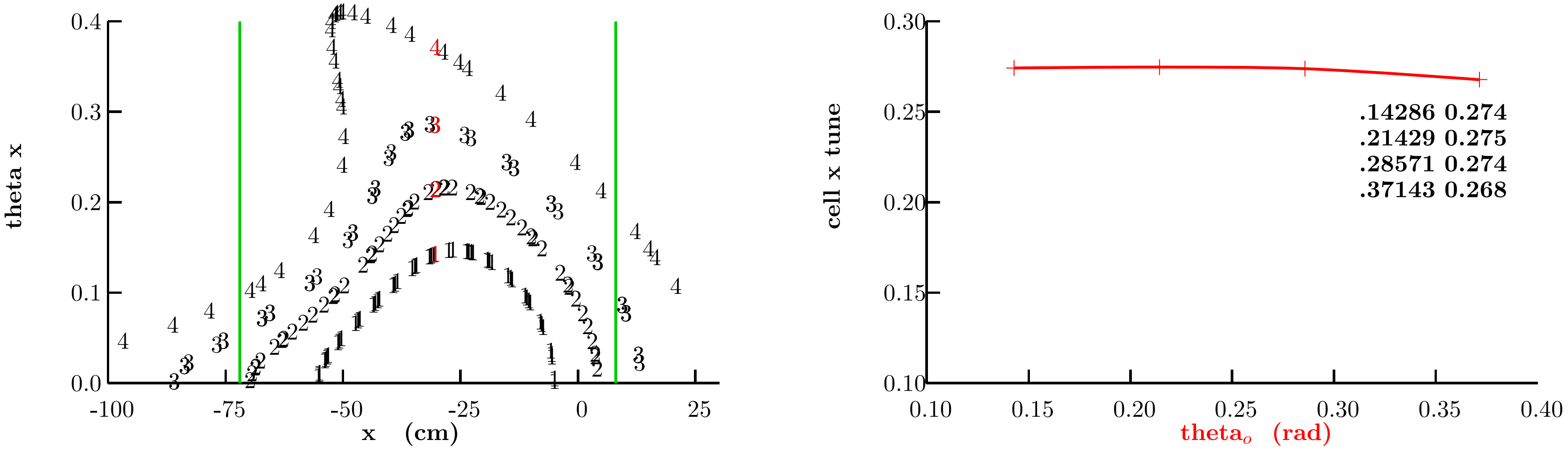

Figure 6: a) x' x phase plot and b) tune vs. initial x amplitude for true zero initial vertical amplitude

The purely dynamic aperture is huge (165 pi mm) but at this aperture the tracks pass outside the physical aperture (shown in green) at plus/minus 40 cm. The tune is seen to be surprisingly independent of this huge amplitude, changing only about 2 % even for an initial amplitude that reaches 0.37 radians (21 degrees).

Unfortunately this huge aperture is non-physical. The motion in y is unstable. The Fig. 7a and b show the x and y phase plots with a very small initial y amplitude; thus breaking the up/down symmetry. We see that the x dynamic aperture now lies within the physical aperture and has a much lower value . The y motion, though initally small, does not remain so. Its phase plot (Fig 7b) appears chaotic becuse it is coupled to the x motion. In Fig. 8 (x and y vs. z) we see that the x motion is largely oscillatory, but has a small amplitude beat. The y motion, with large initial x amplitude, shows a strong beat, with amplitude starting small but growing rapidly. There is thus quite strong coupling between x and y that leads to this serious loss of acceptance.

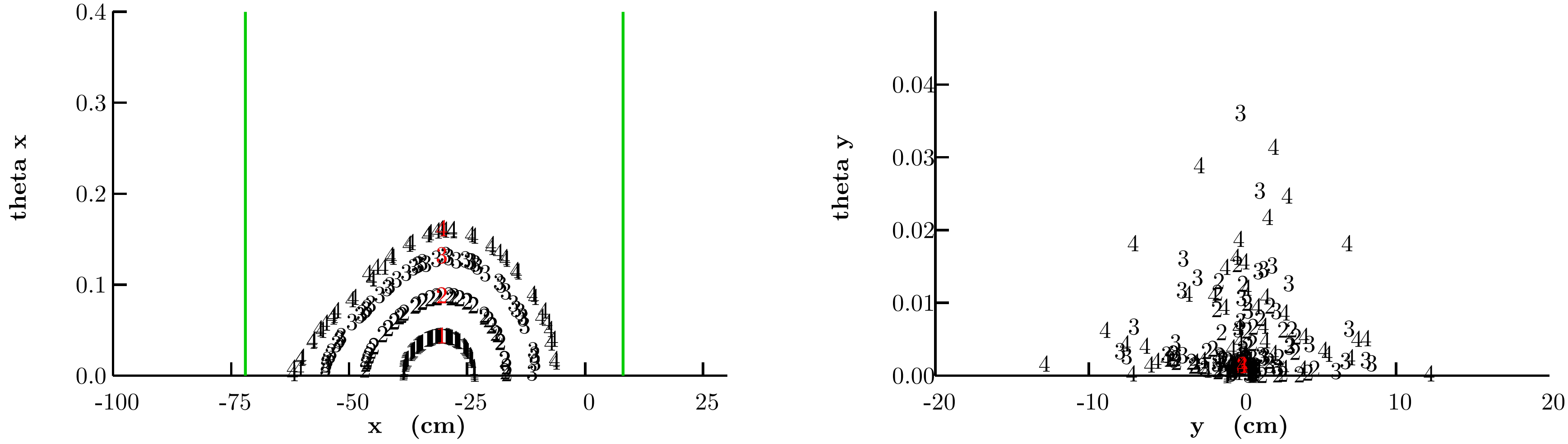

Figure 7: a) x' x phase plot and b) y' y phase plot for very small initial vertical amplitude

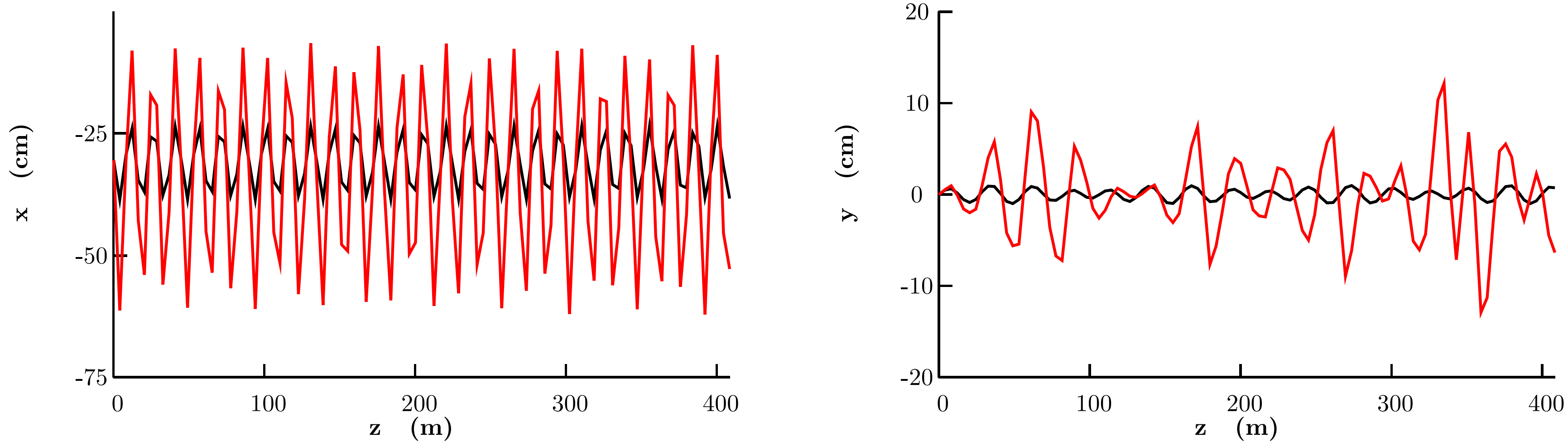

Figure 8: a) x vs z and b) y vs. z for very small initial vertical amplitude, and two initial x amplitudes

When we try to set the initial x amplitude to zero, we cannot remove the intrinsic x assymetries that come from the curvature and field gradients. Thus, as we increase the initial y amplitude, any unstable coupling to x would always be apparent. However, the y phase plots (Fig. 9a) do not in fact show any such strong effects. But the y dynamic aperture is far less than in x : the y acceptance being only 3.8 pi mm. Fig. 9b shows that there is a large amplitude dependent change in the y tune: 27 % for amplitudes of only 0.04 radians.

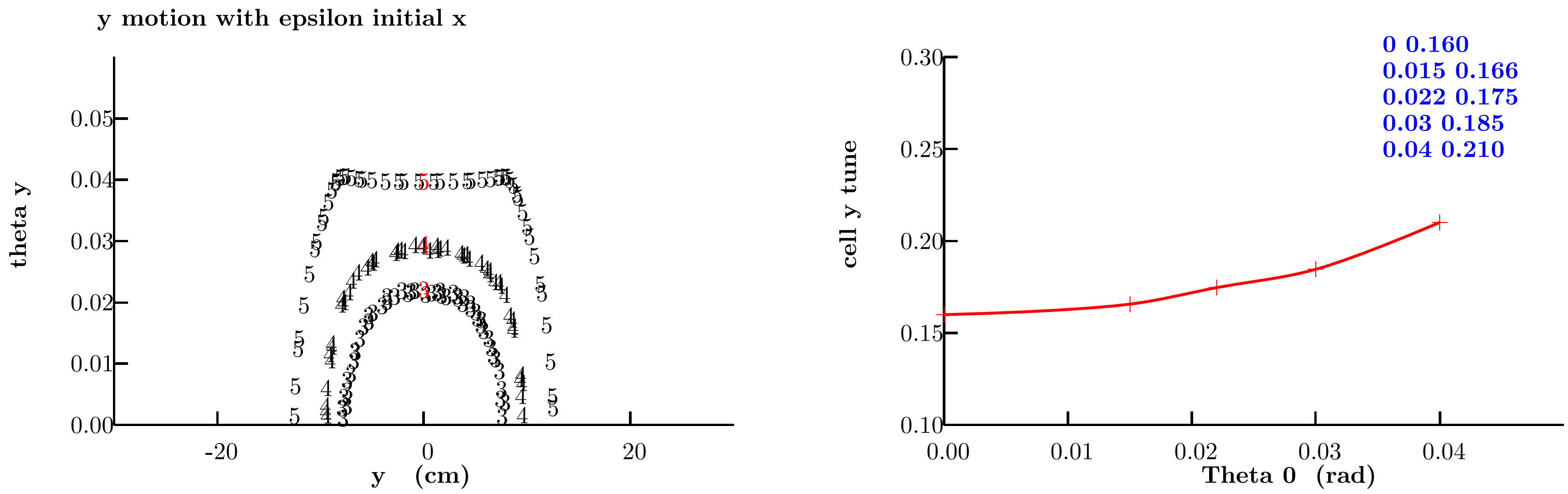

Figure 9: a) y’ y phase plot and b) y tune vs. initial y amplitude for small horizontal amplitude

We have also extended the study to cases with significant amplitudes in both x and y. Fig. 10 shows x and y angular acceptances for a number of cases. Again it is seen that the very large acceptance for the true zero y amplitude (the cross way out on the right) has nothing to do with the true acceptance. The x and y amplitudes of a fitted ellipse (as shown dotted in fig. 10) is probably the best definition of useful acceptances.

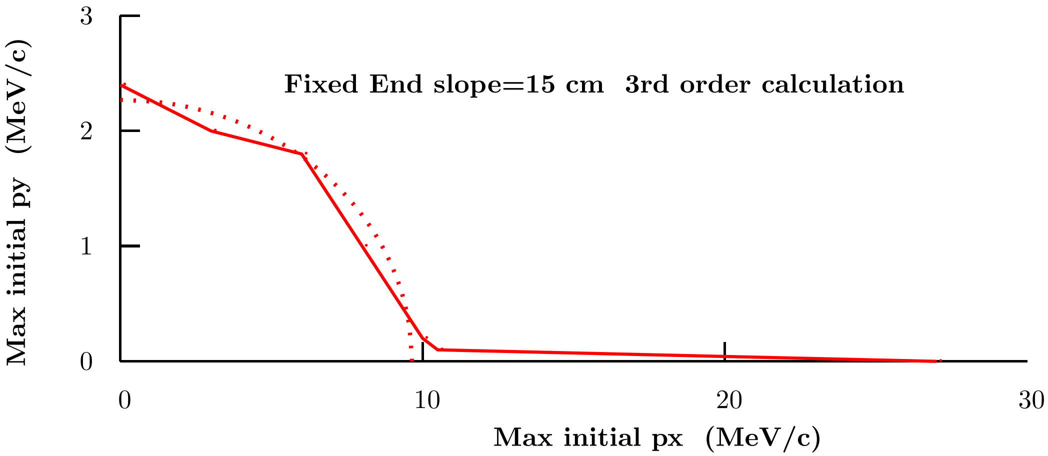

Figure 10: Acceptances with finite x and y initial amplitudes, and (dotted) a fitted ellipse

Note that these results were obtained with only third order field calculations, but they are qualitatively the same as those done to 5th order. Acceptances for 5th order calculations are shown in Fig. 13 and table 5, together with calculations for the other cases considered.

3 Case 2: With Ends Fitted to Arimoto's Plot

The fields used in case 1 differ somewhat from those shown (see fig. 11) by Arimoto[3]. Arimoto's field drop at the end of the defocus magnets does correspond approximately that used in case 1, but the drop between the focus and defocus magnets is more rapid.
So for case 2, we have fit the magnet lengths and shape parameters to Arimoto's field dependence on z. Fig. 12a shows Arimoto's fields (red) and our approximation to them (blue). In Fig. 12b the fields for the two cases are compared.

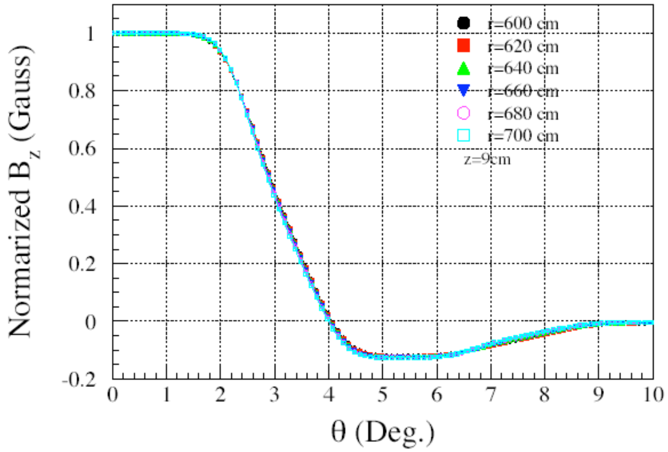

Figure 11: By vs machine azimuth for differing vertical positions as shown by Arimoto

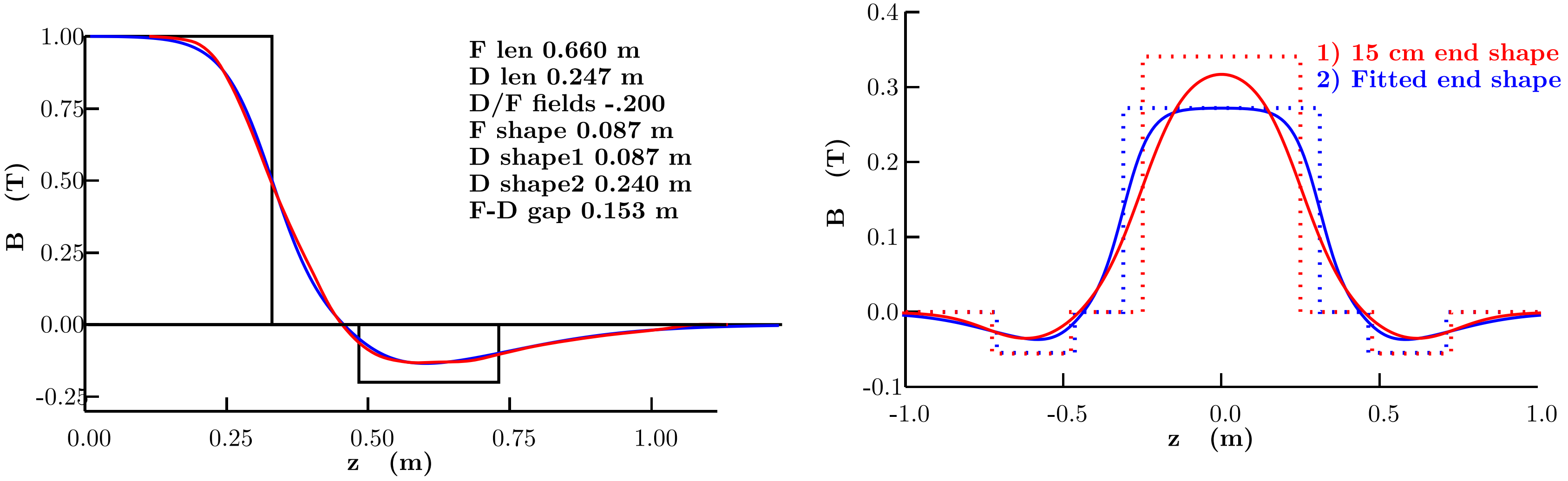

Figure 12: By vs. z plots: a) Arimoto's (blue) & hyperbolic tangent fit (red), b) Case 1 (red) & Case 2 (blue)

Another difference in the proceedure used in case 2 was in the adjustment of parameters to obtain the required tunes. Instead of separately adjusting the nominal fields of the two magnet types (which would spoil the fit) the overall field strength and the length of the focus magnet (which does not affect that fit) were adjusted. Greater care was also taken to obtain the exact tunes quoted by Sato. Tables 4 give the parameters used.

exponent k		4.6
cell	m	4.085
number of cells	m	0
nominal radius	m	6.5
nominal momentum	MeV	68
Shape parameter Γ	m	0.15

	Len m	B T	Grad T/m	Sext T/m^2	Oct T/m^3	Dec T/m^4	Dodec T/m^5
gap	1.3313						
1	.24679	-5.443262E-02	-.2503901	-.4507021	-.3906084	-.1562434	-.0187492
gap	.15341						
2	.622	.2721631	1.25195	2.25351	1.953042	.7812168	9.374601E-02
gap	.15341						
3	.24679	-5.443262E-02	-.2503901	-.4507021	-.3906084	-.1562434	-.0187492
gap	1.3313						

Table 4: x and y Acceptances for case 2 (Fit to Arimoto)

The acceptance was studied, as in case 1, for various of x and y amplitudes. The results for cases 1 and 2, together with those for two more cases to be discussed, are shown in Fig. 13. The parameters of the fitted ellipses are given in table 5.

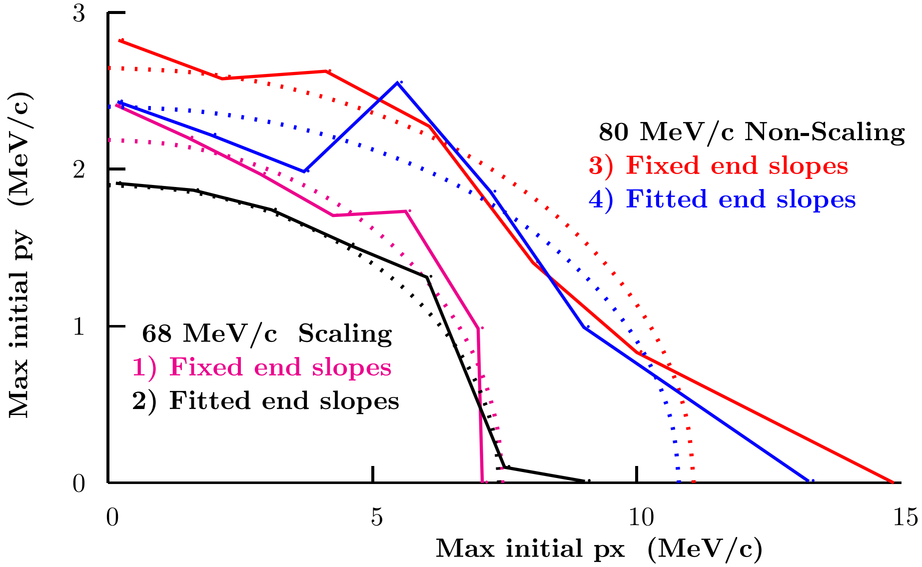

Fig. 13: Acceptances and same area ellipses for four cases: 1) and 2) for scaling lattices; 3) and 4) for non-scaling, linear lattices.

It is seen that Case 1 has significantly greater acceptance than case 2. The difference may arise from the steeper field change in Case 2.

4 Cases 3 & 4: Linear Non-Scaling lattices

We have also looked at cases that used the same z dependencies of the on axis fields, the same gradients, and thus the same central momentum tunes as cases 1 and 2. But for cases 3 and 4, all multipoles beyond the quadrupole are set to zero, thus making the magnets into simple combined (dipole and quadrupole) function magnets. The tunes at momenta other than the central reference value are now no longer constant: the lattice is no longer a "scaling" FFAG. Since the magnets are now "linear", it was hoped to increase the acceptance. Case 3 used the z dependence from case 1; case 4 used that from case 2.

An important question is whether the momentum acceptance is still as large as that for the scaling case. The closed orbits for case 3 (fixed end shapes) are shown in figure 14. Figure 15 shows the tunes as a function of momentum. The closed orbits and tunes for case 4 (fitted end shapes) were essentially identical to those for case 3.

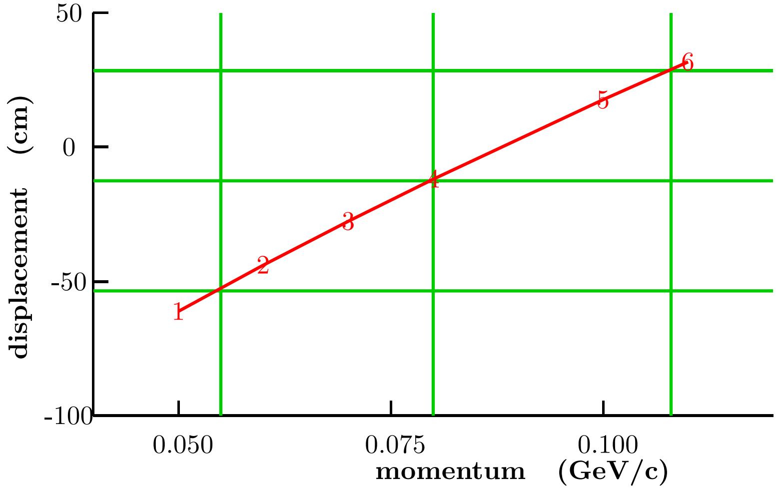

Fig. 14: Closed orbits vs momentum for case 3: Linear lattice with fixed end parameters

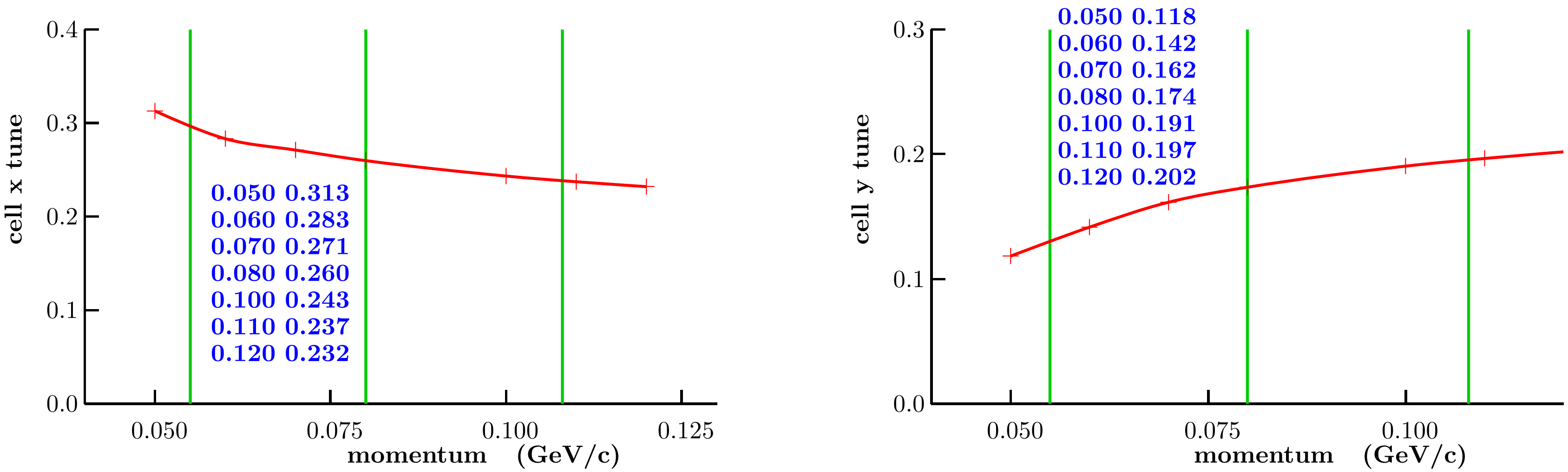

Fig. 15: Tunes a) for x, and b) for y for case 3 with linear fields and fixed end shapes.

It was found that the lowest momentum used for the scaling FFAG case is unstable in both non-scaling cases: the tune plots suggest that this is because of a third order resonance in x. It was thus decided to move the momentum span up 12 MeV, so that the new central momentum is 80 MeV.

The closed central orbit has moved out by approximately 20 cm. Subsequently we scale the fields and dimensions down to 68 MeV and 6.5 m, so that the acceptances are directly comparable to cases 1 and 2.

With this modification, the momentum acceptance, for the same horizontal aperture is a factor of 1.99 (compared with 2.11 for cases 1 and 2). i.e. reduced by only 6%. Alternatively, for the same momentum acceptance the apertures could have to be increased by 6 %.

With this scaling procedure. Fig. 16 shows the scaling (case 1) and non-scaling (case 3) central vertical fields vs horizontal position in the focus magnets. It is seen that the difference in shape is not large, but the linear magnets have about 15% lower fields. This would probably more than offset the cost of the requirement for a 6% larger horizontal apertures.

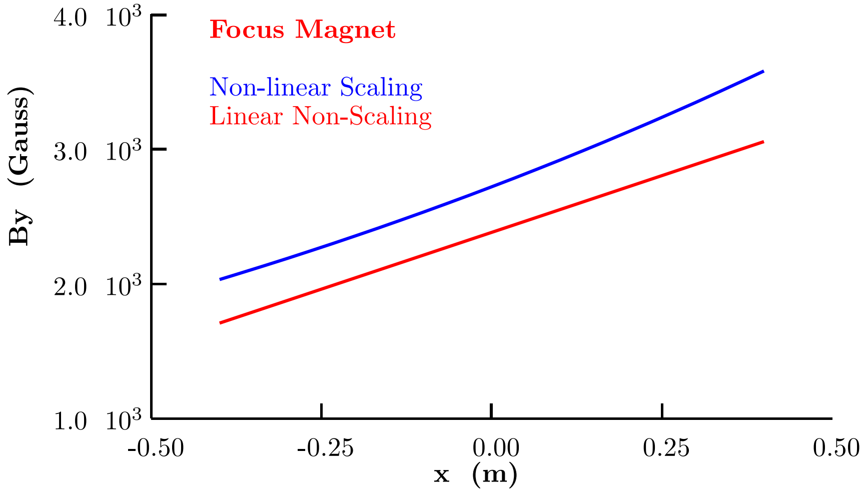

Fig. 16: Central fields vs horizontal position for scaling (case 1) and non-scaling (case 2) magnets.

The transverse acceptance at the central momentum was shown in Fig 13 above, and the results, when scaled to the same reference momentum of 68 MeV/c, are given in table 5.

It is seen that the acceptances are larger for the linear lattices compared with the necessarily non-linear scaling FFAG cases, but not that much larger. One might have expected a greater effect. With no non-linear components in the magnets, why is the aperture not infinite? Simulations of hard edged linear lattices do give significantly greater acceptances ([4]), so it has to be the non-linear effect of the realistic end fields effects that are limiting the acceptance.

The combination of more gentle field changes between the magnets, and the use of linear magnets appears to increase the 4 dimensional acceptance by more than a factor of 2: a non-negligible effect. However, we have not shown that the acceptance is increased at momenta other than the reference. Unlike the scaling case, the acceptances will now be momentum dependent. More study is needed.

Cse	End shapes	B vs x	x	y	xy
2	fitted	scaling	12.57	2.04	25.6
1	14 cm	scaling	12.77	2.70	34.5
4	fitted	linear	21.65	1.95	42.3
3	14 cm	linear	22.78	2.37	54.1

Table 5: Summary of acceptances defined by fitted ellipses

5 Summary and Conclusions

- The ICOOL simulation using 5 multiploles to represent the field index k, gives a good representation of a scaling lattice.
- The observed x acceptances for zero perpendicular amplitudes agree qualitatively with Sato's report at NUFAC04, but other acceptances appear somewhat lower.
- Using a fit to the azimuthal field dependence in Arimoto's NUFACT04 talk gave somewhat less acceptances than with more gentle field end shapes.
- Removing all higher moments, thus making the magnets linear combined function (dipole + quadrupole), gave almost the same momentum acceptance, required somewhat lower peak fields, and gave an over 2 times larger 4D dynamic aperture at the chosen central momentum.
- But we have not studied the acceptance as a function of momentum, as is now required since different momenta have quite different tunes.

References

[1] Sato http://mice.iit.edu/nufact04/sato.pdf

[2] ICOOL http://pubweb.bnl.gov/people/fernow/icool/readme.html

[3] Arimoto http://mice.iit.edu/nufact04/arimoto.pdf

[4] S. Kahn; http://pubweb.bnl.gov/people/kahn/talks/RiversideSimWorkshop.pdf