

# Overview of Recent Progress on 6D Muon Cooling with Ring Coolers

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## *Abstract*

We reference recent works on the use of storage rings for 6D cooling of muon beams. This includes the TETRA Ring, RFOFO Solenoid Ring, Quad-Dipole Ring, Dipole Only Ring, High Pressure Gas Dipole Ring and Lithium Lens Insert Ring. We include representative papers that illustrate these ring concepts. Current simulations suggest the possibility that one or more of these rings could provide adequate 6D cooling for use in a muon collider. A possible scenario involves use of multiple rings in stages – ones cooling both transversely and longitudinally and ending up with a very strong transverse cooler.

## **1. Introduction**

### (i) the Muon Collider Concept

The Muon Collider is a relatively old idea that was revived at a meeting in Napa Valley, Dec. 1992. [1] During 1990s there were 5 dedicated workshops [2] to the development of the Muon Collider and the formation of a Muon Collaboration. [3] These studies focused on the transverse cooling of the muons and spent little time discussing the 6D(6 dimensional) cooling. Around 1998, it was realized that a muon storage ring could produce a powerful Neutrino Factory [4] with the discovery of the neutrino oscillation by SuperK. The interest in a Neutrino Factory grew rapidly. By the time of the 2001 Snowmass APS Meeting, it was clear that the Muon Collider required new thinking about 6D cooling. At that meeting there were two proposals for Ring Coolers to provide 6D cooling. [5] This was the start of a major new effort on 6D cooling of muons.

### (ii) 6D Cooling with Ring Coolers

The basic concept of 6D cooling in a Ring is simple, - a wedge of liquid Hydrogen is placed in a region of high dispersion and muons of different momentum lose more or less  $dE/dx$  depending on the position. The quadrupole ring cooler is easiest to understand. Using an insert regional low beta and high dispersion region is created. [6] The low beta keeps multiple scattering low where the dispersion and the wedge absorber give energy

loss proportional to the muon beam energy – thus “cooling” the muons. All Ring Coolers work in this method to some extent. In the case of the High Pressure Gas Ring Cooler the energy loss is larger for the higher momentum larger radius particles. [7] The overall beta is kept as low as possible.

### (iii) Final Transverse Cooling Stage

The final result of these 6D coolers(to be described later) is a low 6D emittance but the transverse emittance is still larger than required for a muon collider – thus a stage of final transverse cooling is required. This can take place in a dedicated Ring Cooler with Lithium lens inserts. [8] The very low beta in the Lithium lens reduces the heating due to multiple scattering where the  $dE/dx$  in Lithium has modest straggling. Simulations show that such a Lithium Lens Ring Cooler combined with the other 6D coolers could give the required 6D emittance for use in a Muon Collider. [9]

## **Types of Muon Ring Coolers**

Following are several types of muon cooling rings that have been designed.

### **1. Tetra Ring**

Tetra Ring consists of 4 linear acceleration sections with RF cavities inside focusing solenoid magnets, Liq. Hydrogen absorbers in the middle of the sections, 4 bending sections with C-shaped dipole magnets and wedge absorbers in the middle. Circumference is around 40m with 250 MeV/c muon beam. Tetra Ring is the initial muon cooling ring with 6D cooling capability. [10, 11]

### **2. RFOFO tilted solenoid ring with liquid hydrogen absorbers**

RFOFO Ring is an initial stage 6D muon cooling ring that consists of tilted solenoid coils in 12 cells and liq. Hydrogen wedge absorbers. Tilted solenoid magnets provide bending fields for circulating muons in the Ring at 220 MeV/c. Circumference of the RFOFO Ring is 33 m and the maximum longitudinal magnetic field strength is 2.7 Tesla. 201 MHz RF cavities compensate the  $dE/dx$  energy loss of muons going through absorbers. [12-15]

### **3. Dipole-Quadrupole Rings with liquid hydrogen absorbers**

The magnet lattice of this type of cooling ring is composed of dipoles and quadrupoles arranged in a set of symmetric cells. Energy-loss absorbers containing liquid hydrogen are located at the center of each cell, and RF cavities are placed at the centers of the drift spaces between the cells. Figures c1, c2 show schematic representations of one cell of two ring cooler designs.

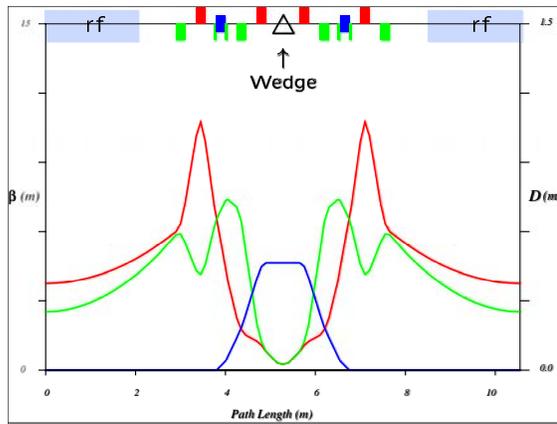
The muons lose total momentum by ionizing hydrogen molecules in the absorbers; the longitudinal component only of the momentum loss is restored (on average) by the RF cavities, this process produces transverse cooling of the muon beam.

Longitudinal cooling is achieved by tilting the edges of the absorbers in a way to make the path length of the higher-energy particles traversing the absorbers be longer than that of the lower-energy particles, so that they lose the most energy. This requires that the lattice be designed to produce *dispersion* in the absorbers, which implies a correlation between momentum offset and radial beam position. The dispersion is produced by the bending in the dipoles, and is focused by the quadrupoles to be flat in the absorbers.

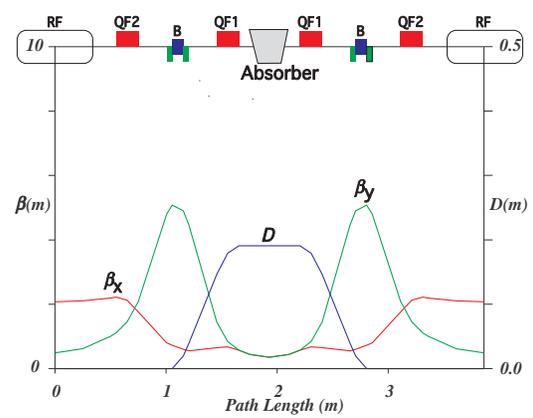
The cooling mechanisms sketched above compete with beam heating caused by the random scattering processes affecting the muon beam in the absorbers. The balance between these processes determines the equilibrium emittances that the beam will approach. Reducing the beta-function values in the absorbers as much as possible minimizes the heating; while maximizing the dispersion there enhances the longitudinal cooling.

Two other considerations are important. Firstly, the peak beta-function values should be minimized in order to obtain large dynamic aperture. Secondly, the ring should be as compact as possible in order to cool the beam rapidly, given the short muon lifetime.

Many rings of this type were designed and their performance simulated with the ICOOL program.



**FIGURE C1** Early example of an 8-cell quadrupole-dipole ring with a liquid-hydrogen wedge absorber.. quadrupoles and dipoles are depicted at the top as colored boxes. The curves show the beta-function and dispersion values.



**FIGURE C2** Cell of a 4-cell quadrupole-dipole ring with a liquid-hydrogen absorber at the center. The quadrupoles QF1, QF2, dipoles B and RF cavity are shown. The curves show the values of the beta-functions and dispersion.

These examples have zero dispersion at the cell ends, which enables inclusion of straight sections into the ring for injection/extraction. Many tradeoffs are involved in designs of this type. The progression from lattices c1 to c2 gave considerable improvement due to lowering peak beta values and compacting the ring.

Besides the lattice design, other parameters such as beam energy, magnetic field and RF gradient are critical.

#### 4. Dipole-only rings with liquid hydrogen absorbers

The next step in seeking better cooling was to make lattices composed entirely of zero-gradient dipoles with tilted edges for focusing. To maintain zero dispersion at the cell

ends, dipoles with reverse bending were included. These examples were not so successful as cooling rings, but one of them was used for the arcs of the lithium-rod ring, discussed below.

By abandoning the dispersion constraint and using symmetric, two dipole cells, rings with improved cooling performance were obtained. An example is shown in Figure D.

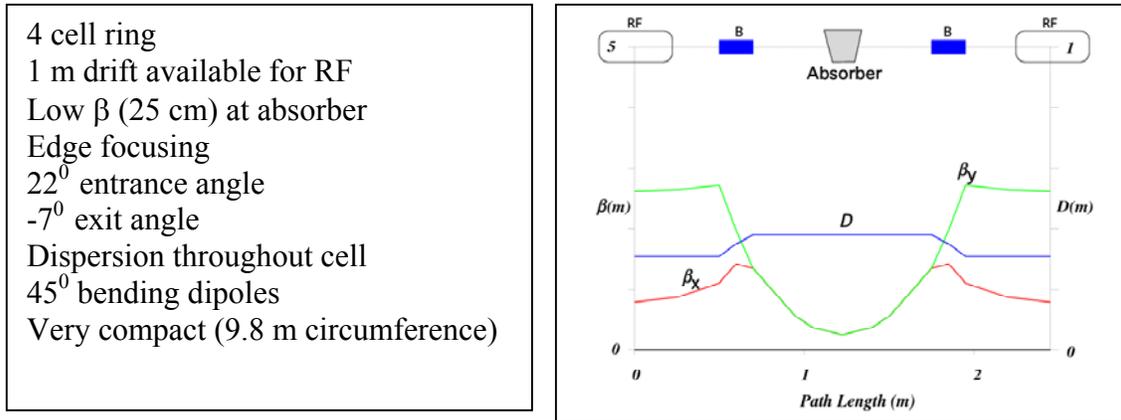
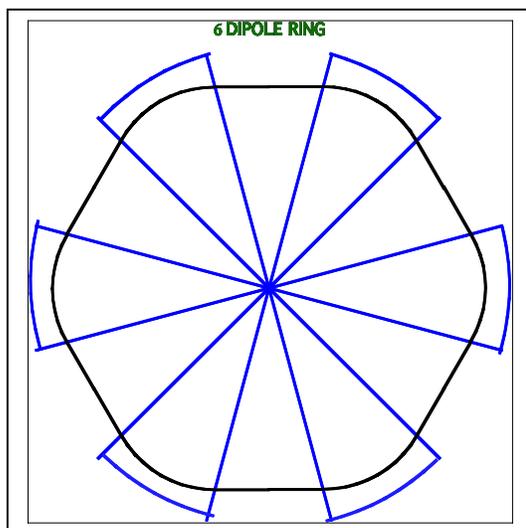


FIGURE D Compact 4-cell dipole-only ring

The two dipoles are the same but their edge angles are arranged with mirror symmetry about the cell center. Also, the separation between adjacent dipoles is different in the RF and absorber spaces. This lattice cools more than the quad-dipole type because its peak beta values and the cell length are lower while the absorber beta value and length are the same.

### 5. Scaling dipole-only rings filled with compressed hydrogen for energy absorption

The examples of Dipole-Quadrupole Rings and Dipole-only Rings resembled synchrotrons or storage rings. Now we turn to cyclotron-like structures, like that suggested in Figure E1.



Key parameters at  $r = 60$  cm  
 $\beta_x = 53$  to  $72$  cm ;  $\beta_y = 60$  to  $64$  cm  
 Dispersion =  $60$  to  $64$  cm  
 Circumference =  $3.91$  m

**FIGURE E1** Schematic diagram of 6 cell weak focusing high-field ring

In the above idealized picture, the blue outlined wedges represent flat-field dipoles, separated by zero field drift spaces. The black curve represents a closed orbit in this magnetic configuration for a particular momentum. The closed orbits for other momenta form a set of similar nested curves, with circumferences that scale with momentum.. It can be shown that the beta-functions and dispersion curves are likewise similar and also scale with the momentum. Furthermore, the betatron tunes are independent of momentum. This is an example of a *scaling* lattice. However, there are other types of scaling lattices,, such as FFAG types, in which the radial dependence is non-linear.

.A major advantage of this lattice is a large dynamic aperture. Another is that the beta-function and dispersion values are fairly uniform with azimuth. These features suggest the attractive option of filling the beam tube with compressed hydrogen gas to serve as an extended energy-loss absorber for cooling.

The parameters of Figure E1 require a superconducting fields of about 2.6 T and beam momentum for the closed orbit of about 170 MeV/c.

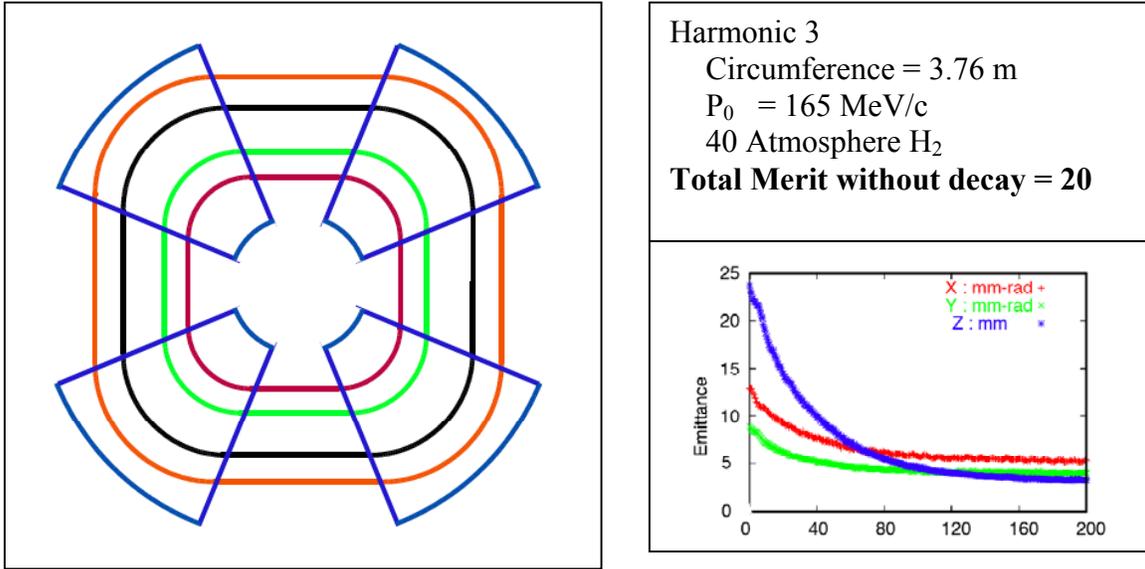
## **6. Gas-filled ring to demonstrate cooling**

The MICE experiment will produce a compelling test of transverse muon cooling. However this experiment is not optimized to cool the energy of the muon beam. So a dedicated Ring Cooler could be imagined to demonstrate the full 6D cooling. An SBIR study of such a ring was granted . Just Stage was approved but failed to obtain full funding. However this studies showed that such a High Pressure Gas Dipole Ring could be constructed to test the concept of 6D cooling. [16,17]

A design scenario to demonstrate 6D muon cooling has been made using the concepts discussed above. In order to make this demonstration economically feasible, the cooling goals were reduced to correspond to a merit factor of at least 10, where

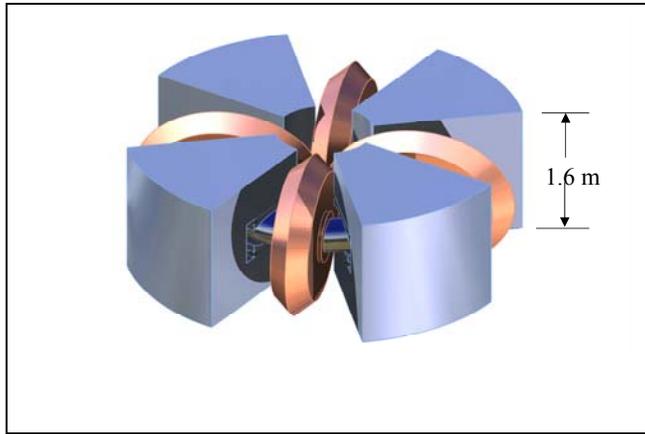
$$Total\ Merit = Transmission \times (e_x e_y e_z)_{initial} / (e_x e_y e_z)_{final}$$

The parameters were set as follows: 1.8T conventional magnets, 200 MHz RF cavities, 40 Atmosphere compressed H<sub>2</sub>. Each harmonic then corresponds to a circumference and momentum that correspond to the 1.8T field. For each harmonic the cooling performance was evaluated using the ICOOL program, and comparisons are made for different harmonics. It was found that for 4 or 6 dipole rings, the harmonic number 3 gave best results. While 6 dipole rings were slightly superior using a hard-edge model for the fields, studies with realistic fields using TOSCA led to the choice of a 4-dipole ring, see Figure E2.



**FIGURE E2** 4-dipole ring: Schematic diagram, parameters and performance for harmonic number 3

Information on the proposed demonstration ring is shown in Figure E3:



Parameter	Value
Dipole Field	1.8 T
Number of Cells	4
Reference Momentum	172.12 MeV/c
Ring Circumference	3.81 m
X Aperture	$\pm 20 \text{ cm}$
Y Aperture	$\pm 10 \text{ cm}$
$P_z$ Acceptance	$\pm 10 \text{ MeV}/c$
Minimum $\beta_x$	38 cm
Maximum $\beta_x$	92 cm
Minimum $\beta_y$	54 cm
Maximum $\beta_y$	66 cm
Hydrogen Gas Pressure	40 Atm @ 300°K
RF Gradient	10 MV/m
RF Frequency	201.25 MHz
Total RF Length	1.2 m
Total Orbit Turns	100

Table 1: Parameters that describe the muon cooling ring.

**FIGURE E3** Proposed gas-filled muon storage ring for cooling demonstration

## 7. Progress in Designing a Muon Cooling Ring with Lithium Lenses

We designed muon cooling rings with a Lithium lens which is made of 2 matching higher  $\beta$  Lithium lenses sandwiching the central lower  $\beta$  Lithium lens.  $\beta$  at the inner 22 cm long Lithium lens is 1.0 cm. The matching Lithium rod with the length of 6.3 cm each, which sandwich the central Lithium lens, has an equilibrium  $\beta$  at 4.0 cm, which swings the  $\beta$  function from the  $\beta$  at 16 cm at the outer end to the  $\beta$  at 1 cm at the inner end of the matching Lithium rod.

The solenoids has 6 Tesla  $B_z$  field where the  $B_z$  direction of solenoids is opposite to each other, and each solenoid is 1.3 m long. Figure 2 shows a schematic diagram of a Lithium lens and straight section which is made of 2 matching solenoids and a set of Lithium lenses. Figure 3 shows the  $\beta$  as a function of  $z$  in the Lithium lens and matching cells with solenoids.

In order to study the muon beam dynamics through a Lithium lens and matching solenoid lattices which sandwich the Lithium lens, we performed tracking simulation with ICOOL tracking code. Original model was designed by using the SYNCH which generates the input data for the tracking code ICOOL. [18, 19]

Figure 4 shows the development of the normalized transverse emittance as a function of  $z$  through 33 sets of 5.9 m long straight channel. In this simulation, the loss of muon  $p_z$  due to the  $dE/dx$  energy loss through the Lithium lens is recovered through a thin RF cavity by adding average  $p_z$  kick. The equilibrium normalized transverse emittance is around 0.3 mm · rad. Figure 5 and 6 show the development of the normalized longitudinal emittance,  $\Delta p/p$  and  $\Delta z$  as a function of  $z$ , and the transmission as a function of  $z$ , respectively.

### **Muon cooling in a cooling ring with Lithium lenses**

We designed a 45 degree bending cell by using two sets of zero-gradient dipole magnets with edge focusing. Wedge absorbers of liq.  $H_2$  are placed in dispersive regions in the bending cells. RF cavities are placed wherever the space is available. Figure 7 shows  $\beta$  vs.  $z$  and  $D$  vs.  $z$  in a muon cooling ring with Lithium lenses in straight sections. The  $\beta$  vs.  $z$  and  $D$  vs.  $z$  in a 45 degree bending cell is shown in Figure 8. We placed 1.7 cm long liq.  $H_2$  wedge absorbers at the center of dispersive regions in the bending cells.

Figure 9 shows the development of the normalized transverse emittance as a function of  $z$  through 8 turns of the muon cooling ring in Figure 1 channel. In this simulation, the loss of muon  $p_z$  due to the  $dE/dx$  energy loss through the Lithium lens or Liq.  $H_2$  wedge absorbers is recovered through a thin RF cavity by adding average  $p_z$  kick. The figure indicates the transverse cooling in the muon cooling ring with Lithium lenses. Figure 10 and 11 show the development of the normalized longitudinal emittance,  $\Delta p/p$  and  $\Delta z$  as a function of  $z$ , and the transmission as a function of  $z$ , respectively.

Table 1 lists parameters of the muon cooling ring with two Lithium lenses in straight channels.

Table 1: Parameters of a muon cooling ring

muon momentum	250 MeV/c
Circumference	37.5 m
straight section length	5.9 m ( $\times 2$ )
Structure of half cell	2 dipoles with edges
number of bending cells	8
bend cell length	3.6 m
length of Lithium lens	34.5 cm ( $\times 2$ )
lowest $\beta$ in Lithium lens	1.0 cm
energy loss	35 MeV/turn
dipole bend angles	44.2, -21.7 degree
dipole edge angles	30/-3, -11/-11 degree
dipole magnetic field	6.5, -3.2 tesla
Cell tunes bend cell	0.72/0.70
Cell tunes straight cell	4.0

We designed a race track muon cooling ring with 35 cm long Lithium lenses in straight channels with  $\beta$  at 1 cm. Bending cells have zero-gradient dipole magnets with edge focusing, and wedge absorbers in dispersive regions. Study is in progress to obtain the 6 dimensional muon cooling in this cooling ring.

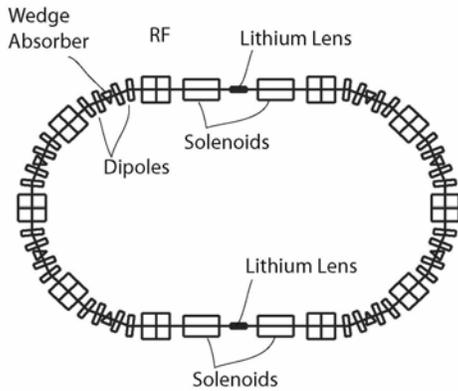


Figure 1 A schematic diagram of a muon cooling ring with Lithium lenses in straight sections.

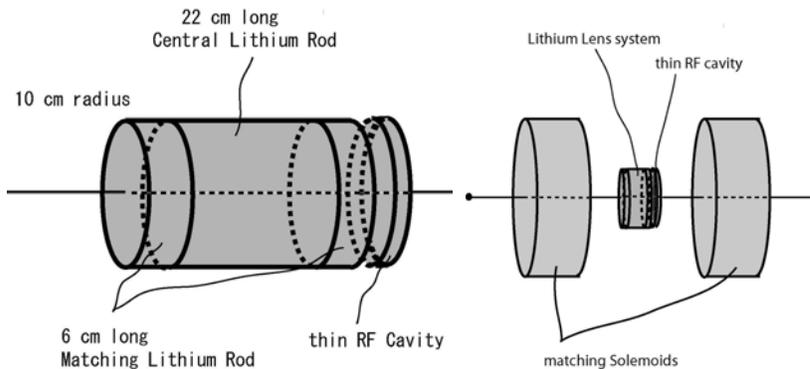


Figure 2 A schematic diagram of a Lithium lens(left) and straight section(right).

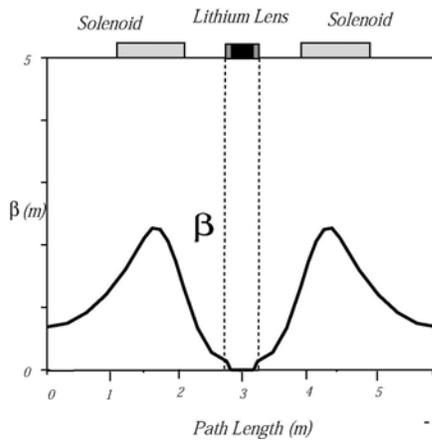


Figure 3  $\beta$  as a function of  $s$  in the Lithium lens and matching cells with solenoids.

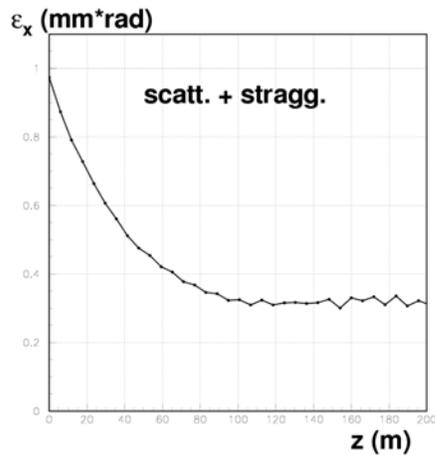


Figure 4 Normalized transverse emittance as a function of  $z$  through 33 sets of straight sections

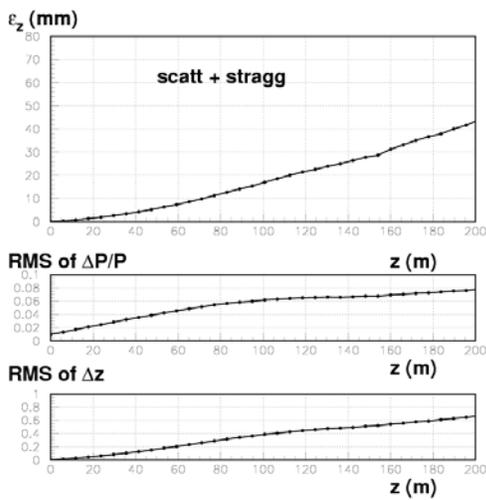


Figure 5 Normalized longitudinal emittance,  $\Delta p/p$  and  $\Delta z$  as a function of  $z$  through 33 sets of straight sections.

### Transmission

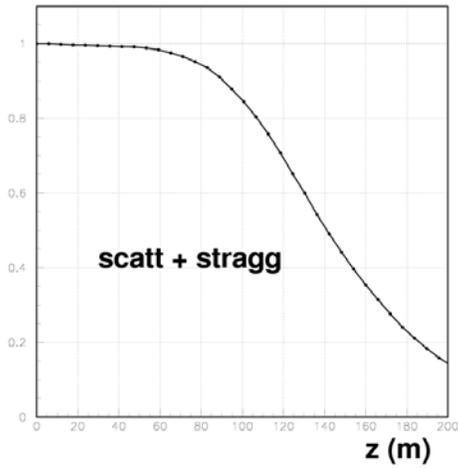


Figure 6 Muon transmission as a function of  $z$  through 33 sets of straight sections.

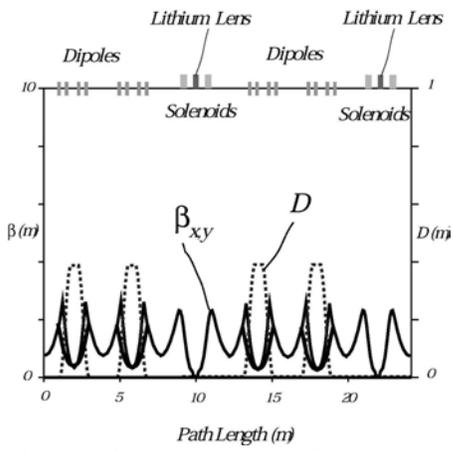


Figure 7  $\beta$  vs.  $z$  and  $D$  vs.  $z$  in a muon cooling ring with Lithium lenses in straight sections.

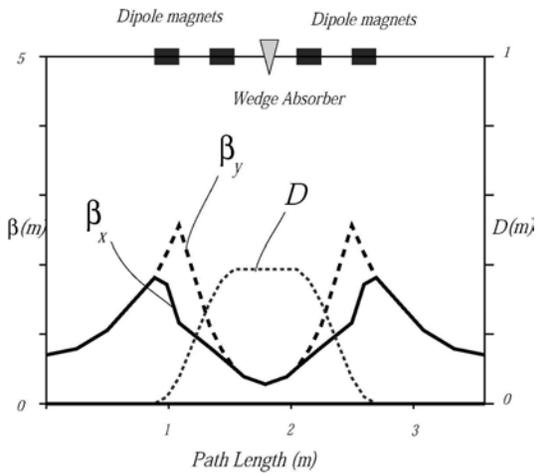


Figure 8  $\beta$  vs.  $z$  and  $D$  vs.  $z$  in a 45 degree bending cell.

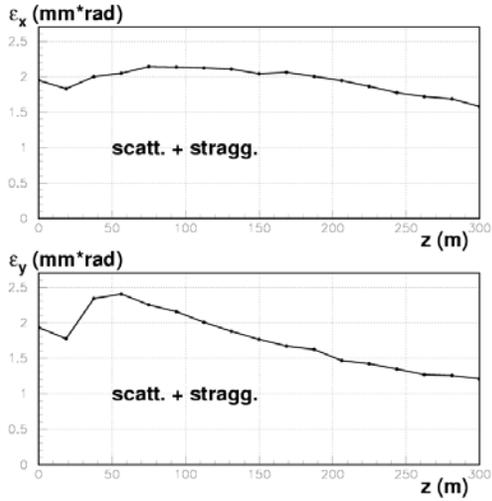


Fig. 9 Normalized transverse emittance as a function of  $z$  through 8 turns of a muon cooling ring.

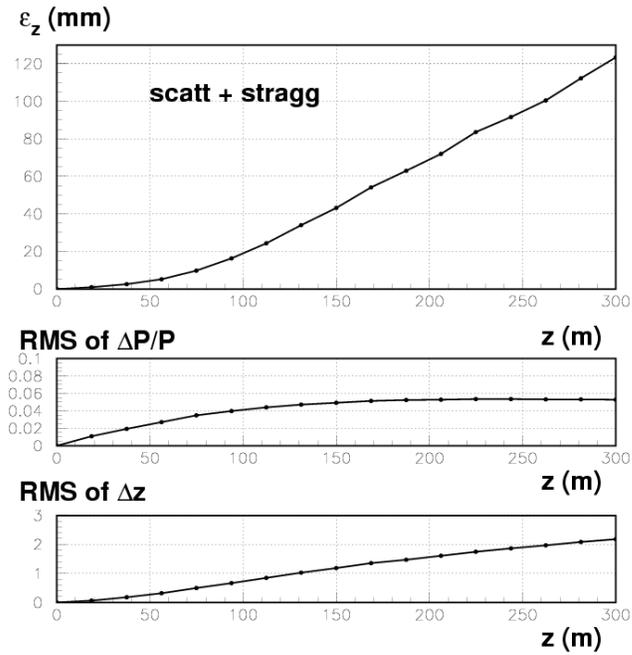


Fig. 10 Normalized longitudinal emittance,  $\Delta p/p$  and  $\Delta z$  as a function of  $z$  through 8 turns of a muon cooling ring.

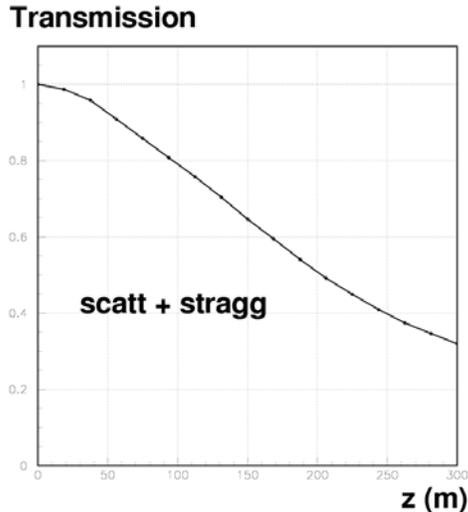


Fig. 11 Muon transmission as a function of  $z$  through 8 turns of a muon cooling ring.

## References

- 1) Proceedings from the First Workshop on Muon Colliders  
(Napa, California, 1992), Nucl. Inst. Methods, Vol. A350, pp. 24-56, 1994.
- 2) P. Chen and K. MacDonald, Summary of the Physics Opportunities Working Group,  
AIP Conference Proceedings 279, Advanced Accelerator Concepts, 853 (1993)  
Proceedings of the Mini-Workshop on  $\mu^+\mu^-$  Colliders: Particle Physics and Design,  
Napa CA, Nucl. Inst. and Meth., A350 (1994); Ed. D. Cline  
Physics Potential and Development of  $\mu^+\mu^-$  Colliders, 2nd Workshop, Sausalito,  
CA, Ed. D. Cline, AIP Press, Woodbury, New York (1995)  
Proceedings of the 9th Advanced ICFA Beam Dynamics Workshop, Ed. J. Gallardo,  
AIP Press (1996)  
Proceedings Symposium on Physics Potential and Development of  $\mu^+\mu^-$  Colliders,  
San Francisco, CA, December 1995, Supplement to Nucl.Phys. B, Ed. D. Cline and D.  
Sanders,
- 3) Robert B. Palmer, et. al., "Mu+Mu- Collider Feasibility Study" ,  
Snowmass 1996
- 4) M.Alsharo, et. al.,  
"Recent Progress in neutrino factory and muon collider reserach within the  
muon collaboration"  
Phys. Rev. ST Accel. Beams 6, 081001 (2003)
- 5) H.Kirk, D.Cline, Y.Fukui, A.Garren  
"Progress towards a Muon Ring Cooler"  
Snowmass M1 workshop Proceedings, July 2001
- 6) Y.Fukui, D.Cline, A.Garren, H.Kirk  
"Muon Cooling Rings for the neutrino factory and  $\mu^+\mu^-$  collider"  
ICAP2002 proceedings
- 7) H.Kirk, S.Kahn, F.Mills, D.Cline, A.Garren  
"A Compact 6D Muon Cooling Ring"  
PAC05 proceedings

- 8) Y.Fukui, D.Cline, A.Garren, H.Kirk  
"Progress in Designing a Muon Cooling Ring with Lithium Lenses"  
PAC03 Proceedings
- 9) D.Cline  
"Development of muon ring coolers, neutrino factories and supersymmetric Higgs factory"  
J.Phys.G: Nucl. Part. Phys. 29 No8(August 2003)
- 10) R.Palmer, V.Balbekov, J.S.Berg, S.Bracker L.Cremaldi R.Fernow J.Gallardo  
R.Godang, G.Hanson, A.Klier, D.Summers  
"Ionization Cooling Ring for Muons"  
Phys.Rev.ST-AB 8,061003(2005)
- 11) S.Kahn, R.Fernow, V.Balbekov, R.Raja, Z.Usubov  
"Tetra Muon Cooling Ring"  
AIP04 Proceedings 721, 387(2004)
- 12) R.Palmer, V.Balbekov, J.S.Berg, S.Bracker L.Cremaldi R.Fernow J.Gallardo  
R.Godang, G.Hanson, A.Klier, D.Summers  
"The RFOFO Ionization Cooling Ring for Muons"  
NuMu Note 314, Submitted to Phys. Rev. ST-AB, April, 2005
- 13) R.Fernow, J.S.Berg, R.Palmer  
"Muon Cooling in the RFOFO Ring Cooler"  
PAC03 Proceedings
- 14) R. B. Palmer, L. Reginato, D. Summers  
"An Induction Kicker for Muon Cooling Rings"  
NuMu 256, Sep. 2002
- 15) R.C. Fernow, J.S.Berg, J.C.Gallardo, R.B.Palmer  
"Muon Cooling in the RFOFO ring"  
NuMu 273, April 2003
- 16) D. J. Summers, S. B. Bracker, L. M. Cremaldi, R. Godang, D. B. Cline, A. A. Garren,  
G. G. Hanson, A. Klier, S. A. Kahn, H. G. Kirk, R. B. Palmer  
"6D Ionization Muon Cooling with Tabletop Rings"  
DPF2004, Riverside Conference, NuMU Note 306.
- 17) S.Kahn, H.Kirk,F.Mills, D.Cline, A.Garren  
"Design of a Magnet System for a Muon Cooling Ring"  
PAC05 proceedings
- 18) A. A. Garren, A. S. Kenney, E. D. Courant, A. D. Russel, and M. J. Syphers,  
"SYNCH - A Computer System for Synchrotron Design and Orbit Analysis, User's  
Guide", SSCL-MAN-0030, 1993
- 19) R. Fernow,  
"A Simulation Code for Ionization Cooling of Muon Beams"  
Part. Accel. Conf., Proc. 1999, p. 3020.