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Use of a Helical Channel with a Large Slip Factor for Bunch Recombination

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Abstract. Cooling scenarios for a high-luminosity Muon Collider require bunch recombination for optimal luminosity. In this report we note that the tunable slip factor of a helical transport channel (HTC) makes it a desirable component of a bunch recombiner. A HTC with a large slip factor is desirable for the bunch recombining transport, while more isochronous transport may be preferred for RF manipulations prior to bunch recombination. The effectiveness of a bunch recombiner that starts from a string of muon bunches is presented; the bunch centers are displaced in energy at 10 MeV intervals, with each bunch having 6-D emittances equivalent to that at the end of an HCC. Eleven bunches spread from 160 to 260 MeV are demonstrated to have kinematics at the end of the channel that favor them to be merged into a single bunch with greater than 99% efficiency, ignoring decays which would be roughly 3% over the 52 m long channel (L_z = 37m with pitch angle 45º) for 200 MeV muons. Future work will examine how to transform the mono-energetic bunches at the end of the HCC into those with displaced energies at which this study has started.

Introduction

For a μ^+-μ^- collider, muons from a high intensity production system must be cooled into short, intense bunches for maximal luminosity.[1, 2] The muons result from the decay of pions produced from bunches of protons focused onto a target. The muons are produced within a very large phase space that must be compressed and cooled to obtain high-luminosity parameters. Reference [3] presents a scenario for this capture and cooling process. Longitudinally, the muons are initially captured into a string of RF bunches and kept within those bunches as they asre cooled to much reduced emittances, both longitudinally and transversely. At that point, it is desirable to recombine the bunches into a single one with large longitudinal emittance. That single bunch would then receive further cooling toward collider densities.

Figure 1 shows an overview of the muon collider system and Figure 2 shows the progression of transverse and longitudinal emittances through the muon cooling system. As displayed in those figures and detailed in [2], the muons are produced with initial emittances of ~0.02m transverse and ~0.4m longitudinally (~0.03/bunch) and are captured in ~12 bunches (200MHz). The bunches are cooled to ~0.0015m transverse and ~0.002/bunch longitudinally, where they are then merged into a single bunch. (step 5 of Figure 2). The merged bunch is further cooled and emittance exchanged to ~0.07m longitudinal and 0.00003 transverse emittances, suitable for high-luminosity collisions.

The merger into a single bunch was imagined to require a very long RF and transport section, with large losses from μ decay alone. In this report we note that the large tunable slip factor that is possible in a helical transport channel enables performing this bunch combination in a much more compact system that should be more efficient. The work described here furthers the original concepts presented for bunch recombination [4] and advances the 1-D simulation previously performed into 3-D.

HTC with a Large Slip Factor for Bunch Recombination

In the previous study [4], the string of mono-energetic muons from the end of the HCC utilized a frequency slightly incommensurate with the bunch spacing for the purpose of displacing the muon bunches to different energies prior to entering the region with the large slip factor where bunch merging occurs. The first attempt at evolving the prior 1-D study to 3-D that is reported here does not address this “bunch preparation.” Instead, our efforts will focus on the main issue of bunch merging and determining its performance, assuming the preparation can be done. We also assume that momentum matching across the bunches that are at different energies will be satisfied. Specifically, for the given lattice that is designed for a particular reference energy, the bunch centers at energies different from the reference will travel at different radii and corresponding pitch angles ($\kappa = P_\perp/P_z$) to satisfy the longitudinal periodic length common to the helical channel itself, the reference muon, and muons at non-reference bunch centers. Mathematically [5],

$$p(r) = \frac{\sqrt{1+\kappa^2}}{k}\left[B_z\big|_{z-axis} - \frac{1+\kappa^2}{\kappa} b_\phi(r) \right] \qquad (1)$$

is satisfied for the reference muon as well as muons at each bunch center, where:

- p is the muon momentum
- r is the cylindrical radius
- $\kappa = P_\perp/P_z = 2\pi r/\lambda$ is the tangent of the pitch angle of the muon trajectory
- $k = 2\pi/\lambda$ where λ is the period of the helical channel
- $B_z|_{z\text{-axis}}$ is the solenoid field on axis
- $b_\varphi(r)$ is the radial dependent azimuthal dipole field

It is not possible to invert equation (1) for r(p) analytically, but numerical solutions can be found once values are assigned to all parameters. In the case studied here,

- $\kappa = 1$
- $\lambda = 1$ m; $k = 2\pi$ m^{-1}
- $r|_{\text{on reference}} = 1/(2\pi)$ m = 0.159 m
- $B_z|_{z\text{-axis}} = 5.7$ T, ($B_z|_{\text{on reference}} = 5.0$ T)
- $b_\varphi(r)|_{\text{on reference}} = 0.72$ T

- $\delta b_\varphi/\delta\rho$(ref)= -1.2 T/m.

where $\delta b_\varphi/\delta\rho$(ref) was determined by the chosen slip factor of 0.43 in [4] and the subsequent relations in equations (2) and (3) that are derived in [5]:

$$\eta_{HC} = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left[\frac{\hat{D}\kappa^2}{1+\kappa^2} - \frac{1}{\gamma^2}\right] \cong 0.43 \tag{2}$$

$$\hat{D}^{-1} = \frac{\kappa^2 + (1-\kappa^2)}{1+\kappa^2}\left[\frac{B\sqrt{1+\kappa^2}}{pk} - 1\right] - \frac{\left(1+\kappa^2\right)^{3/2}}{pk^2}\frac{\partial b}{\partial a} \cong 0.59 \tag{3}$$

Given the values above, numerical solutions for the inverse a(p) are found as shown in Figure 3(a). Momentum is converted to kinetic energy in Figure 3(b), since that is the more convenient quantity discussed in this study. Figure 4 shows event displays of trajectories of the reference muon with kinetic energy 200 MeV as well as two others with 160 and 260 MeV, both traveling at their bunch centers. Each track travels one period over the same longitudinal distance, which is the period of the helical channel itself.

The length of the channel is determined by the difference in time (5 nsec) and energy (10 MeV) of neighboring bunches by the (linearized) equation of motion:

$$\delta(c\tau) = \eta_{HC}\,\frac{\delta E}{m_\mu c^2}\,z \tag{4}$$

resulting in z = 37 m for the length of the channel where bunches merge. This is a dramatic improvement over the bunch recombination strategy presented in [3], where a 340m transport is required for bunch recombination. [6]

Within each bunch, muons are simulated to match emittance values at the end of the HCC that are more current [7] than those described in the previous section, which were based on older published literature [2]. Specifically, the spreads used in this study are:

- Δp = 2.664 MeV/c
- Δt = 0.355 nsec
- $\Delta x = \Delta y$ = 0.0885655 mm
- $\Delta(dx/dz) = \Delta(dy/dz)$ = 0.00688

Figure 5 shows four snapshots of longitudinal phase space of muon bunches with kinetic energies 100 to 300 MeV with 10 MeV and 5 nsec spacing that traverse the 37 m long helical bunch merger channel. At the end of the channel, more than 90% of the muons starting with kinetic energies 170 to 270 MeV are within 2.5 nsec of the reference and ~60-70% are within 1.25 nsec.

Addition of RF to HTC with a Large Slip Factor for Increased Bunch Recombination Efficiency

Careful examination of the longitudinal phase space evolution within bunches in Figure 5 reveal rotation due to the large slip factor, which is responsible for the bunch merging (between bunch centers) in the first place. To counteract this rotation within the bunches, we studied the effect of applying RF such that the bunch centers are at zero crossing (no net acceleration) and with voltage gradient large enough to counter the effect of enlarged time spread due to the large slip factor. To maintain zero crossing across bunches that are being merged, it is necessary to change the frequencies of the RF cavities as the muons traverse down the channel. The algorithm for the frequencies simply used the two nearest neighboring bunches, as the rational is that those nearest to the reference should matter the most. We also imposed a upper limit of 1.3 GHz (or lower limit of 0.77 nsec bunch spacing). Figure 6 shows in magenta the frequencies calculated this way for each RF cavity that is spaced 10 cm apart. The aperture for the assumed cavity/coil combination has a radius of 18 cm for frequencies 633 MHz or lower, where the 18 cm radius is driven by studies on the helical channel [8], while higher frequencies utilized an aperture determined by the lowest frequency mode (zeros of J0 Bessel function). Above 1.3 GHz which occurs at z >~ 31 m, no RF was used, so only coils with radius 18 cm are utilized. The resultant aperture along the channel is plotted in magenta in Figure 7.

Muons with distributions identical to those analyzed without RF (Figure 5) are subjected to the helical bunch merger with RF just described and the longitudinal phase space progression is shown in Figure 8. It is readily apparent that bunch merging efficiency has been increased.

Attempts at Reducing RF Infrastructure and Cost While Maintaining High Bunch Recombination Efficiency

The layout of the frequencies in the channel shown in magenta in Figure 6 along with the enhanced bunch merging efficiency (compare Figure 5(d) to Figure 8(d)) suggests possible cost savings at expense of reduced performance. In our first attempts, we avoid the obvious evolutionary approach (notice small fill factor of blue and yellow with respect to magenta coverage in Figure 6) and instead investigate if desired RF manipulations can be achieved by using only targeted frequencies 200, 400, 800, and 1300 MHz, as these are frequencies often discussed within the muon collider community. The algorithm for quantizing frequencies is that any cavity having its original frequency within 5% of a targeted frequency will have its frequency set to that of the target. The resultant frequency layout is shown in yellow in Figure 6; the corresponding aperture of this layout is shown in yellow in Figure 7. Note the smaller apertures for the 800 and 1300 MHz cavities. The voltage gradient was increased to 3.07 MV/m to compensate for the reduced fill factor such that the overall averaged gradient remained the same at 0.5 MV/m. Evolution of the longitudinal phase space of this 200/400/800/1300 MHz setup is shown in Figure 9.

A second attempt at reducing RF cost/infrastructure was to eliminate the use of the 800 and 1300 MHz cavities in the previous setup with the motivation of keeping the aperture as large as possible (see light blue points in Figure 7). RF gradient is further increased to 4.23 MV/m to maintain the same average gradient of 0.5 MV/m. Evolution of the longitudinal phase space of this 200/400 MHz setup is shown in Figure 10.

Figure 11 displays the longitudinal phase space at the end of the channels studied and provides a qualitative measure. Figure 12 attempts to quantify the bunch merging efficiency by calculating the fraction of muons originating in each bunch that are within 1.25 (2.50) nsec of the reference in Figure 12a(b) at the end of the channel. The case where each RF cavity has its own frequency (magenta) results in the highest efficiency where 11 bunches spread from 160 to 260 MeV are demonstrated to have kinematics favoring them to be merged into a single bunch at greater than 99% efficiency. Aside from this best case, it is not clear if adding RF in these cost reducing scenarios that utilize only a restrictive set of frequencies helps or not, although an evolutionary approach should produce a continuum of configurations with reduced performance and cost. The best way to score the results is to capture these muons at the end of the channel by turning on RF with 200 MHz and holding on to the muons through a helical channel with a low slip factor to demonstrate that they have indeed been caught in a single RF bucket. We have not yet designed such a helical channel with a low slip factor with κ = 1, as it requires a relatively large field (~10 T) to maintain transverse stability. Designing such a lattice (with κ reduced from 1 if necessary) is next on our agenda as it also serves the bunch preparation as well.

Summary and Future

We have presented a 3-D evolution of a previous 1-D evaluation of a bunch merging concept based on transport in a helical channel with a large slip factor. This study began with the assumption of bunches existing at different energies and time displacements that are equidistant across neighboring bunches. The concept has been enhanced to incorporate RF to counteract rotation within each bunch with result for 11 bunches spread from 160 to 260 MeV in 10 MeV intervals to have kinematics favoring them to be merged into a single bunch at the end of the channel with greater than 99% efficiency. Attempts to drastically reduce RF cost/infrastructure were also carried out by using only a restrictive set of frequencies, but any resultant benefit from the added cost was inconclusive, although an evolutionary approach should produce a continuum of configurations with reduced performance and cost. Quantifying the results requires enhancements to determine whether muons are caught in RF buckets.

The enhancement requires a helical channel with a low slip factor and κ equal to or near 1, which will also help serve in the design of the bunch preparation (initial conditions from which this study began). Thus, the next step is to design a helical channel with a low slip factor and κ equal to or near 1 that has reasonable magnetic field values that will complete the demonstration of bunch merging in a helical channel over lengths much shorter than competing techniques, which are much longer and cost more in dollars and muon decay losses.

Beyond demonstrating that this technique is more efficient than its competitors, optimization of this channel will include varying the slip factor η and its associated channel length to find an optimal bunch merger rate. Also, based on other work not shown, there may be further gains to be made by varying Bz of the helical channel and investigating whether or not increasing Bz simply raises the kinetic energy operating point of the reference or if it widens the useful dynamic kinematic range of muon bunches to be merged.

Acknowledgments

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8. K. Yonehara, private communication.

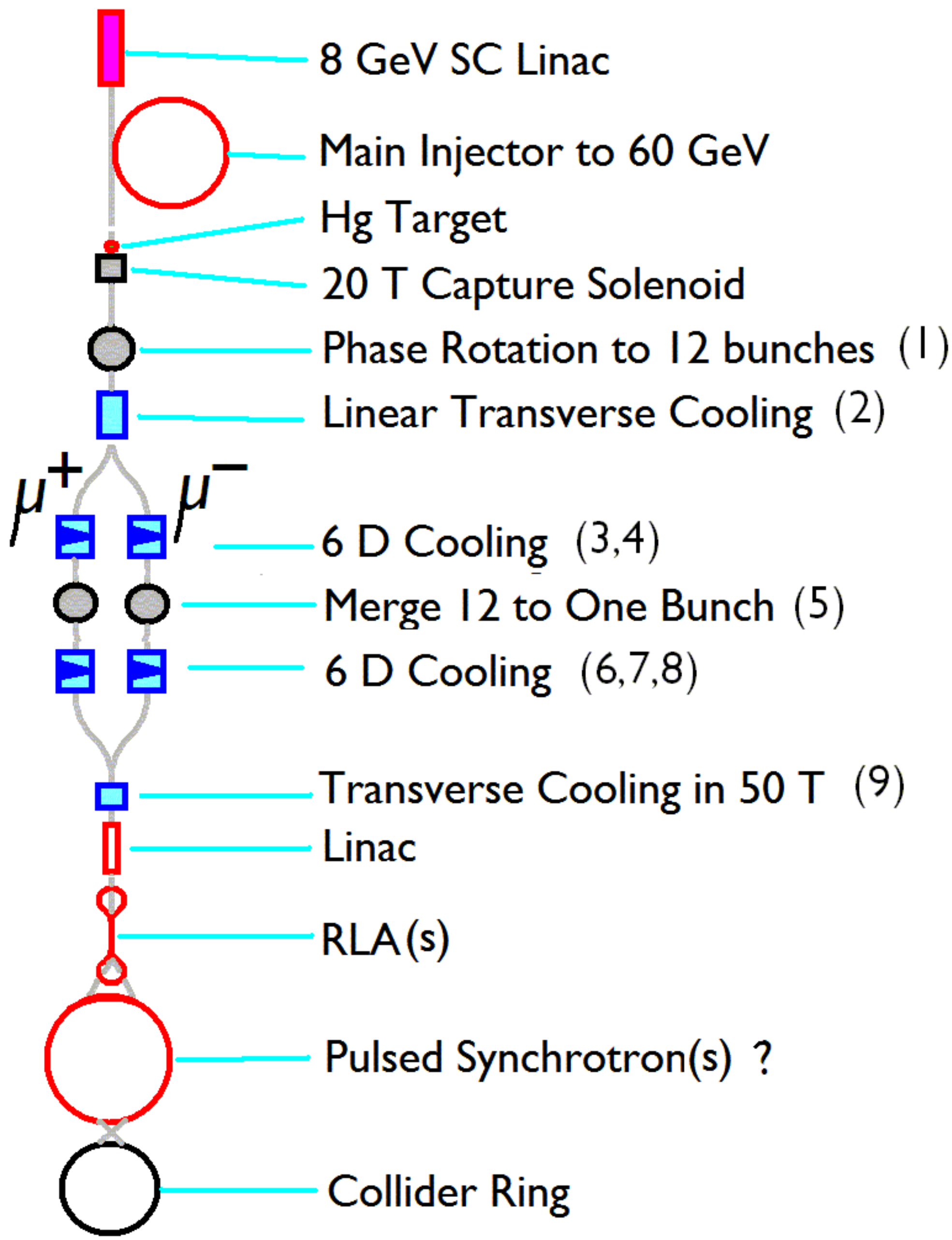


Figure 1. An overview of the muon collider system, showing the muon production, phase-energy rotation and cooling of 12 muon bunches, bunch recombination to 1 bunch, followed by more cooling and acceleration into a storage ring. [2]

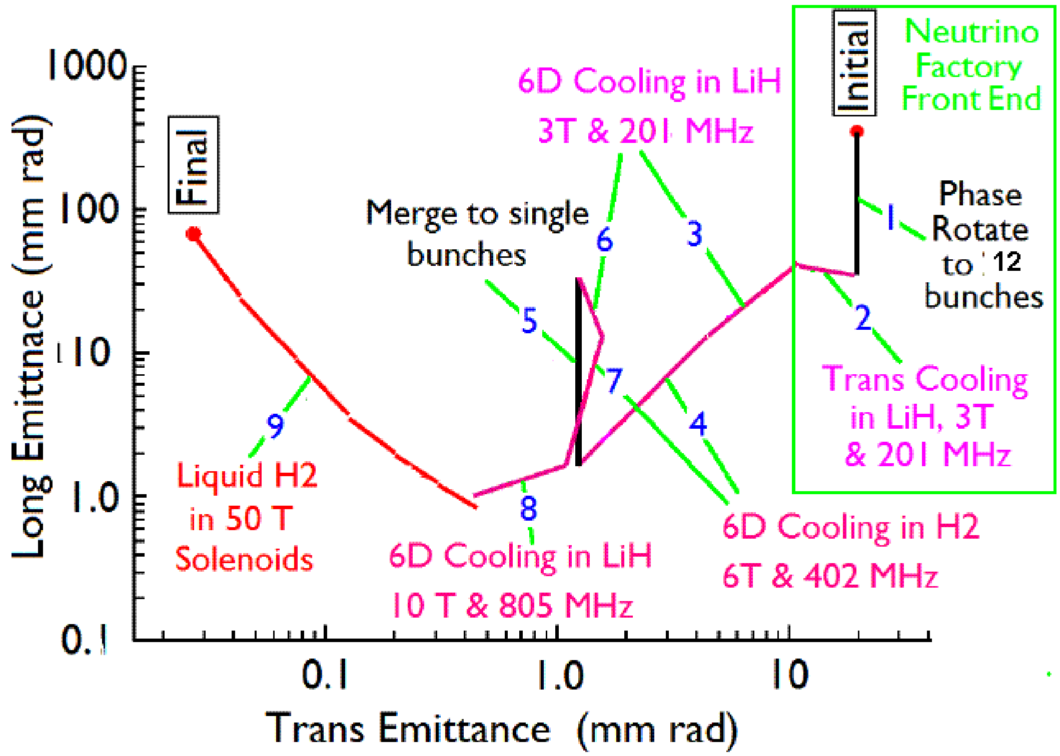
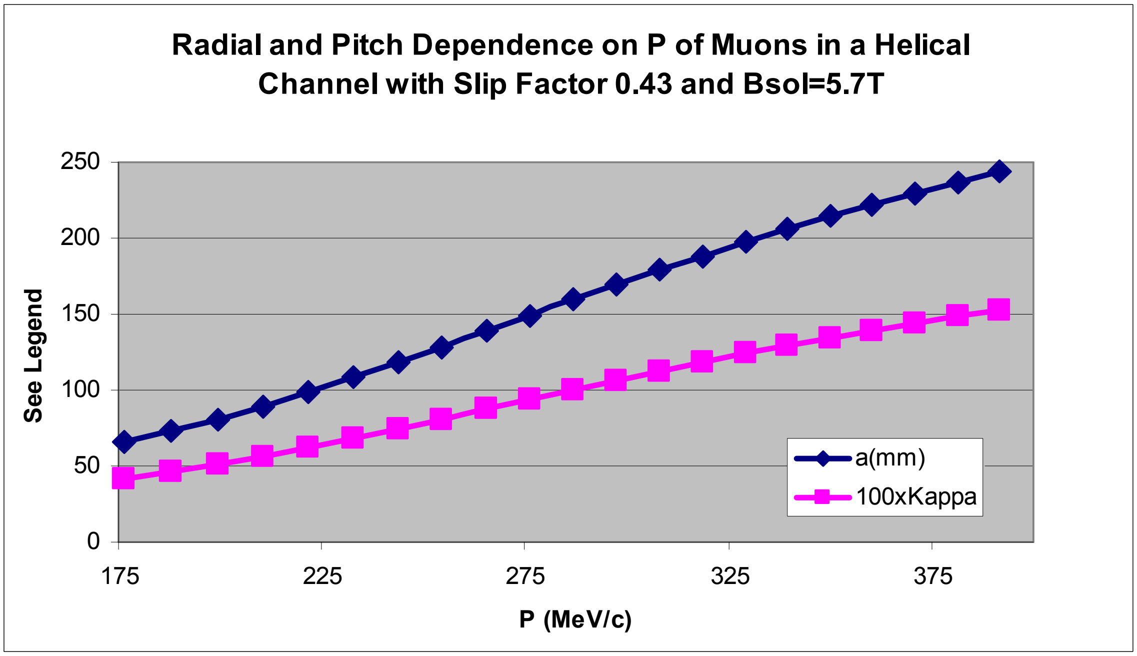
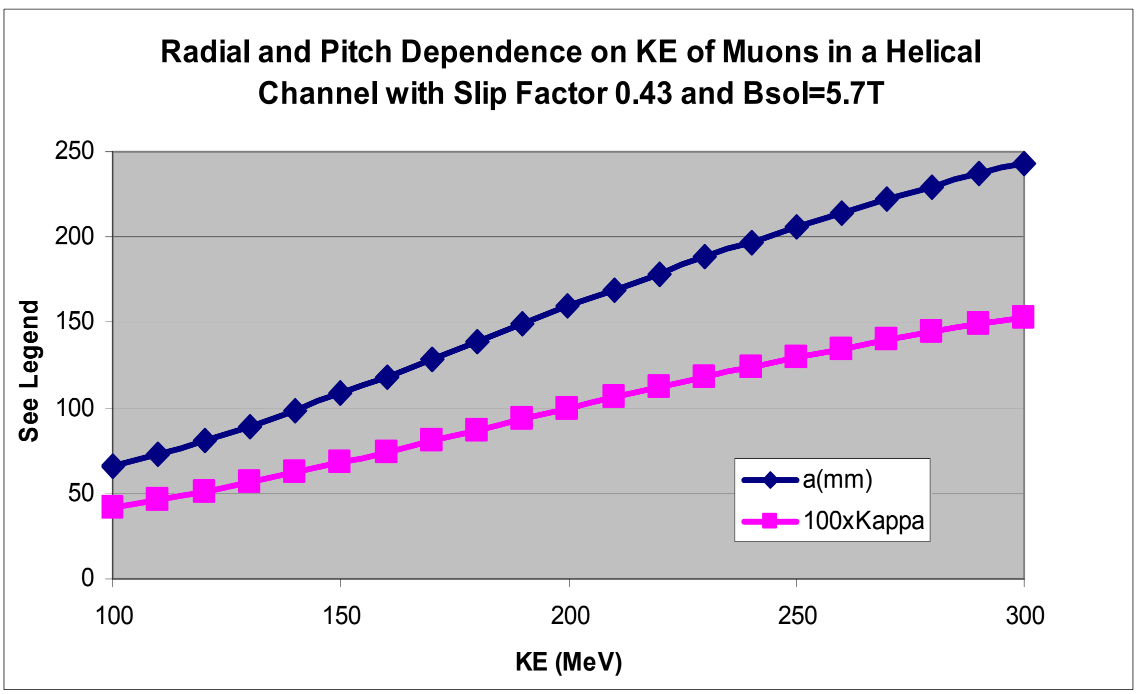


Figure 2. Progression of transverse and longitudinal emittances through a cooling system for a muon collider. The initial emittances of ~0.02m transverse and 0.4m longitudinally (~0.03/bunch) are captured in ~12 bunches, then cooled to ~0.001m transverse and ~0.002m/bunch longitudinally. The merger of muon bunches to a single bunch is shown as step 5. The merged bunch is further cooled to ~0.001m longitudinal and 0.0004 transverse, and then cooled and emittance exchanged to ~0.07m longitudinal and 0.00003 transverse emittances. [2]



(a)



(b)

Figure 3. Radii and κ of bunch centers in a helical channel that is momentum matched as functions of momentum in (a) and kinetic energy in (b).

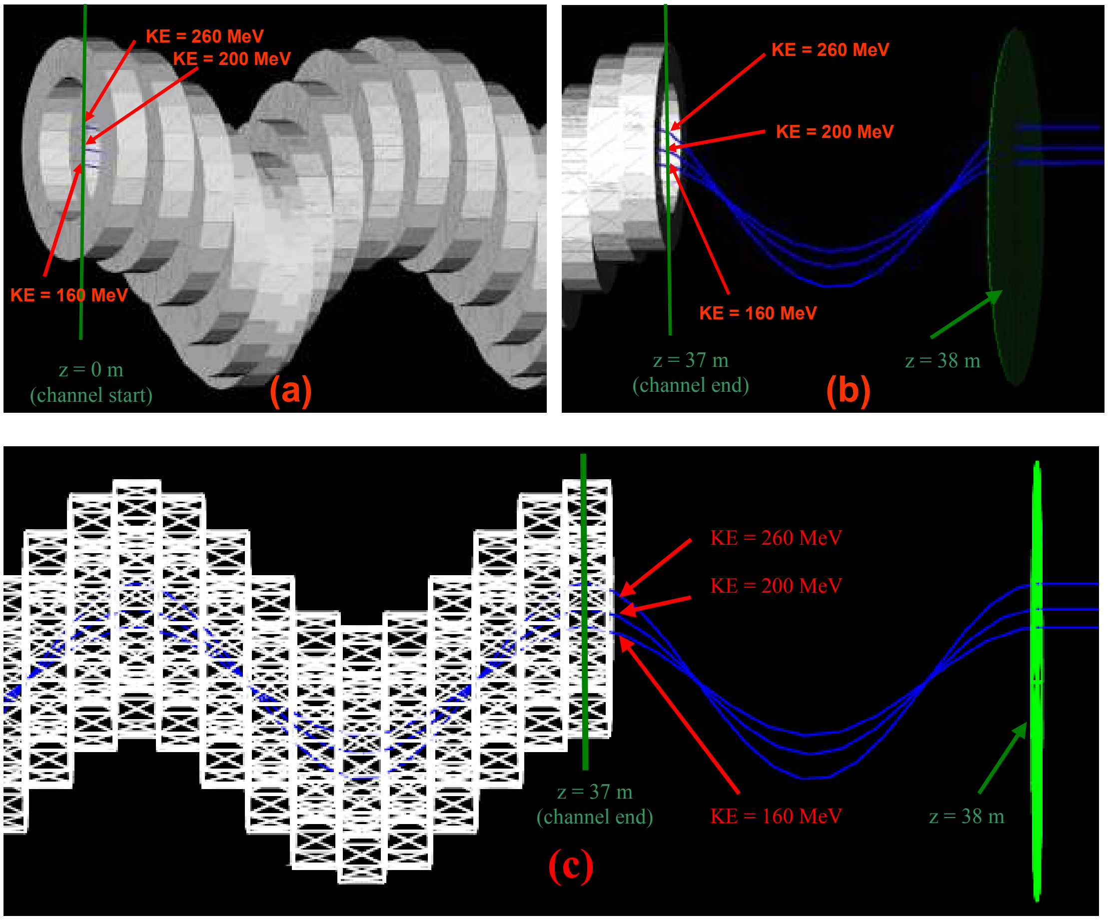


Figure 4. Display of muons with kinetic energies 160, 200, and 260 MeV that are momentum matched to the helical channel at the start in (a) and end in (b) and (c). Muon tracks past the end at z=37 m are shown to better illustrate the trajectories and the green disk in (b) and (c) is placed at z = 38 m, which is one period past the channel end; each muon travels a full helix period between z = 37 m and 38 m.

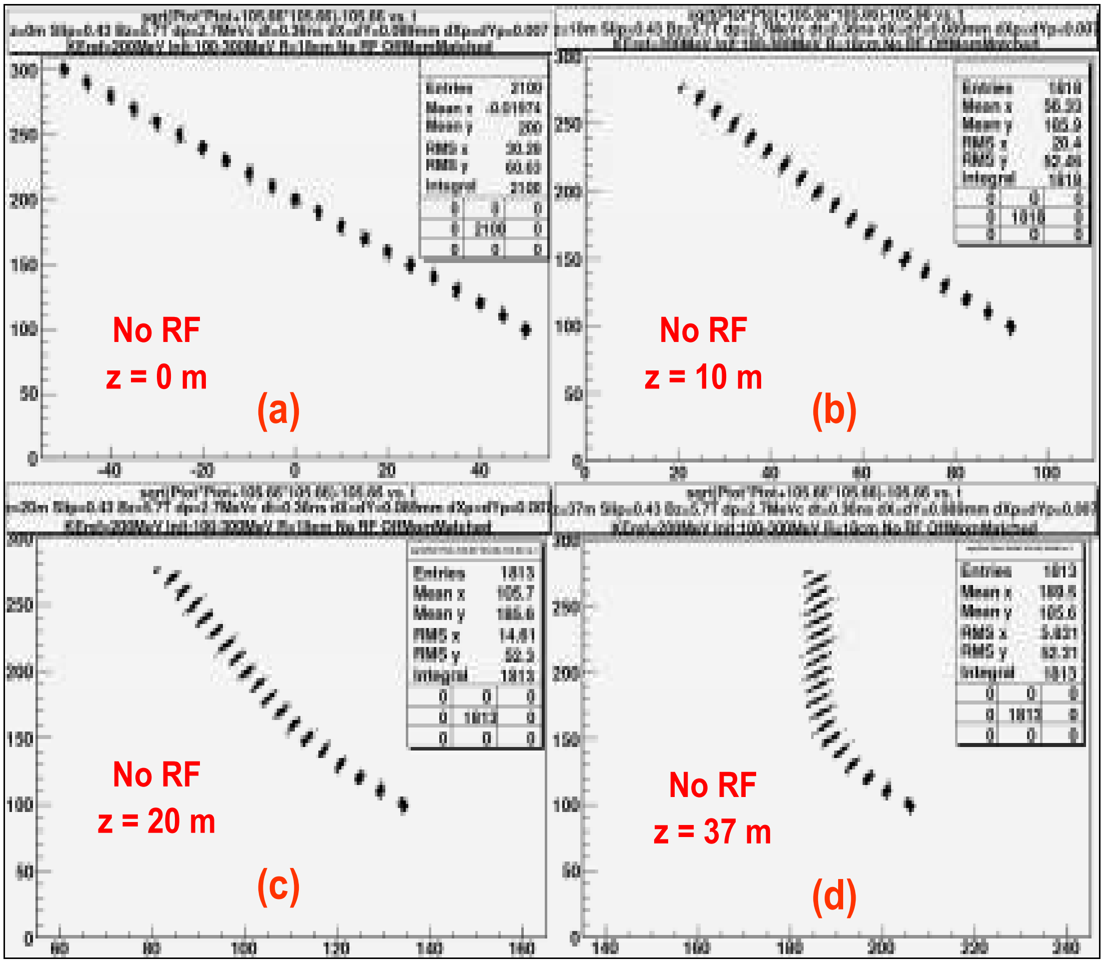


Figure 5: Kinetic energy (MeV) versus time (nsec) for muon bunches in the helical channel without RF at the start at z = 0 m in (a), z = 10 m in (b), z = 20 m in (c), and end of channel at z = 37 m in (d). Neighboring bunches are separated by 10 MeV and 5 nsec with energy range from 100 to 300 MeV.

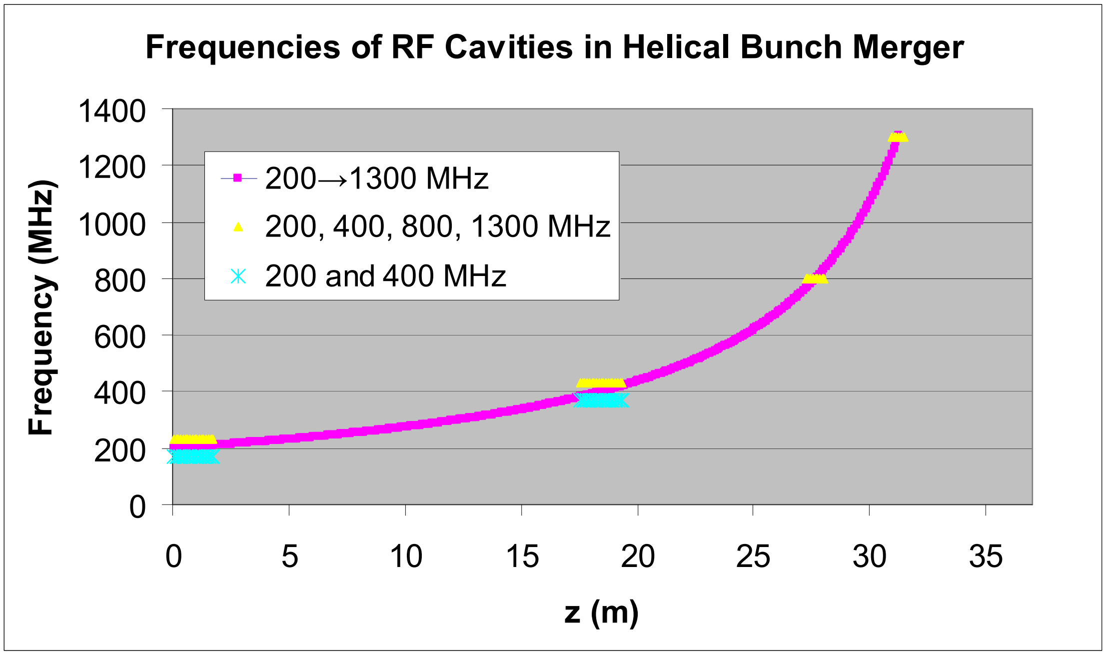


Figure 6: Frequencies of RF cavities versus longitudinal position in the Helical Bunch Merger Channel for the various RF schemes. Frequencies at 200 and 400 MHz have been displaced for sake of clarity for displaying other nearby frequencies.

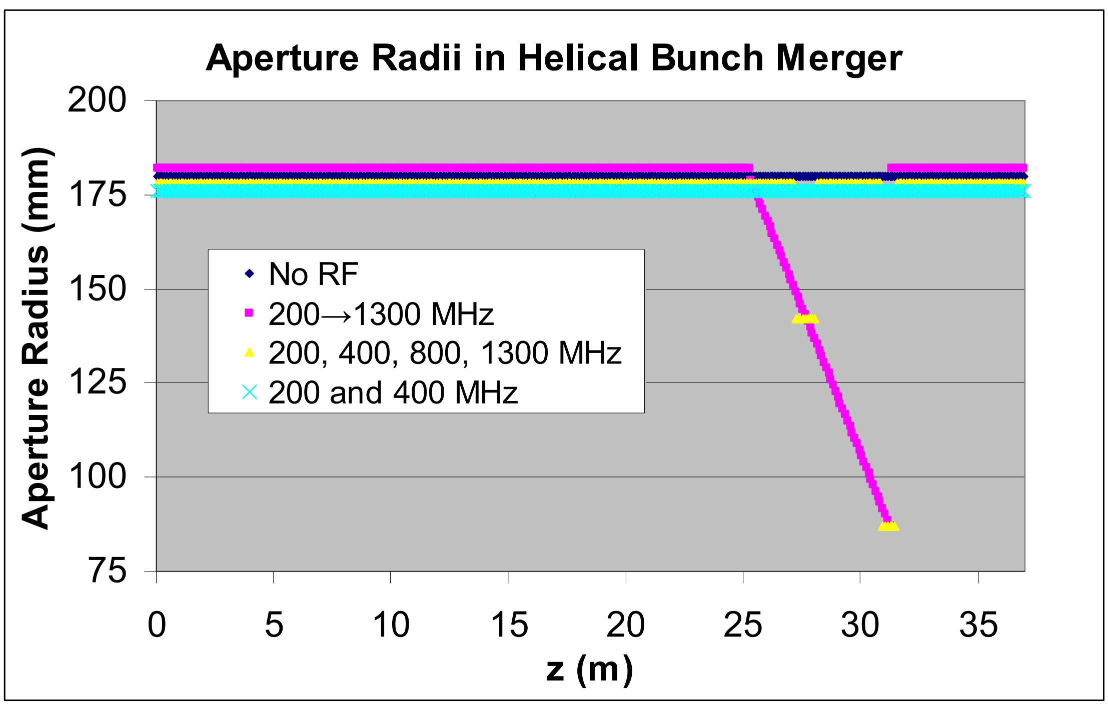


Figure 7: Aperture radii versus longitudinal position in the Helical Bunch Merger Channel for cases without RF as well as the RF schemes studied. Radii at 180 mm have been displaced for sake of clarity to elucidate the schemes which share that radius.

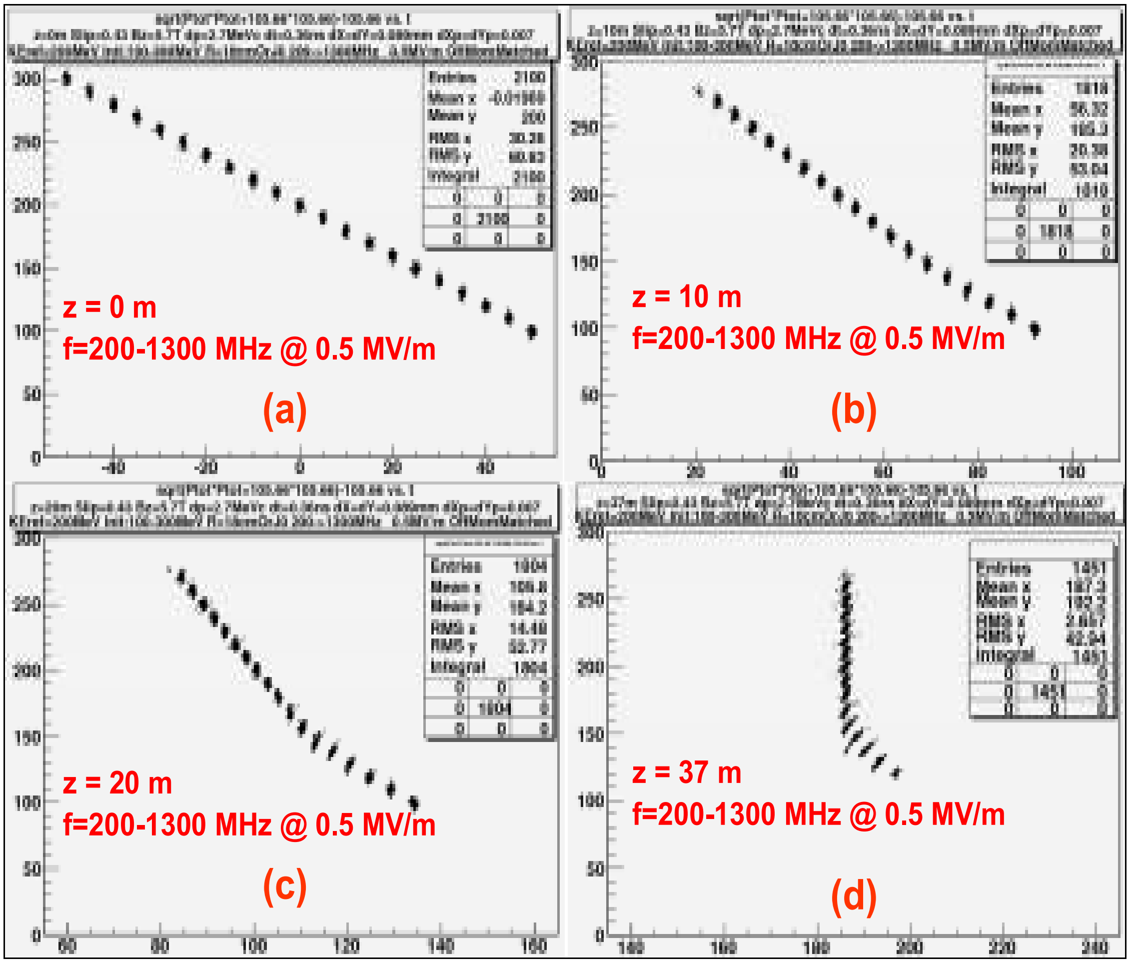


Figure 8: Kinetic energy (MeV) versus time (nsec) for muon bunches in the helical channel with RF where each cavity has frequency tuned for reference neighbors with frequencies between 200 to 1300 MHz. KE vs. t at channel start at z = 0 m in (a), z = 10 m in (b), z = 20 m in (c), and end of channel at z = 37 m in (d).

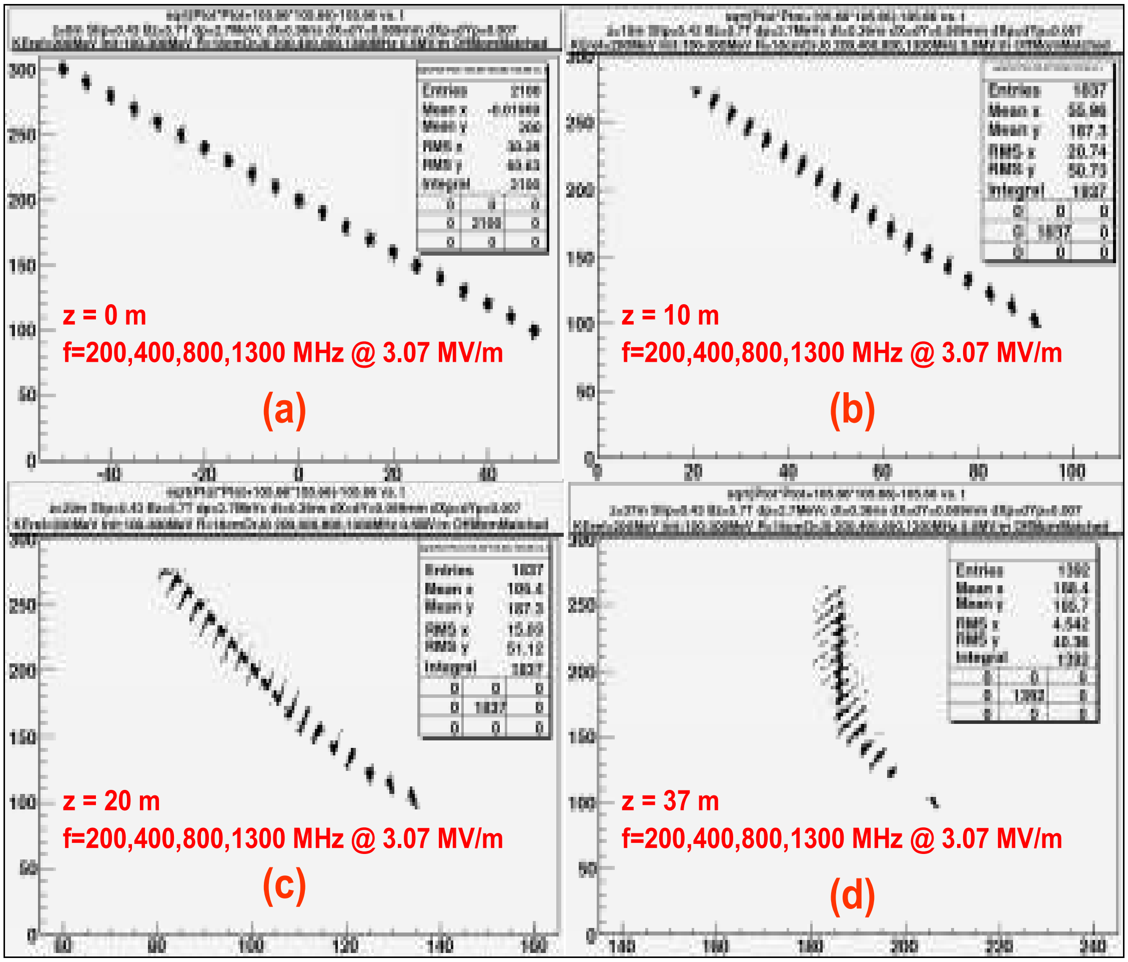


Figure 9: Kinetic energy (MeV) versus time (nsec) for muon bunches in the helical channel with RF where only discrete frequencies of 200, 400, 800, and 1300 MHz are used. KE vs. t at channel start at z = 0 m in (a), z = 10 m in (b), z = 20 m in (c), and end of channel at z = 37 m in (d).

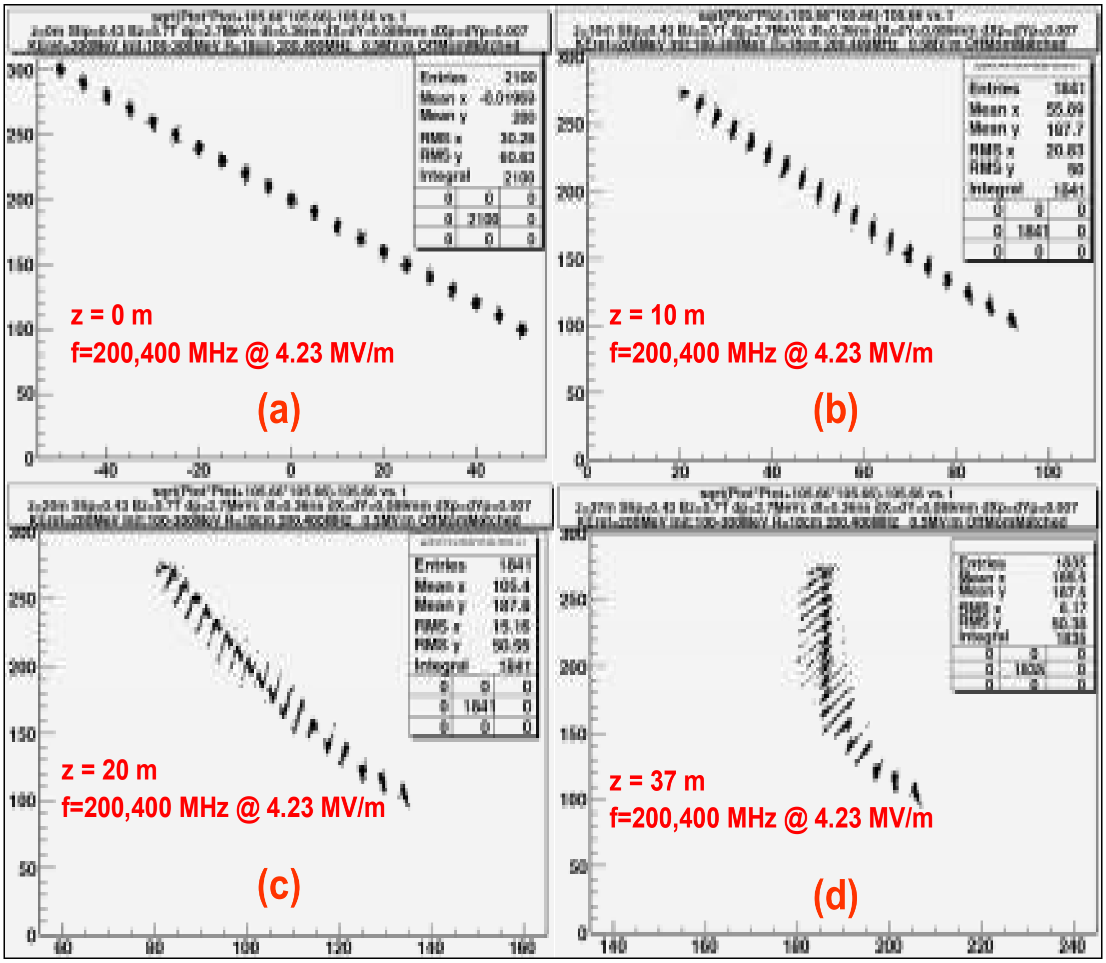


Figure 10: Kinetic energy (MeV) versus time (nsec) for muon bunches in the helical channel with RF where only discrete frequencies of 200 and 400 MHz are used. KE vs. t at channel start at z = 0 m in (a), z = 10 m in (b), z = 20 m in (c), and end of channel at z = 37 m in (d).

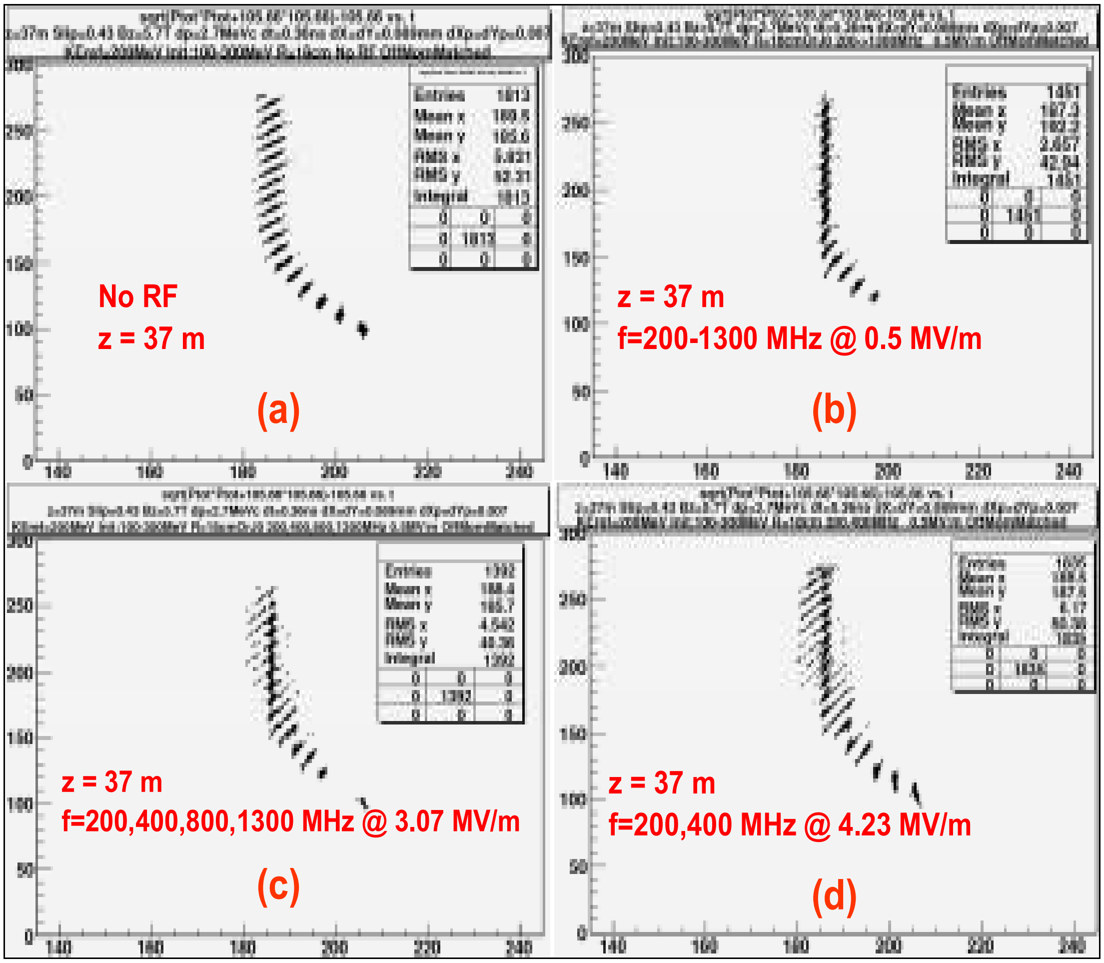
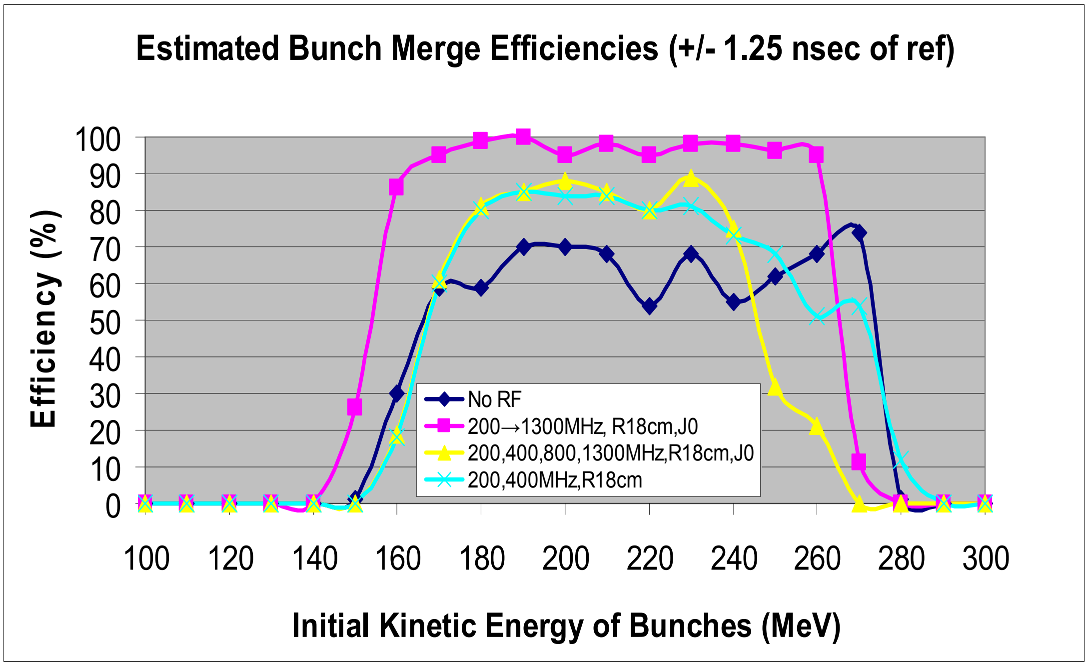
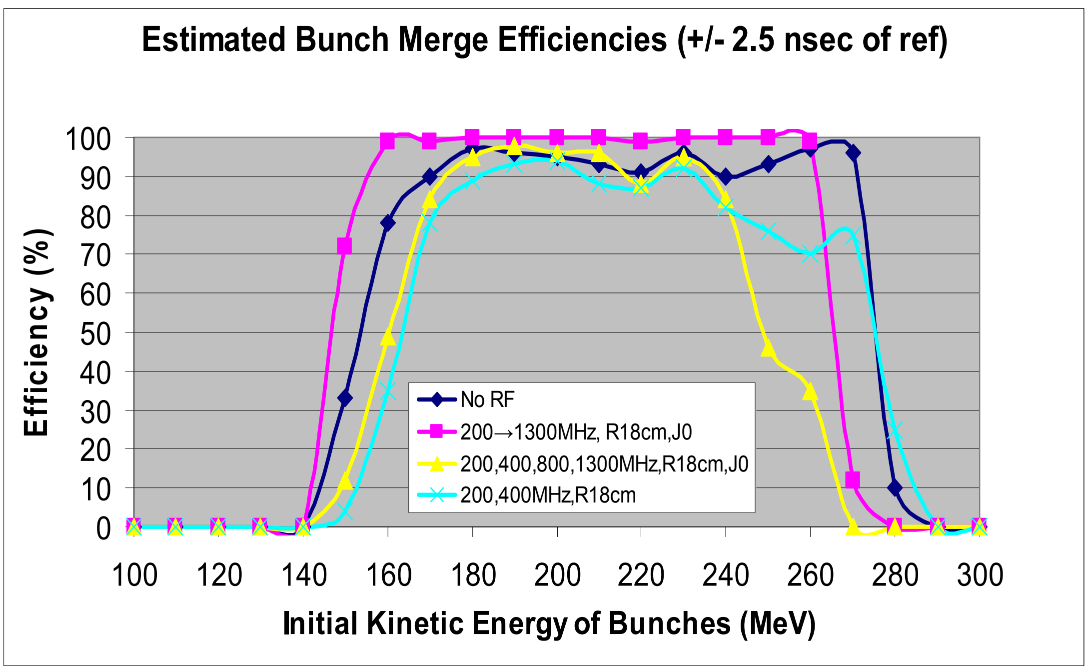


Figure 11: Kinetic energy (MeV) versus time (nsec) for muon bunches at the end of the helical channel without RF and the various RF schemes.



(a)



(b)

Figure 12: Estimated efficiency of helical bunch merger at end of channel for muon bunches with various kinetic energies at start of the channel without RF as well as the various RF schemes. Estimated bunch merge efficiency is based on the fraction of muons that are within the stated timing window centered at the reference muon.