Beam-Cavity Interaction for High Pressure Hydrogen Gas Filled Cavities

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Introduction:

In this note, we will investigate some of the physics processes taking place in an RF cavity that is filled with high pressure H gas that has been cooled to LN2 temperature. The measurements made by Muonsinc on the breakdown of such cavities [1] is shown in the Fig. [1] below.

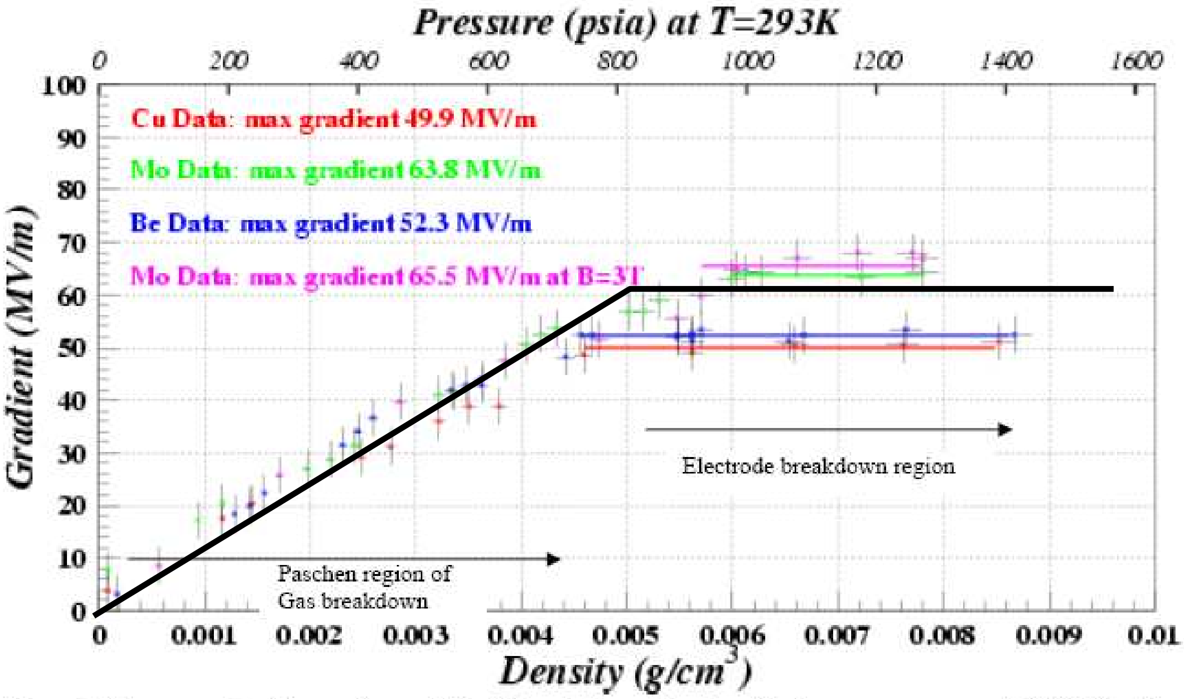

Figure 3: Measurements of the maximum stable TC gradient as a function of hydrogen gas pressure at 800 MHz with no magnetic field for three different electrode materials, copper (red), molybdenum (green), and beryllium (blue). As the pressure increases, the mean free path for ion collisions shortens so that the maximum gradient increases linearly with pressure. At sufficiently high pressure, the maximum gradient is determined by electrode breakdown and has little if any dependence on pressure. Unlike predictions for evacuated cavities, the Cu and Be electrodes behave almost identically while the Mo electrodes allow a maximum stable gradient that is 28% higher. The cavity was also operated in a 3 T solenoidal magnetic field with Mo electrodes (magenta); these data show no dependence on the external magnetic field, achieving the same maximum stable gradient as with no magnetic field.

Fig. 1

The possibility of obtaining very high gradients is very attractive for muon cooling channels. We will abstract from this data a single constant which is normally designated as E/P where E is the field gradient in V/cm and P is the pressure in millimeters of Hg. If we take a straight line thru the origin and the point 60 MV/m and density of .005 grams/cm^3, then one can calculate that E/P 13.2. The units are Volts/cm and the pressure in torr that would give H gas a density of .005 grms/cm^3 at T = 273 K. A second and better variable is given by E/n where n is the number of molecules/cm^3 and since n is proportional to P, the behavior is the same, but makes clear that the only gas variable is the spacing between molecules. The RF breakdown measured at 800 MHz is about the same as the DC measurements reported in the literature and fits the behavior described by Paschen. The limiting value for breakdown that corresponds to E/P = 13.2 is E/n = 4 10^{-16}

Breakdown occurs on the linearly rising part of the curve when an avalanche forms. This happens when the probability for a free electron in the gas to make a second electron is greater than one. This can only take place when the mean free path of the electron is long enough so that its kinetic energy picked up from the field exceeds the ionization potential of the hydrogen molecule. The threshold is 15.37 eV and the cross section increases linearly after about 16 volts as shown in the Fig. [2] below from [3].

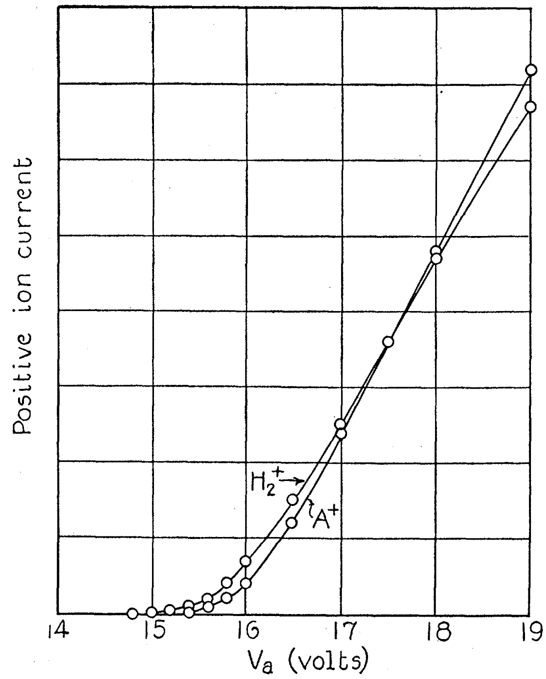

Fig. 2. Typical ionization potential curves showing the positive ion current as a function of the electron velocity (uncorrected).

The actual cross section as measured by [3] is shown in the Fig. [3] on the following page. The curve gives the probability per cm for ionization in Hydrogen gas at 1 mm pressure. We are not directly interested in this data as the régime proposed for cavities in the cooling channel is such that secondary ionization processes do not occur. It obviously does apply to the linear part of the Paschen curve where E/P is 14 or greater and where break down does occur. The data collected in this note should be useful when we try to interpret future experiments.

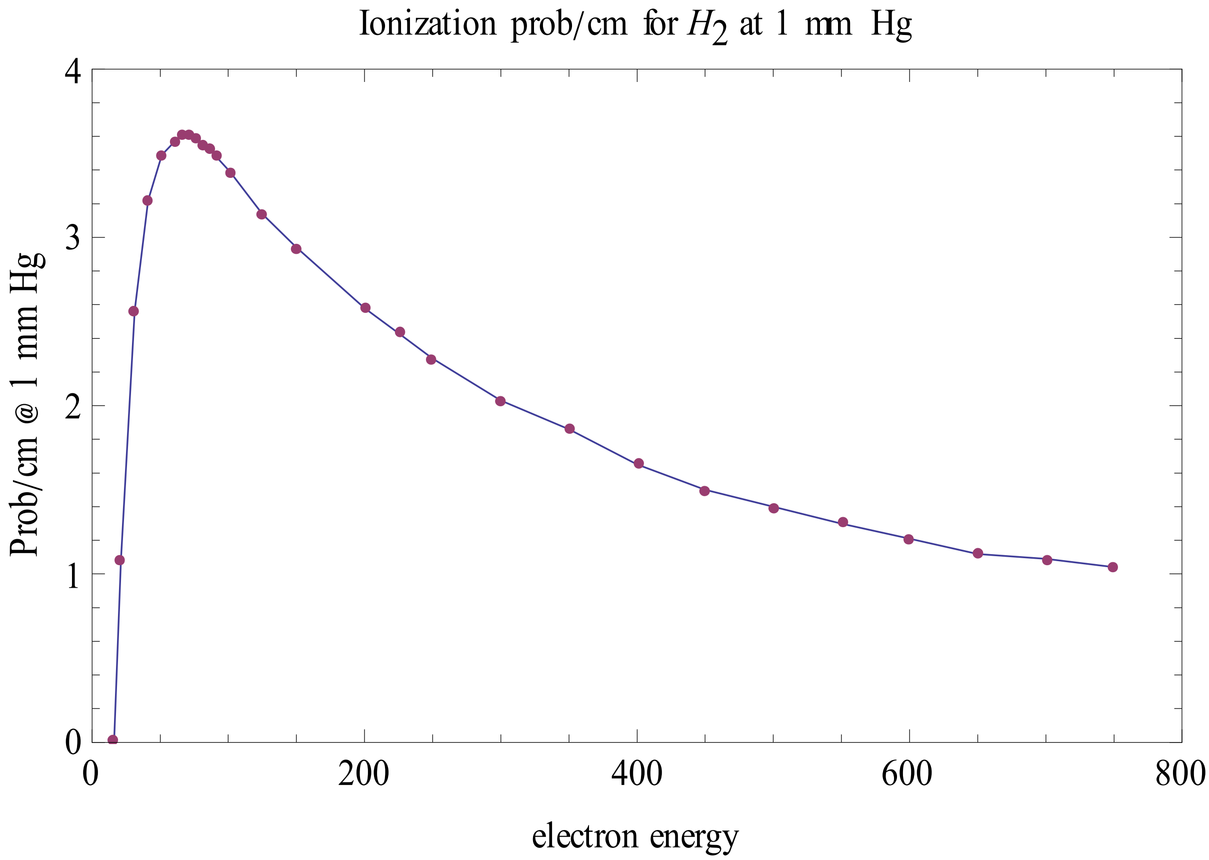

Fig. 3

Let's now consider the region where E/P <14 and we have a beam thru the cavity. First of all it is worth noting that breakdown does occur on the plateau region where the gradient is high enough to inject a strong source of electrons pulled from the metal electrodes. This swarm of electrons increases the local field and E/P is great enough to support an avalanche and the cavity breaks down. We need a picture of what happens when an intense beam of muons passes thru the cavity.

Description of beam transit thru cavity

We consider first a closed cell cavity with parallel plates and a uniform field and the beam goes thru at the peak of the RF cycle as a delta function. We will pick dE/dx for hydrogen as 4 MeV/grm/cm^3 and use the measured value of 35 eV/ion pair to calculate the ionization density. A cartoon is shown on the next page, Fig. [4]. The green represents the region of ionization caused by the transition of the beam pulse and immediately after the electrons drift in the field toward the upper plate. The positive ions are so massive that their mobility is very small and they essentially stay in place. As the column of electrons drift upward, they leave a disc of positive ions near the lower plate. They also collide with the gas ions and loose energy, making a random walk in the vertical direction. The energy lost in the collision process takes energy out of the field in the cavity. When the rf voltage crosses zero, we will have the situation shown in the next picture, Fig. [5]. Energy has been dissipated and there is a layer of positive charge near the bottom plate equal to the charge of electrons collected on the upper plate of the cavity. During the next half cycle the electrons drift downward leaving an area of positive charge near the top plate and again loosing energy. We should also note that there is an axial magnet field of the order of 5 T that will confine the plasma in the radial direction. We now need to get quantitative and calculate the magnitude of the ion densities and their mobilities.

Beam passing thru cavity at voltage peak:

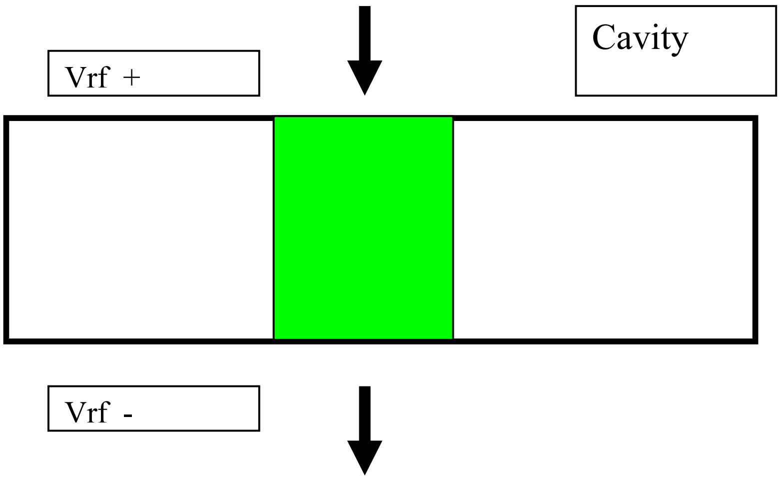

Fig. 4

One quarter cycle later:

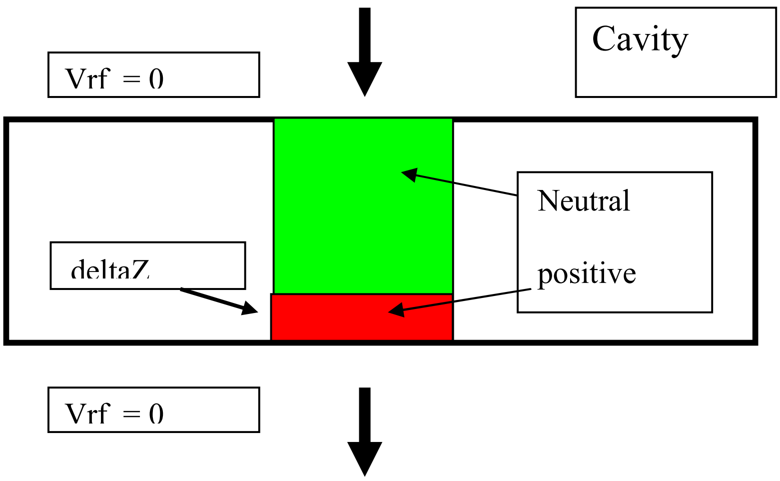

Fig. 5

The table on the next page is representative of the environment that we must consider.

Example: **Table 1.**

```
P= 200  Bo=13.6`  betaPerp= 0.0981
emit= 0.000422  minEmit= 0.000209
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Nbeam= $1.\times10^{11}$ rhoGas= 0.03`

RF Gradient V/m =$25.\times10^{6}$,Frf Hz= $400.\times10^{6}$

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1. beamRadius, cm	0.643745
2. H molecule density	9.033×10^{21}
3. av. molecule Spacing in microns	0.000480245
4. muons/cm^2 =	7.68108×10^{10}
5. Averge μ spacing microns	0.0360819
6. Radius 2 eV electron, Bo T field, microns	0.350413
7. spacing between ions along track, microns	2.91667
8. electron path length to ionize, microns	0.6148
9. tforIonization ps	0.522413
10. positive ion density/cm^3	2.63351×10^{14}
11. plasma Frequency	9.14864×10^{11}
12. EoverP, KV/cm/torr =	0.919481
13. Mobility=	0.0187169
14. electron velocity cm/sec=	1.0207×10^{6}
15. deltaZ, microns	9.77355
16. q/cm^2 cavity,No. electrons	1.38349×10^{11}
17. plasma density x deltaZ/2	1.28694×10^{11}
18. plasma Charge/ Cavity Charge	0.930212
19. deltaW/cm^3 / cycle	0.00686796
20. Qeffective	2.53138
21. E/n (V/cm /Molecules/cm^3	2.78977×10^{-17}
	Null

Description of the above parametrs:

P=beam momentum MeV/c Bo= axial field in T betaPerp= focus from Bo
Emit= actual beam emittance minEmit = equilibrium emit Nbeam = # muons in pulse
rhoGas = density in grms/c^3

The beam is assumed to be a simple cylinder of uniform density. The tforIonization is the time it would take for a free electron to accelerate to 15.3 eV. deltaZ is the distance an electron moves thru the gas in ½ cycle (not 1/4)! In line 18, the charge in the volume deltaZ is compared to the charge on 1 cm^2 of the cavity electrode, ie 8.87 10^{-12} E /100. Finally in line 20 the effective Q of a unit volume of space is calculated.

Discussion of the example

First of all it is crucial to understand how the mobility of the electrons is obtained. The mobility μ is defined by the equation $v = \mu E$. It is measured by observing the drift velocity of electrons in a field E and is normalized to the value observed for a gas pressure of 1 Torr. To see the physics, consider a swarm of electrons moving under the effect of E. An electron will be doing random collisions with the molecules and changing direction. However it will have a coherent drift given by:

(1) $v = \frac{1}{2} a t = \frac{1}{2} e/m \, E t = \frac{1}{2} e/m \, E \langle \lambda / V_r \rangle$

We start with an electron whose random velocity is Vr but is zero in the z direction. It accelerates under the action of the field until it hits a molecule and is deflected by 90^o or more. The distance it travels is λ and its velocity is Vr, the random velocity of the swarm. The distance it travels is given by the total cross section and the gas density:

(2) $\lambda = 1 / N \sigma$

Combining with (1) gives

(3) $v = \frac{1}{2} e/m \, \langle 1 / (N \sigma V_r) \rangle \, E = \mu E$ and is proportional to E /P or E/N.

We note that E / P is only a good variable if Vr, the random velocity of electrons in the swarm , is a constant and not dependent on density. In fact, Vr is not the kinetic velocity where the electrons are in equilibrium with the gas molecules with each degree of freedom having an energy of ½ kT. In the case of high field, the swarm random velocity of the electrons is much above that of the molecules. The energy from the coherent motion in the field is fed into random motion by scattering. So the electron swarm has mean kinetic energy of several eV. The following figures give the availale data on mobility of electrons in hydrogen.

The first curve by Bartels [4] discloses a big dependence on density for both He and H in the region covered. Remember that the region of interest is for E/P considerably greater than 1.0 and this is off the curve. I have extrapolated the limiting curves for He at low density and high density up to E/P = 1.0. It is apparent that they are coming together. The data for H is not as extensive and brings into question what the high pressure mobility for H really is. The table at right converts the number density into gas density and to the equivalent P in Torr. The gas density in this experiment reaches about .02 which is in the region of interest. However, the important point is that the mobility is approaching the low density value,i.e. the topmost curve The same effect is shown in the work of Bartels [5] for H but again for lower densities. A number of effects come in at low values of E/P: The electron swarm comes into equilibrium with the ambient gas and there are molecular states that can temporarily capture the electron and thus impede its motion.

The result was finally explained by Braglia and Dallacasa[2]. When the mean energy of the electron becomes low enough the wavelength of of the electron becomes greater than the spacing between molecules and so one can no longer treat things on the basis of the electron colliding with one atom at a time. The theory worked out fits the data well in the region of very small E/P where the electrons have come into equilibrium with the ambient environment. I have not been able to find a treatment for "hot" electrons. (the following two pages show data)

Bartels [4]

Density Dependence of Electron Drift Velocities 61

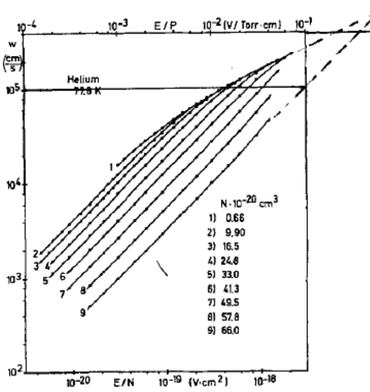

Fig. 1. Electron drift velocity versus E/N in helium at 77.6 K and at nine densities. The points are experimental values. In the scale of E/P is 1 Torr $= 3.3\times10^{16}$ cm^{-3}

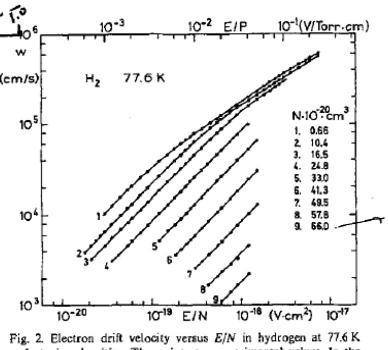

Fig. 2. Electron drift velocity versus E/N in hydrogen at 77.6 K and at nine densities. The points are experimental values. In the scale of E/P is 1 Torr $= 3.3\times10^{16}$ cm^{-3}

At the highest density of 6.6×10^{21} cm^{-3} the drift velocity is only 0.52% of the value at low density. Here the drift velocity of the electrons is only about

N/cm^3	grms/cm^3	E?P
6.6×10^{19}	0.00022044	1876.39
9.9×10^{20}	0.0033066	28145.8
1.65×10^{21}	0.005511	46909.6
2.48×10^{21}	0.0082832	70506.6
3.3×10^{21}	0.011022	93819.3
4.13×10^{21}	0.0137942	117416.
4.95×10^{21}	0.016533	140729.
5.78×10^{21}	0.0193052	164326.
6.6×10^{21}	0.022044	187639.

Fig. 6

Pack and Phelps [6]

Bartels [5]

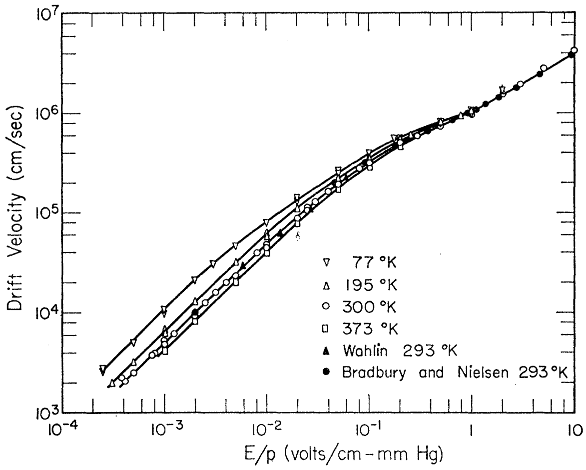

FIG. 7. Electron drift velocity as a function of E/p in hydrogen at 77°K, 195°K, 300°K, and 373°K. For $E/p<3\times10^{-3}$ the electrons are in thermal equilibrium with the gas at each temperature.

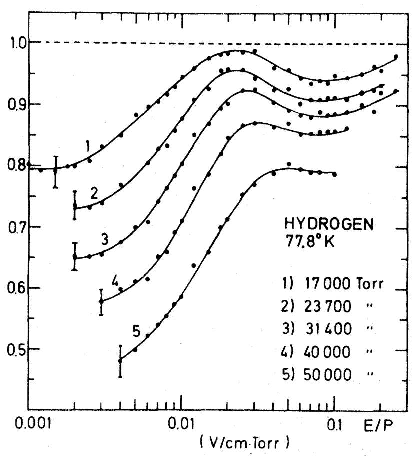

FIG. 1. The quotient q of the drift velocity at high pressures and the drift velocity at low pressure (here 2000 Torr) as a function of E/P.

Fig. 7

Bradbury and Nielson [7] The large E/P is at low pressure and gives E/P with no density effect.

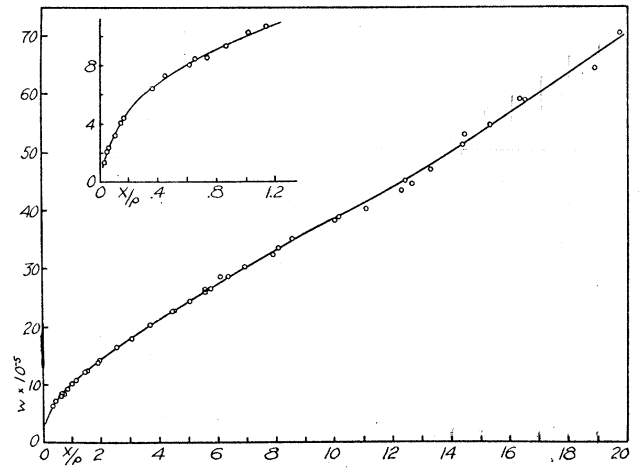

FIG. 4. Drift velocity of electrons in hydrogen as a function of X/p.

Fig. 8
Heylen [1]

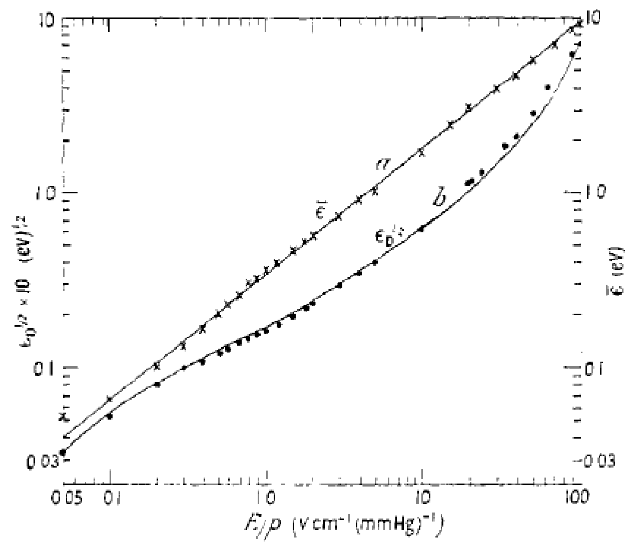

Figure 2. Electron mean energy: crosses, Crompton and Sutton (1952); curve a, according to equation (5). Electron drift velocity: points, Bradbury and Nielsen (1936) and Gill and von Engel (1949), curve b corresponds to equation (6)

Fig. 9

Heylen [1] has fit the data at low density with an empirical equation that is good to 16% in the region 0.1 < E/P < 100. He gives two curves shown on the bottom of the last page. One is for the mean random energy for an electron in a swarm and the other is for the mobility. The fits are:

$$\epsilon_m = 0.357\ (E/P)^{0.71}$$

$$\mu = 1.72\ 10^{-2}\ [\ 1 - 2.4\ \ 10^{-2}\ (E/P)^{0.71}\ ]^{-1.75}\ (E/P)^{-.53}$$

(There is an error in his eq[6]….it should be $(E/P)^{0.71}$ not -0.71). The fits to the data are shown in the last figure on the previous page. These fits have been used in calculating the motion of the electrons for the example cavity. The velocities at an E/P = 1 are 10^6 cm/sec. and the mean electron energy is several electron volts.

We can use this data to investigate the questions raised by the observed density effect. Using the first equation, we can compute the mean wavelength of the electrons as a function of E/P. The figure below shows the results.

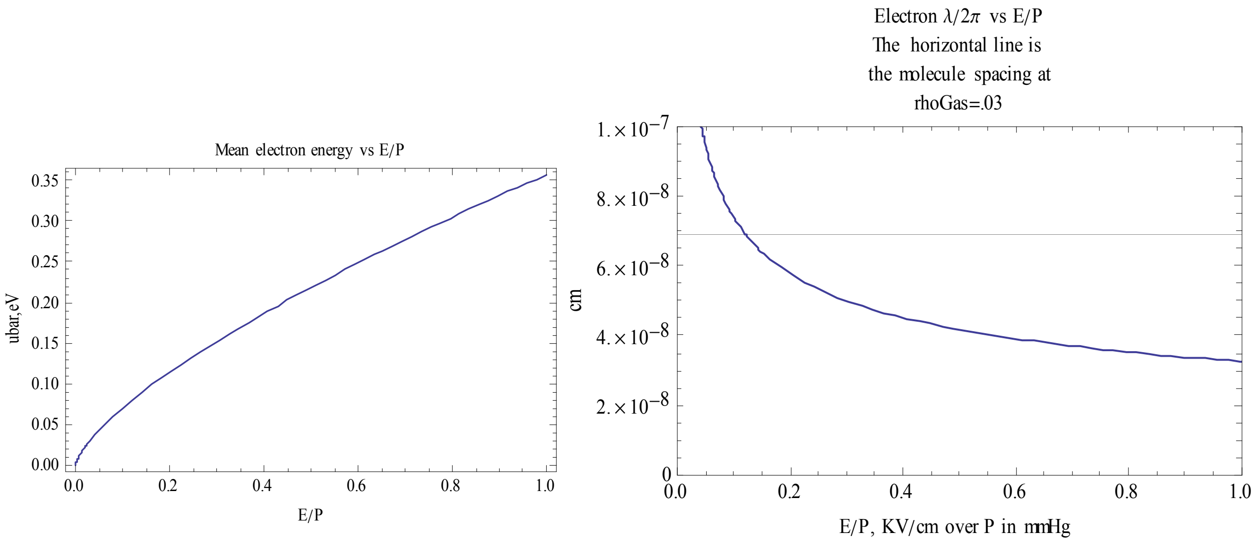

Fig. 10

To calculate deltaZ, the length the electrons drift in ¼ cycle, we integrate the drift velocity over time

$$\text{deltaZ} = \int_0^{T/4} \mu[E_0 \, \text{Cos}[\omega\, t] \,/\, P]\ E_0 \, \text{Cos}[\omega\, t]\ dt$$

Where the expression for the mobility is given above and deltaZ is shown in line 15 of the table.

In a similar fashion we can calculate the energy loss in a cubic cm of the plasma and the answer is in line 19:

$$dW = \int_0^T \mathrm{velocity}\ \mathrm{force}\ dt = \int_0^T \mu\, E\, e\, E\, dt$$

In addition to the energy loss/cycle, we can compute the electrostatic energy stored in a cubic cm of space and comparing this with the energy loss, we can compute an "effective Q" of this volume. This is not the Q of the cavity because the cavity has energy stored outside the region of the plasma, but it does give and indication of the intensity of interaction between the beam and cavity.

The plots below are for the conditions listed in figure [ ] except that the gas density is varied from .001 to .04. Ignore the region below .004 because breakdown will occur at these low densities since Vrf is fixed at 25 10^6 /cm.

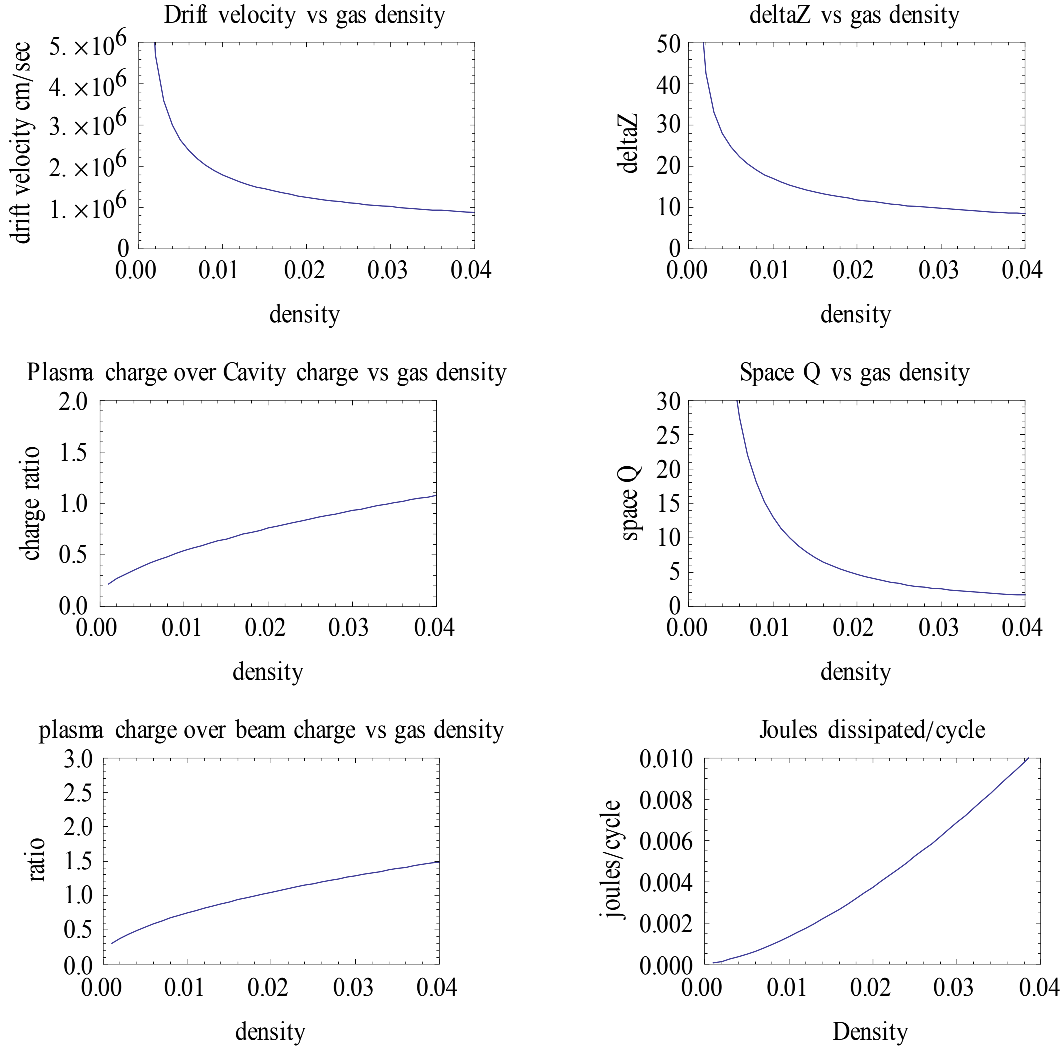

Fig. 11

The drift velocity comes from the fit made by Heylen and is the velocity at the peak of the RF cycle. deltaZ is calculated by integrating the velocity over a half cycle. The plasma charge over the cavity charge is an indication of the cavity loading by the beam. The actual cavity loading is less than this by the ratio of the beam area to the effective high field area of the cavity. Charge will flow in from the adjacent areas and the effect is to increase normal beam. The best indication of this is the lower left hand plot shat shows the plasma charge over the beam charge. In the useful region, this ratio is of the order of 1 and so the gas effectively doubles beam loading effect in this example. The last curve shows the energy dissipation per cycle per cm^3 and will cause damping in the cavity.

Recombination

The question of recombination of the plasma has been raised. If the recombination is extremely fast, the effects discussed above will be absent and if it is very slow, it can influence subsequent cycles. First of all, two body recombination with the emission of a photon is orders of magnitude too slow to be of any effect. For a two body process, the recombination rate is defined by the equation

$$dn_1 / dt = r\, n_2\, n_1$$

where n_1 and n_2 are the concentrations of the two species participating in the reaction. The value of r has been measured and calculated for hydrogen and is between 10^{-11} and 10^{-12} cm^3 sec^{-1}. For the example with the ion density = electron density = 2 10^{14} , the initial lifetime is of the order of milliseconds. Note that if $n_2 = n_1$ the equation can be integrated:

$$n_1(t) = n_1(0) / (1 + n_1(0)\, r\, t)$$

This is not exponential and ions can last for very long times. The quantity $1 / n_1 \; d n_1 / dt$ gives the fractional rate that n_1 is disappearing and can be used for comparing various rates.

The main process for recombination is a 3 body process where the electron is captured into an excited state of the molecule which is then de-excited through further collisions. The process is complex and the numbers that exist are generally for plasma at low pressure. The numbers give recombination rates that are maybe a factor of 10 faster than the direct recombination which place the initial time constant in the range of 100 μsec.

A case where attachment to a heavy molecule could possibly help would be thru the introduction of something like SF_6 which has a great affinity for electrons. It also has a radiation length that is much shorter than H and so must be used sparingly! The radiation length per molecule is about 1% that of H. However, the introduction of this amount of material in H would only double the radiation length and one could expect a short capture time for the mixture. The attachment rate of electrons to SF_6 has been measured in a mixture of N_2 and SF_6 to be between 10^{-8} and 10^{-7} [8]. If it is the same in hydrogen then a 1% mix would lead to recombination rates of the order of picoseconds. A drawback could be that under radiation, SF_6 forms F^- and a whole string of other nasty ions which could combine with H into even nastier things. It isn't clear whether or not the vapor pressure is high enough at 77 K make an effective mixture. Its vapor pressure at 160 K is .01 bar. I wasn't able to find values at lower temperature. One should search for more appropriate gasses.

Mathematica Program

I have written a Mathematica program that calculates many of the numbers that will pertain in a cooling channel. An example of its output is shown in Table 1. It is available for asking. It can be used as a tool to help explore where gas filled cavities might be used in a cooling channel.

Experimental program

There are many questions! I list some of my comments here:

1. The primary data will come from the first exposure of a gas filled cavity to beam. At a gas density of .03, we will be at 6 times the Pashen limit observed at a density of .005. Measuring the Q at low beam intensity should reveal whether the loss factors calculated here agree with the observations. At high intensity the charge calculations can be checked. It will be important to have a fast scope coupled to the cavity to observe exactly how the cavity reacts to beam during the rf cycle. We have a digitizer that was purchased for the Tevatron and has a BW of several GHz and enormous storage that would be ideal for these studies. Contact Bob Flora.
2. Having clearing electrodes could be useful for measuring the clearing times of the ions. Note that these times are very sensitive to gas purity as small quantity of something like oxygen can have a large effect.
3. A scintillation light fiber exposed to the hydrogen Balmer lines in the cavity could be useful.
4. Blasting the cavity with a 10 μSec dirty beam is a great first test, but dos not very accurately represent a small very highly cooled beam that will exist near the end of the channel. So if this works in the initial test, some thought needs to be given to how we can generate a better beam. 10^{12} particles in 10 μSec is very different than if they are in a single bunch.
5. An open cell cavity must also be tested. In this case the complications at the end are much harder to model and have not been considered here. However the losses in the ionized region will still be there and will damp the cavity.

References

Below are a number of references that are useful.

Published Papers

Mobility

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2. G. L. Braglia and V. Dallacasa "Theory of electron mobility in dense gases" PR A 26, 902, (1982).

3. John T. Tate and P. T. smith "The efficiencies of ionization and ionization potentials of various gases under electron impact" PR 39, 207, (1932)
4. A. Bartels "Pressure dependence of electron drift velocity in hydrogen at 77.8 K" PRL 28, 213, (1971).
5. A. Bartels "Density dependence of electron drift velocities in Helium and Hydrogen at 77.6K" Applied Physics 8, 59, (1975).
6. J. L. Pack and A. V. Phelps "Drift velocities of slow electrons in Helium, Neon, Argon, Hydrogen, and Nitrogen" PR 121, 798, (1960).
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8. P.G. Datskos, L.G. Christophorou, and J.G. Carter " Attachment of low energy electrons to "hot" SF_6 molecules" ORNL Report

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2. Physics of Ionized Gases. Boris M. Smirnov
3. Electron
4. Avalanches and Breakdown in Gases. H. Raether