Rf Capture Variations for the Muon Storage Ring Neutrino Source

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Abstract

We present possible scenarios for capturing the long bunches of a muon source into a string of high-frequency bunches for cooling and acceleration into a muon storage ring for neutrinos. A matched solution using double-harmonic rf appears appropriate to the Palmer neutrino source scenario[3]; however, different scenarios (i. e., with single stage rf rotation) may prefer an adiabatic solution to minimize phase space dilution.

Introduction

In various possible scenarios for high-intensity muon collection following a π-producing target, the muon bunch is rf rotated to an extended length in order to reduce the momentum spread to within acceptable limits. In the muon collider scenario, the bunch has an rms bunch length of 1.5m (~10m full length) with a momentum spread of ~10% at a mean momentum of p ≅ 200 MeV/c[1], and this initial beam was also used for a version of a neutrino source.[2] For the neutrino source, other scenarios with much longer bunches (~80m full width) and smaller momentum spread[3] are obtained by extending the rf rotation section with, i. e., an induction linac section.

With a long bunch, a very low-frequency rf system is required to capture and maintain the beam in a single bunch for cooling. In ref. [1] a 20 MHz rf system is used for initial cooling. μ^+-μ^- collider scenarios require maintaining the beam in a minimum number of bunches. Such low-frequency high-gradient rf systems are likely to be technically difficult and expensive. However μ-storage ring neutrino sources do not require a small number of bunches, and only require the beam to be formed into a string of bunches which fit within the circumference of the muon storage ring.

Therefore, we propose to impose a high-frequency rf system upon the long bunches from the rf rotation system and capture the beam into a string of higher-frequency buckets that can be more affordably cooled and accelerated to storage ring energies.

The optimum parameters for this capture depends on the features of the complete scenario, in particular, the rf rotation scenario and the cooling system, and the problems in integrating the capture system with the other systems. In the present paper we present results of studies of rf capture for a variety of scenarios. In general, it is possible to capture ~2/3 of the particles in long muon bunches into acceleration and cooling system, provided there is accurate matching of longitudinal phase space throughout the system. The matching process and constraints are discussed below, and various scenarios for capture are described. Phase space dilution can be controlled by accurate phase space matching and/or adiabatic bunching.

Equations of motion and separatrices

In the present discussion we concentrate on the longitudinal motion in the capture process, and in initial simulations only longitudinal motion was included in the studies.

The equations of longitudinal motion in a linac are:

$$\frac{d\Delta E}{ds} = eV'(\cos(\phi + \phi_s) - \cos\phi_s) \cong -eV'\sin\phi_s\,\phi \qquad (1)$$

$$\frac{d\phi}{ds} = \left(\frac{1}{\beta_0} - \frac{1}{\beta}\right)\frac{2\pi}{\lambda_0} \cong \frac{1}{\beta^3\gamma^3}\frac{2\pi}{\lambda_0}\frac{\Delta E}{mc^2} \qquad (2)$$

where V' is accelerating gradient (which is zero outside rf cavities), λ_0 is the rf wavelength. This second equation should be modified in the case of a noncollinear transport by the inclusion of the nonisochronous transport element $M_{56}' \equiv 1/\gamma_t^2$ (or $\alpha_p = 1/\gamma^2 - 1/\gamma_t^2$), to

$$\frac{d\phi}{ds} \cong \frac{1}{\beta^3\gamma}\left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}\right)\frac{2\pi}{\lambda_0}\frac{\Delta E}{mc^2} = C_l\Delta E\ , \qquad (3)$$

where $M_{56}' = \eta/R$ and η is the dispersion and R is the local bending radius, and the symbol C_l is introduced as a shortened notation for the equation coefficient.

The first equation assumes the beam energy centroid E_0 is accelerated at

$$\frac{dE_0}{ds} = eV'\cos\phi_s\,. \qquad (4)$$

With ionization cooling an energy loss term equal to -dE/ds is added to this centroid motion, as well as an increase in energy spread due to energy straggling. The mean energy E_0 remains constant if $dE/ds = eV'\cos\phi_s$.

If only this mean energy loss and regain is included(no straggling), then the equations of particle motion are integrable for ΔE, ϕ, and particle trajectories move along orbits such that:

$$\frac{(\Delta E)^2}{2} = \frac{eV'\lambda_0 mc^2\beta^3\gamma}{2\pi\alpha_p}\left[(\phi_o - \phi)\cos\phi_s + (\sin(\phi + \phi_s) - \sin(\phi_0 + \phi_s))\right]\ , \quad (5)$$

where ϕ_o is a constant of a trajectory and $\Delta E(\phi)$ maps out a particle trajectory. The separatrix (separating trapped from untrapped particles) is obtained from eq. 4 with $\phi_0 = -2\phi_s$. Figure 1b shows the separatrix and an interior orbit ($\phi_0 = -\phi_s$) at "typical" ionization cooling parameters, used in cooling scenario segments: $eV' = 10$ MeV/m, $\phi_s = 60°$, $E_{kinetic} = 100$ MeV, $\alpha_p = 1/\gamma^2$, and $\lambda_o = 1.714$m (175 MHz rf). The separatrix extends over $\Delta E = \pm 40$ MeV, which is not much larger than typical energy spreads ($\Delta E_{rms} \cong 15$ MeV) in the cooling scenarios.

A useful parameter is the longitudinal motion oscillation length-the distance over which the longitudinal motion undergoes a full oscillation. For small amplitude oscillations this can be written as:

$$\lambda_{osc} = 2\pi\sqrt{\frac{\beta^3\gamma\lambda_0 mc^2}{2\pi\alpha_p eV'\sin\phi_s}}\ . \qquad (6)$$

This is 24.7m at the above parameters. Adiabatic changes must be slow compared to this length.

800 MHz example

As a first example we consider the μ^+-μ^- Collider-based μ-storage ring scenario of ref. 2. That scenario follows the Collider scenario of ref. 1 for initial collection and cooling of the $\pi\rightarrow\mu$ beam. This beam is rf rotated from the production target to obtain a $\Delta E \cong 20$ MeV, $\sigma_z = 1.5$ m bunch (9m full bunch length), using ~30MHz rf systems. The beam is then inserted into a cooling system using 20 MHz rf, which cools the beam transversely by almost an order of magnitude ($\varepsilon_{\perp,N}$ cooled from ~0.02 to 0.003 m-rad). The beam is then small enough transversely to fit within 800 MHz rf systems. For example a 5T solenoid field would set the betatron function $\beta_\perp \cong 0.26$m for 200 MeV/c muons, which would set the rms beam size at $\sigma_x \cong 2$cm for the cooled emittance. 800 MHz cavity designs with apertures of ~8cm have been developed by the muon collider collaboration, so the cavity aperture could be ~$4\sigma_x$, which is above the minimum acceptance criterion of 3σ.

The beam could then be transferred from the cooling system into a 800 MHz rf system (with ~5T solenoid focusing. A 800 MHz rf system with 20–30 MV/m can capture the large-momentum spread, ~10 m long (full-width) bunched beam into a string of 800 MHz bunches and accelerate it to higher energy, with minimal phase space dilution and beam loss from uncaptured particles.

A scenario for this capture and acceleration is described in ref. 2, an acceleration scenario can be constructed to take the beam from end of cooling to higher energies. Fig. 2 displays some simulation results of that scenario. In this example a continuous beam with mean kinetic energy of 200 MeV/c (and rms energy spread of 29 MeV) is injected into a 800 MHz linac with mean acceleration of 15 MV/m and a stable acceleration phase which ramps from 80° to 40° In 14m steps, with 10 steps in the initial acceleration scenario. This rf system captures the beam and accelerates it to ~1.29 GeV,at which ~70% of the initial distribution is captured and accelerated. The captured beam has rms phase spread of 20° (2cm), with an energy spread increased to 105 MeV, and rms emittance per bunch of 0.23 cm-GeV. This is less than the initial emittance of the full beam of 0.30 cm-GeV/bunch. Because the capture buckets are well-matched to the energy spread of the initial beam there is little emittance dilution in the capture, and mainly particle loss of uncaptured phase space.

At GeV energies and higher, muons are highly relativistic and there is little phase motion of the beam. Minimum energy spread is then obtained by on-crest acceleration In figure 2, the acceleration is continued to 10 GeV in a linac (or isochronous recirculating linac), with 20MV/m acceleration and phases shifting from 40° to -14°. In this process the energy spread increases to 0.25 GeV, while the bunch length reduces to ~1cm (see Fig. 2).

We note that this particular capture scenario is well matched to capture and acceleration of a large energy spread beam. Insertion of smaller energy spreads (i. e., $\Delta E = 0.02$ or 0.01 GeV)does not greatly reduce the final energy spreads or emittance, and beam capture rates are not greatly increased. The general result of introducing smaller-emittance initial beams is to increase the dilution of the beam phase space; final beam emittances are relatively independent of initial conditions.

175MHz -adiabatic capture

The above scenario uses 800MHz rf, which requires relatively small transverse beam sizes in the capture region. To obtain these it needs strong focusing and prior cooling of the beam, which requires a long low-frequency rf system. Also the scenario is somewhat matched to the initial section of a μ^+-μ^- Collider, and some changes in that section might be redesigned for optimal neutrino source design. A medium-frequency rf system, matched to the beam sizes of the muon beams before cooling, may be better for both cooling and initial beam acceleration.

A scenario to obtain a reoptimized neutrino source is presented in ref. [3]. In that paper a two-stage rf system using 30 MHz capture rf plus an induction linac and a short transverse cooling segment is used to obtain a long bunch with relatively small energy spread. In ref. 3 a ~80m long bunch with kinetic energy of E= 100 MeV and an rms width of ΔE = 3 MeV is obtained. At this point the beam would have a transverse emittance of $\varepsilon_{\perp,N} \cong 0.01$ m-rad, which, with focusing fields of ~1.25T, would obtain beams with an rms size of $\sigma_x \cong 7.3$cm. These beams would fit inside the apertures of 200 MHz (or less) cavities, which can have apertures up to 4× that of 800 MHz. Since ref.3 also contains a plan to use CERN 350 MHz cavities for later muon acceleration it is natural to choose a capture and cooling rf frequency that is a subharmonic (175MHz), and that frequency is used in the present study.

At E=100 MeV, the energy spread is not well matched to the rf bucket size of 175 MHz rf, and a simple insertion of the beam into high-gradient rf would greatly dilute the phase space. A gradual increase of rf field as the bunch is transported along the linac can capture the bunch with minimal phase space dilution. However, an adiabatic capture at the scenario parameters would require a relatively long length; realistic lengths (<100m) give relatively large phase space dilution.

But if the beam energy is reduced, the parameters for adiabatic capture become more practical. As an example, we present a scenario with simulation results depicted in fig. 3. The central energy is reduced to 25 MeV, where the energy loss is assumed to occur through energy loss in a material. In the energy loss the energy spread increases to 4 MeV (in this example). The beam is then inserted into a bunching and accelerating linac, where the rf voltage is gradually increased from 1MV/m to 6MV/m while the synchronous phase increases from 90° to 72° over an 84m length. In the process 70% of the beam is captured in an accelerating bucket with the central beam energy increasing to 106 MeV while the energy spread increases to 12 MeV. The longitudinal emittance per bunch of the accelerated beam is 0.14 cm-GeV, compared to 0.20cm-GeV for the uncaptured initial beam. This indicates no phase space dilution in the adiabatic capture; the emittance decrease simply represents the uncaptured portion of the beam.

The initial beam energy, linac length, and rf increase program can be varied somewhat to obtain optimized matching in this capture and energy increase sequence.

175 MHz - matched capture

Rather than an adiabatic capture (with gradual increase of the bunching field) the bunching can be designed as a matched transport which takes the longitudinal phase space from the long bunch to a shorter bunch. Since in this case the initial beam is quasi-continuous (that is, extends over many rf wavelengths), an rf system can only transport in matched format a portion of that beam; the optimization procedure is to obtain linear bunching over a maximal portion of the beam. Following a suggestion of Palmer, we consider obtaining extended linear bunching by adding a second harmonic component (350 MHz) to a primary 175 MHz acceleration.

In this section we construct and analyze some initial scenarios for matched capture. In these scenarios, a linear energy ramp is placed over an extended portion of the beam by a double-harmonic rf (sample parameters: 6MV/m at 175 MHz + (–1.5 MV/m) at 350 MHz for 3m). This is followed by a drift section (9m in this example), where the ramped beam converges toward a minimum bunch length which is longitudinally matched to the acceptance of a cooling channel. The initial cooling channel would follow. In an initial approximation this channel would have a mean energy loss with an accelerating and bunching rf; the stable phase of the accelerating rf is matched to the center of the bunch, with the mean cooling energy loss matched to the central acceleration ($dE/ds = eV' \cos \phi_s$). In an initial approximation the energy straggling through the cooling material is ignored, and we track the longitudinal capture motion through ~2 synchrotron periods (~60m).

Figure 4 shows results of some simulations of this process. The initial beam corresponds to that of scenario of ref. 3, with parameters matched to the beam at the end of the second rf rotation; a long bunch with kinetic energy 100 MeV and rms energy spread of 3MeV. The beam is bunched and matched into a 175 MHz cooling rf system with 12 MV/m of cooling rf and dE/ds = 6MV/m.

In the simulation ~83% of the initial distribution is captured into a cooling bucket. The central ~70% of the initial beam is captured into the center of the cooling bucket with little phase space dilution, while the remainder of the captured beam appearing as a filamented halo about the central core. The rms energy spread of the beam captured by the bucket is 15 MeV, while the emittance has increasedfrom 0.14 to 0.2 cm-GeV, due to the filamented halo. The core of the beam shows a more modest emittance increase from 0.10 to 0.12 cm-GeV. In practice the energy straggling in the cooling section could expand the core to fill the rf bucket, and most (but not all) of the halo would be lost with the rms energy spread increase.

The Fig. 4 simulation obtains a beam with a matched core and a relatively large extended halo within the cooling bunch. Also the initial energy spread is quite small and would require a relatively long rf rotation scheme to obtain from the initial short-bunch, large ΔE muon distribution. To explore dependence on ΔE, we simply doubled the initial energy spread (to ΔE = 6 MeV) and tracked the resulting distribution. Fig. 5 shows simulation results. The beam capture rate is almost identical to the previous case, and, since the longitudinal beam area is more closely matched to the cooling bucket acceptance, there is less phase-space dilution. The beam capture is more properly matched, with less separation into core and halo, than that seen in the previous case.

However, the beam does more completely fill the cooling rf bucket and the beam would therefore be more vulnerable to losses due the energy straggling (rms energy width growth) in the cooling absorbers. An extended transverse cooling channel for this case would have a correspondingly greater need for some longitudinal cooling (or reduction of heating) than the previous small ΔE-case. However, both cases are vulnerable to the straggling effect, and more complete simulations of the cooling channel with straggling are needed to evaluate this effect, and determine what losses may occur and what improvements in the cooling scenario are needed.

The matched bunch compression shown here is most closely appropriate to the needs of the scenario of ref. 3, and some variation of this should be used if that is the final scenario. Other scenarios may, however prefer the adiabatic match of fig. 3, particularly if the transverse cooling the the energy decrease is desirable, and minimal phase space dilution is required.

References

[1] The Muon collider collaboration, Phys. Rev. Spec. Topics Accel. Beams 081001 (1999).

[2] S. Geer, C. Johnstone and D. Neuffer, "Design Concepts for a Muon Storage Ring Neutrino Source," FERMILAB-Pub-99/121 (1999).

[3] R. Palmer, Muon Collider Note 46, 9/3/99 (1999).

Fig. 1 rf bucket – separatrix and an interior orbit (in ΔE, ϕ space) for E_{total}=205.66 MeV, V′=10MV/m, f_0=175 MHz, ϕ_s = 60°.

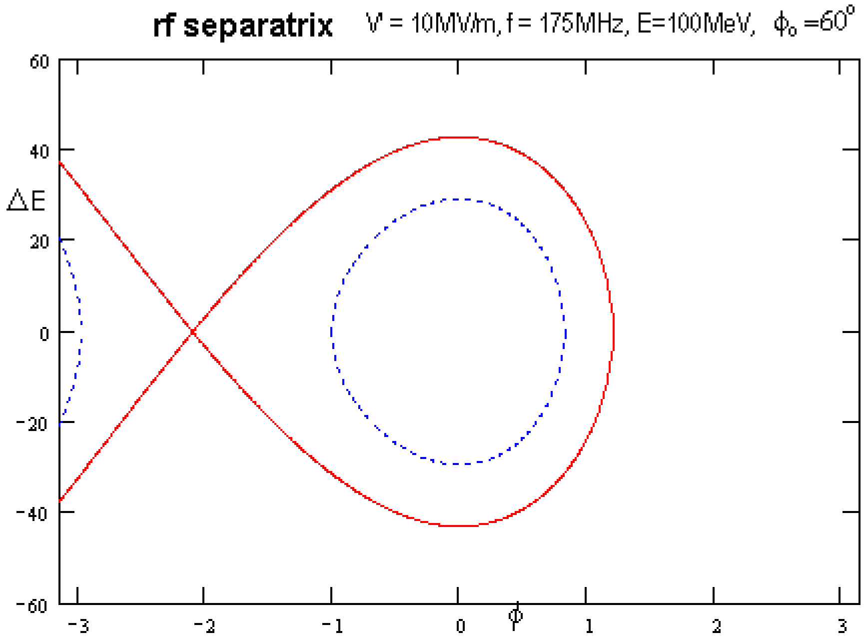

Figure 2. Capture with acceleration in rf buckets at 800 MHz.

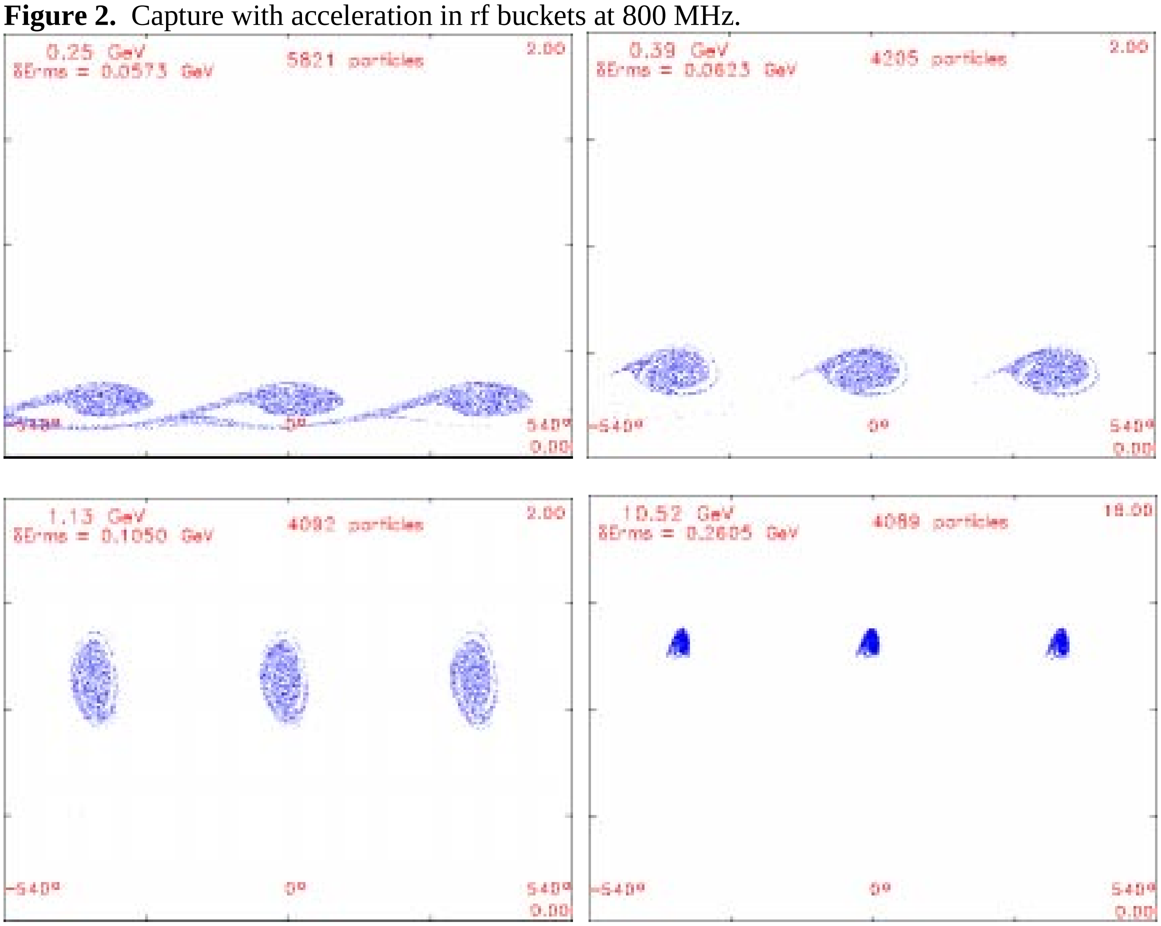

Figure 3 Adiabatic capture at 175MHz – In this simulation the beam is decelerated to 25 MeV kinetic energy and from there the beam is adiabaticly captured by a gradually increasing rf voltage.

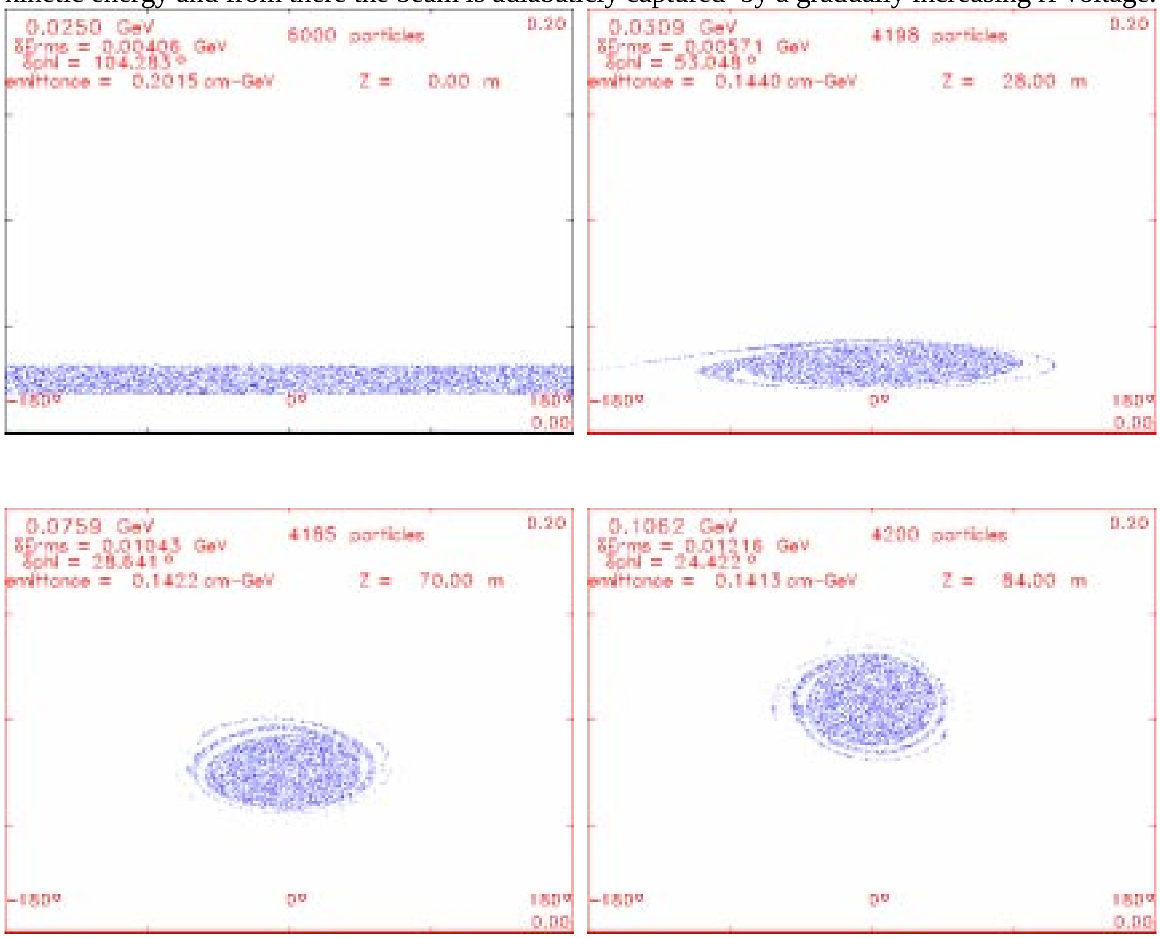

Figure 4 Matched capture with 175MHz rf (with second harmonic) of 100 MeV beam into the cooling rf bucket. The simulation tracks the beam through the bunching rf, drift, and two synchrotron oscillations of the cooling system.

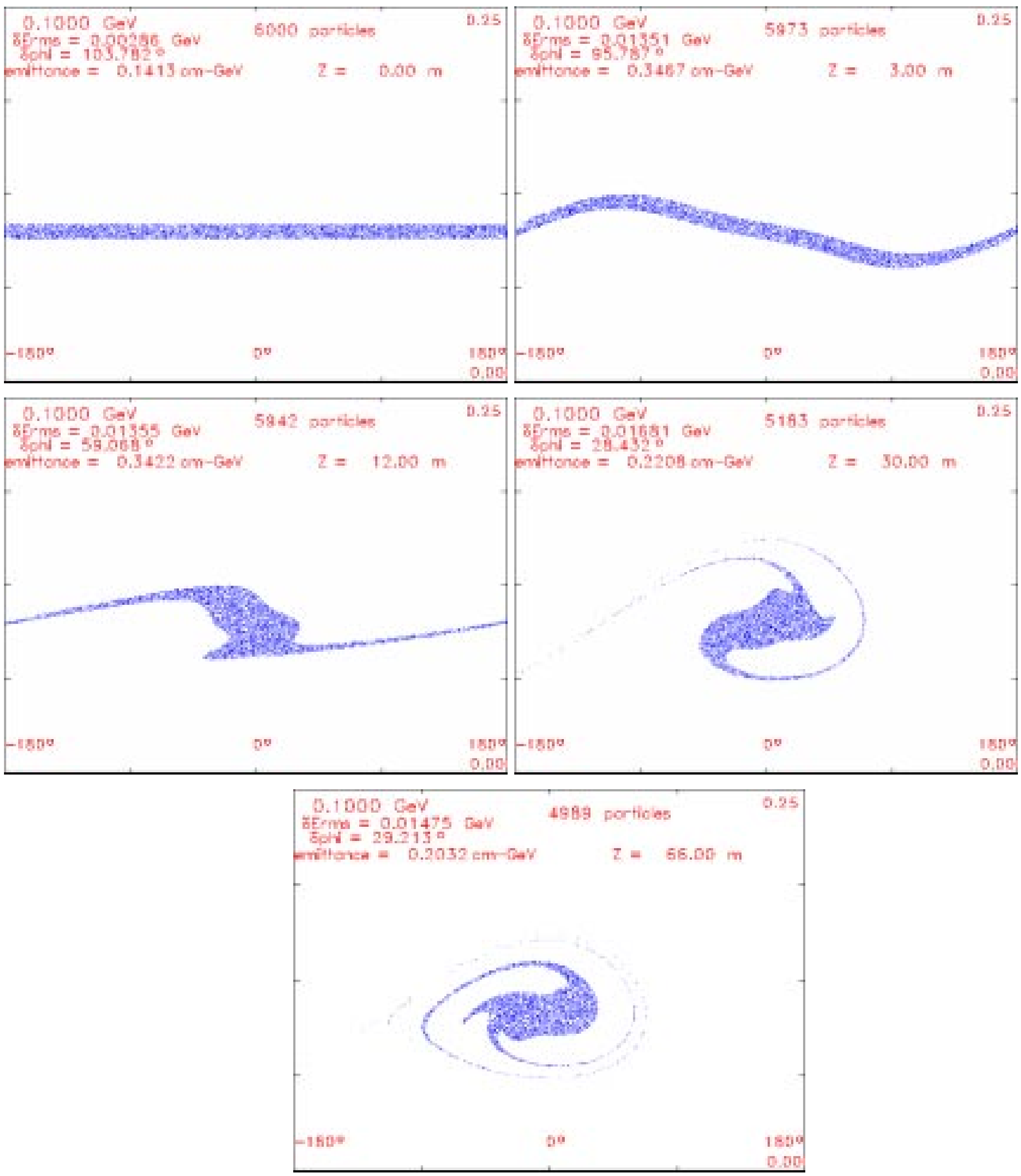

Figure 5 – matched capture with larger ΔE at the same parameters as fig. 4. Note that the initial beam is more closely matched to the cooling rf bucket size.