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Bunch Coalescing in a Helical Channel

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Abstract. A high-luminosity Muon Collider requires bunch recombination for optimal luminosity. In this report, we take advantage of the large slip factor in a helical transport channel (HTC) to coalesce bunches of muons into a single one over a shorter distance than can be achieved over a straight channel. To reduce complications associated with matching out of upstream and into downstream subsystems, we designed the bunch coalescing subsystem with a value for the slip factor that is representative of an existing helical cooling channel (HCC) design. Alternate designs have been developed to investigate the tradeoffs between simplifications in reducing the number of RF cavities versus performance. All key components of the bunch coalescing subsystem have been simulated in 3-D. Excluding the initial acceleration of all bunches to the desired operating energy (not simulated), the coalescing subsystem that is designed to merge 9 bunches has a horizontal length of ~105m and is able to achieve efficiencies of 99.7%, 98.4%, and 94.2% for 9, 11, and 13 bunches, respectively, where each bunch has emittances expected at the end of a HCC. Designs to merge more bunches will be longer, but should achieve comparable efficiencies. The simplified designs incorporating fill factors for RF cavities of ~25% and ~50% obtained efficiencies of 96%, 94-95%, and 90-91% for 9, 11, and 13 bunches, respectively. The efficiencies above do not include decay losses, which would be ~8% for muons with kinetic energy of 200 MeV. Following the RF capture into a single bunch, a series of radial wedges in the helical channel may be needed to reduce the longitudinal emittance (via emittance exchange afforded by allowable growth of transverse emittance) as well as reduce the operating energy to that of the second HCC. The amount of longitudinal cooling and energy reduction for the single bunch is dependent on the acceptance of the downstream HCC, which is yet to be designed. Results of the coalesced single bunch provide a starting point for an iterative process between the bunch coalescer and HCC with aim to create the shortest and most efficient integrated design that transforms a string of hot muon bunches into a single cooled bunch that is ready for extreme cooling (emittance exchange), acceleration, and collision with its particle counterpart at the energy frontier.

Table of Contents

1 Introduction

A muon collider provides a possible path for the U.S. to return to the high energy leadership. One configuration of a muon collider [1] is shown in Figure 1. The start of the muon collider is considered at the impingement of protons on a high-Z target, a mercury jet in this case. Pions produced at the target are captured in a tapered solenoidal field (not shown at the target) and are allowed to simultaneously decay into muons and drift to develop an energy-time correlation. Muons are adiabatically bunched into RF buckets followed by an energy-phase rotation that utilizes the energy-time correlation that was created upstream; the output from the energy-phase rotation is a string of mono-energetic bunches of muons. The string of bunches are cooled transversely in the Linear Cooler, followed by further 6D cooling in the Helical Cooler. The cooled string of bunches are then merged into a single bunch, which is re-cooled in 6D to produce a single bunch of muons that is ready for the final stage of cooling that typically involves emittance exchange to reduce transverse emittance to maximize luminosity at cost of growing the longitudinal emittance. Beyond all cooling stages are sets of accelerators to take the muons to their final energy after which they will collide in ~1000 turns. The study reported here exploits the benefits of a helical channel by investigating the coalescing of muon bunches in such a channel.

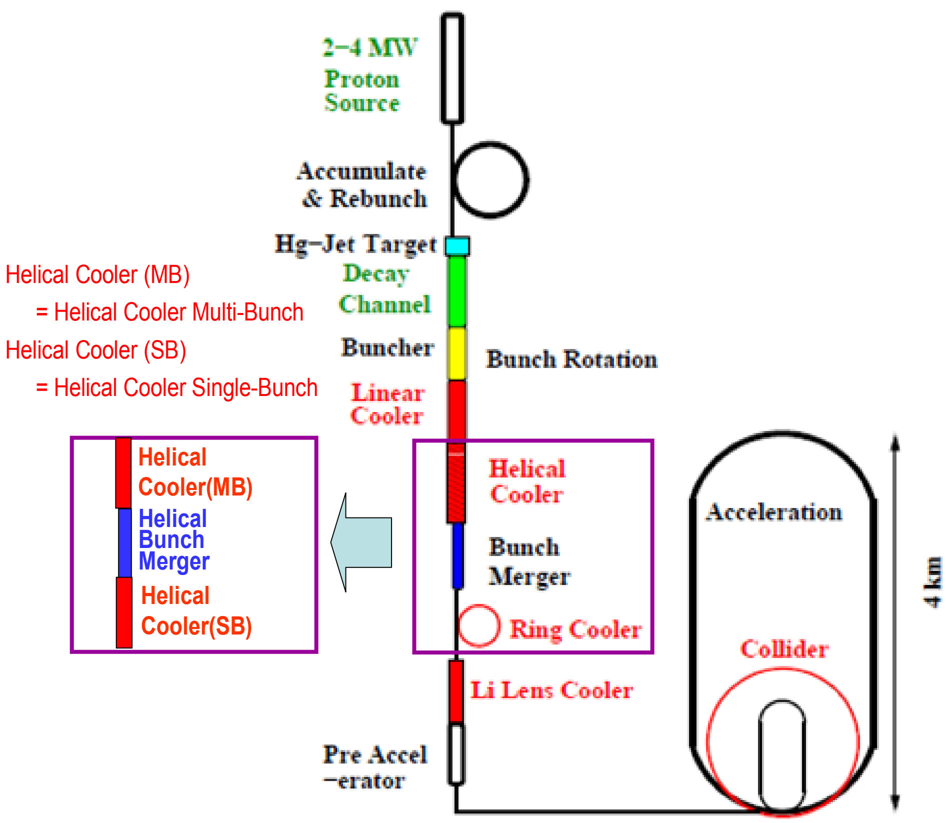

Figure 1: Muon Collider layout. To the right of the arrow is a configuration of a muon collider [1]. This study investigates coalescing muon bunches in a helical channel with assumption of a multi-bunch Helical Cooler upstream and a single-bunch Helical Cooler downstream of the Helical Bunch Merger (or Coalescer) , as shown to the left of the arrow.

1.1 Motivation and Advantages of Bunch Coalescing in a Helical Channel

There are two primary reasons for coalescing muon bunches in a helical channel: 1) Shorter bunch merging distances can be achieved than those of a straight channel and 2) It provides a natural match into and out of a helical cooling channel (HCC).

Shorter coalescing distances compared to a straight channel is afforded by the larger absolute values for the slip factor attainable in a helical channel rather than relying solely on kinematics of the particles. Table 1 quantifies the difference for muons with a variety of kinetic energies. We note that in a straight configuration, bunch coalescing can be achieved with channels of decreasing length and decay losses for lower energies, but are still longer and have more decays than the helical channel for energies of interest.

Bunch coalescing in a helical channel provides the most natural match into and out of the cooling channel that allows the maximum cooling per unit distance, the helical cooling channel (HCC). Because the HCC implements continuous emittance exchange and transverse cooling (resulting in net longitudinal cooling), it is able to cool muons over a shorter distance compared to competing cooling schemes. Thus, we seek to leverage together the channels that are shortest for cooling and bunch coalescing: the HCC and helical bunch coalescing channel.

Table 1: Coalescing distances muon decay losses in straight and helical channels for muon bunches that have initial energy-distance relationship of dE/d(ct) = 4.09 MeV/m. The slip factor for the helical channel is $\eta_{HC} = 0.43$. $L_{z,straight} = (\beta^3\gamma\ m_\mu)/\ (\eta/(dE/d(ct))|_{bunches})$ is the distance of the straight channel. $L_{z,HC} = 1/\eta_{HC}\ m_\mu/(dE/d(ct))|_{bunches}$ is the longitudinal distance of the helical channel. L_{HC} is the length along the reference, which follows a helical trajectory. The energy of the reference particle is 200 MeV (kinetic) and its row is highlighted. The drift lengths here pertain to the drift portions that align the bunches, given the initial energy-distance relationship amongst the bunches; bunch preparation to put different bunches at different energies and downstream RF capture are not included.

KE (MeV)	P (MeV/c^2)	Straight: $\eta=(1/\gamma_T^2 - 1/\gamma^2)$	$L_{z,straight}$ (m)	$L_{z,HC}$ (m)	L_{HC} (m)	Decay loss in a straight (%)	Decay loss in a helical channel (%)
100	176	-0.264	120	60	85	10	7.4
120	199	-0.219	174	60	85	13	6.6
150	233	-0.171	277	60	85	17	5.7
175	260	-0.142	385	60	85	21	5.1
200	287	-0.119	517	60	85	25	4.6
225	313	-0.102	674	60	85	29	4.3
250	340	-0.088	858	60	85	33	3.9

Going beyond the choice to use the shortest bunch coalescing scheme that fits the shortest cooling channel, this study incorporates a design decision that further reduces or eliminates matching sections and their associated inefficiencies by investigating bunch coalescing efficiencies using magnetic fields representative of an existing HCC design [2,3]. However, the design of the upstream multi-bunch HCC and downstream single bunch HCC are not addressed in this investigation.

1.2 Initial Conditions

As mentioned above, the design goal is to have a bunch coalescing subsystem that naturally fits between an upstream HCC that cools multiple bunches of muons and a downstream HCC that cools a single bunch of muons. Therefore, we will take the expected output emittances from a known HCC design [2,3] as our initial distribution of muons. In particular, we will use normalized emittances $\varepsilon_{T,N} = 0.34$ mm-rad and $\varepsilon_{L,N} = 1.1$ mm, as shown in Table 2. Also highlighted in the table are the input emittances to the HCC. These values will be used as artificial emittance targets at the end of the bunch coalescer. They are artificial, since the downstream HCC will likely have a different design that is optimized for accepting muons out of the bunch coalescer.

Table 2: HCC Parameters

Target emittances at end of coalescing subsystem in this study

HCC Parameters for 200 MHz case

	Z	b	b'	bz	ν	E	κ	λ	ε_{μ}	ε_T	ε_L	ε_{6D}
unit	m	T	T/m	T	GHz	MV/m		m		mm rad	mm	mm³
	Channel length	@ ref	@ ref	@ ref	RF		$p_{\perp}/p_z$		Transmission	RMS normalized		
0									1.0	21	23	8900
1	100	1.2	-0.21	-4.2	0.2	16	1.0	1.0	0.75	1.9	4.3	9.4
2	91	1.8	-0.42	-6.0	0.4	16	1.0	0.7	0.62	0.86	1.8	0.99
3	86	3.1	-1.29	-10.7	0.8	16	1.0	0.4	0.41	0.32	1.0	0.08
4	24	4.2	-2.29	-14.0	0.8	16	1.0	0.3	0.38	0.34	1.1	0.07

Assumed emittances at start of coalescing subsystem

9

2 Helical Bunch Coalescing Subsystem

The helical channel parameter that drives the design for bunch coalescing is the slip factor. The flexibility of the helical channel allows for this slip factor to be varied along the channel. In fact, an earlier study [4] performed a 1-D simulation of the bunch preparation that puts different bunches at different energies in a channel with a small slip factor of 0.05, followed by a drift portion that aligns the bunches in time that used a larger slip factor of 0.43. A later study [5] simulated in 3-D the drift portion, assuming that the bunches could be properly prepared with suitable energy correlations. However, as mentioned above, to reduce or eliminate matching sections into and out of HCCs and avoid their associated inefficiencies, this study will perform all operations using magnetic fields representative of an existing HCC design, whose slip factor is 0.43. These operations performed between the end of the upstream multiple bunch HCC and downstream single bunch HCC are:

1. Acceleration to desired initial energy
   - Accelerates bunches out of HCC to higher energies where bunch coalescing works better.
2. Creation of linear energy-time correlation
   - Creates an energy-time correlation amongst the bunches, so that they can be aligned at the end of the drift region.

3. Drift to align bunches in time
    - The bunches become aligned during the drift, as described by its slip factor.
4. Capture into a single RF bucket
    - RF is turned on at the point where the bunches are aligned.
5. Deceleration and longitudinal cooling
    - The bunches will likely need to be decelerated to match the downstream single bunch HCC.
    - Longitudinal cooling may also be needed.

Values of helical channel parameters, other than slip factor, used throughout the above operations are:

- λ = longitudinal spatial period = 1 m
- r_{ref} = reference radius = 16 cm
- $\kappa = p_T/p_z = 2\pi r_{ref}/\lambda = 1$ = tangent of the pitch angle of the reference trajectory
- $B_z|_{z\text{-axis}} = 5.7$ T, $B_z|_{\text{on reference}} = 5.0$ T
- $b_\varphi(r)|_{\text{on reference}} = 0.72$ T
- $\delta b_\varphi/\delta\rho(\text{ref}) = -1.2$ T/m.

where value of $B_z|_{\text{on reference}}$ was chosen for transverse stability and $\delta b_\varphi/\delta\rho(\text{ref})$ was determined by the chosen slip factor of 0.43 in [4] and the subsequent relations in equations (1) and (2) that are derived in [6]:

$$\eta_{HC} = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left[\frac{\hat{D}\kappa^2}{1+\kappa^2} - \frac{1}{\gamma^2}\right] \cong 0.43 \qquad (1)$$

$$\hat{D}^{-1} = \frac{\kappa^2 + (1-\kappa^2)}{1+\kappa^2}\left[\frac{B\sqrt{1+\kappa^2}}{pk} - 1\right] - \frac{\left(1+\kappa^2\right)^{3/2}}{pk^2}\frac{\partial b}{\partial a} \cong 0.59 \qquad (2)$$

2.1 Acceleration to Desired Initial Energy

Bunch coalescing works best at higher energies due to the more linear relationship between the relativistic parameter β and energy. This is best illustrated via simulations, as shown in Figure 2. The simulation used a helical channel with slip factor of 0.43 and bunch coalescing appears to be more efficient for muons with kinetic energies between ~170 to ~270 MeV. (Muons with energies above this range were lost due to instabilities in the channel.) As muon bunches that exit the upstream HCC have kinetic energy of 120 MeV (p = 200 MeV/c), we will assume that uniform acceleration of all bunches to kinetic energy of 200 MeV (p = 287 MeV/c) is possible without growing the emittances. This shift in energy has not been simulated in this study, but we assume its successful operation going forward.

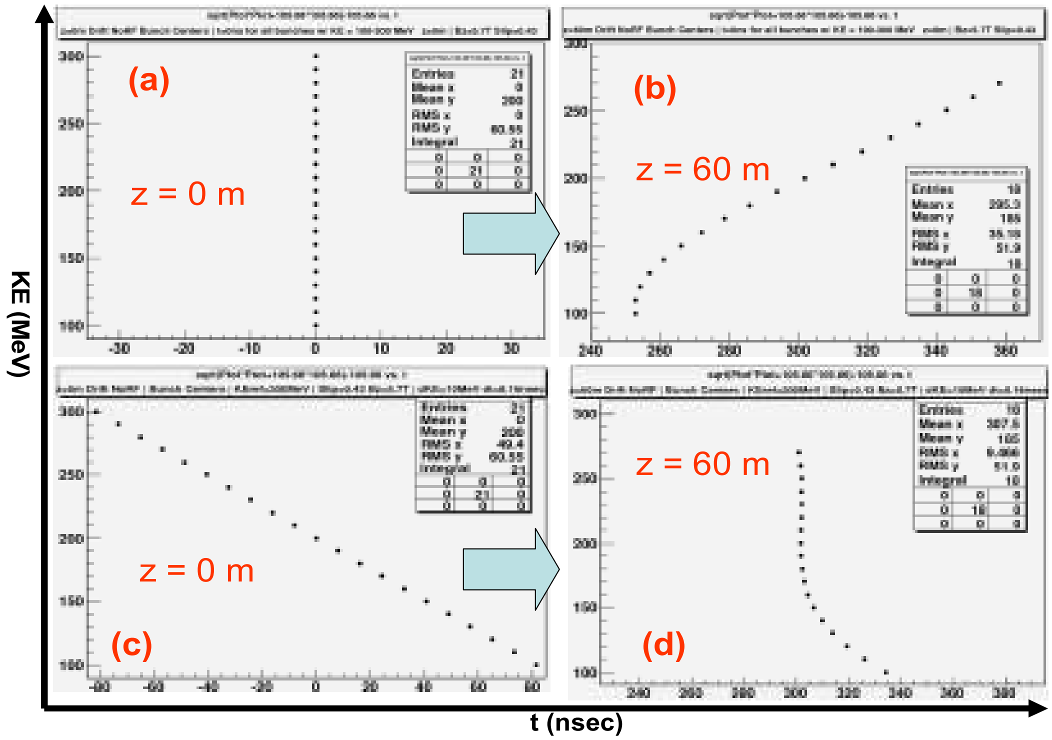

Figure 2: Simulations illustrating kinematics that motivate use of higher energies to coalesce bunches of muons. All bunch centers start at z=0m at the same time (t=0 nsec) in (a). After 60 m of longitudinal free drift in a helical channel, a linear E(t) relation has been developed at the higher energies in (b). In (c), bunches are initiated with a linear E(t) relation at the start of the channel at z=0m. After 60 m of longitudinal free drift in the helical channel, the higher energy bunches are aligned in time in (d).

2.2 Creation of Linear Energy-Time Correlation

Bunch coalescing requires the different bunches to have different energies so that as they traverse the downstream drift region with a slip factor $\eta = \beta^2(\Delta\tau/\tau)/(\Delta E/E)$, bunches that are initially displaced with appropriate ΔE-$\Delta\tau$ separations will be merged at the end of the drift. The method applied to create the desired energy-time correlation is an adiabatic approach, where a modest RF gradient is used (V'_{max} = 1 MV/m) with frequencies varying along the channel. In designing this coalescer to merge 9 bunches, the frequency at any particular location is determined by the time difference between the center reference bunch that is not being accelerated and the fourth bunch ahead of it that is being accelerated with a phase of 30º; hence, there are $4^1/_{12}$ number of wavelengths between these four bunches. The resultant range of frequencies is 204.17 to 271.84 MHz and its profile along the channel is shown in Figure 3. To study the dynamics of the bunches beyond for which it is designed, we applied these set of frequencies to 13 bunches with emittances expected out of the HCC described earlier and accelerated to 200 MeV kinetic energy, as shown in Figure 4. The end of this section is chosen at 40m where onset of particle losses begin at the end buckets, as well as consideration for the energy difference between the two end bunches as this will drive the requirement for the acceptance of the second HCC that operates on the single bunch. The gross characteristics of this section are collected in Table 3. Note that the convention in this paper is that the longitudinal location z = 0 m is at the start of the section that creates the energy-time correlation.

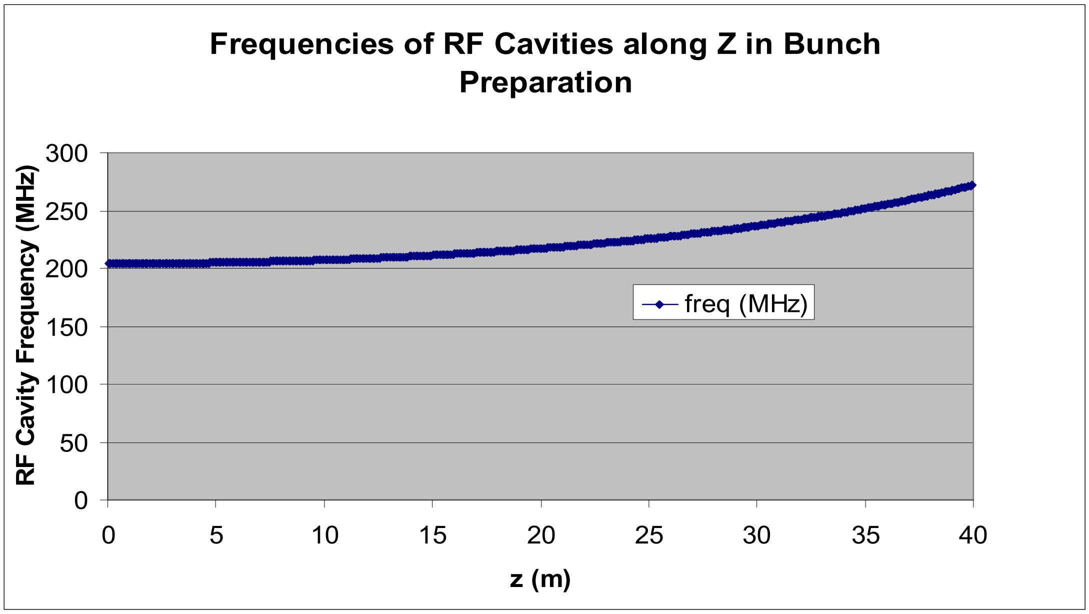

Figure 3: Frequencies of RF cavities in creation of linear energy-time correlation.

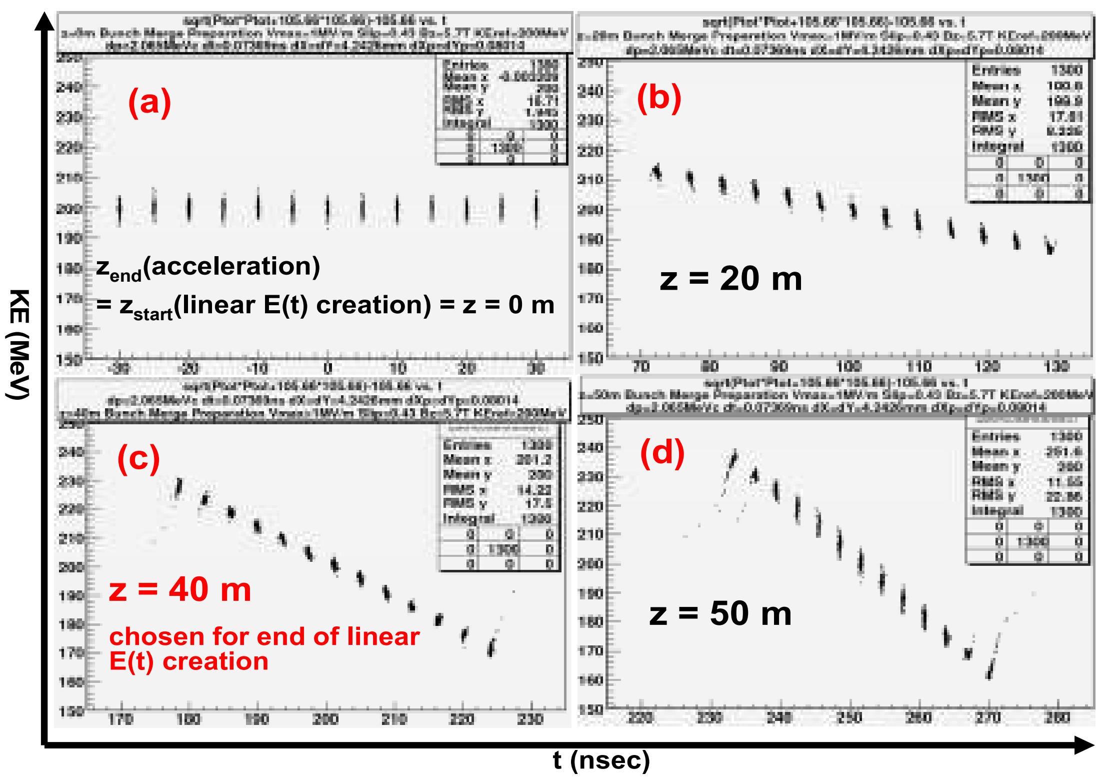

Figure 4: Creation of linear energy-time correlation amongst bunches after acceleration to 200 MeV of kinetic energy. The string of bunches at end of the initial acceleration to 200 MeV is shown in (a). Build up of linear energy-time correlations at 20m, 40m, and 50m are shown in (b), (c), and (d), respectively. The end of this section is chosen at 40m, in part due to the onset of particle losses at the end buckets. Note that the RF bucket area for the end bunches is ~17% of that of the center non-accelerating bunch.

Table 3: High level features of section that creates linear energy-time correlation

Creation of Linear Energy-Time Correlation Features
Length is 40m
$V'_{max} = 1$ MV/m
204.17 MHz ≤ frequency ≤ 271.84 MHz

2.3 Drift to Align Bunches in Time

The primary section in this bunch coalescing scheme is that for the drift of the bunches to be aligned in time. The distance over which the bunches are aligned is determined in the linear approximation by the dE/dt slope of the initial bunch string and the slip factor of the channel. In particular,

$$L_z = \frac{1}{\eta_{HC}} m_\mu c^2 \frac{c}{\left|\frac{dE}{dt}\right|} = 60m \tag{3}$$

where:

- L_z is the length over which bunches are aligned
- η_{HC} is the slip factor defined in a helical channel, given previously in equation (1)
- dE/dt is -1.229 MeV/nsec, obtained from the two nearest bunches around the center reference bunch shown in Figure 4(c)

The drifting of the bunch string was simulated in 3-D with results shown in Figure 5, which verifies the calculated drift distance for bunch coalescing of 60 m. Recalling that this particular bunch coalescer is designed to merge 9 bunches, Figure 6 shows the reduction of $\sigma_{rms}(t)$ as the outer bunches that are beyond design are removed from the calculations. Values of $\sigma_{rms}(t)$ for 13, 11, and 9 bunches are 1.33ns, 0.85ns, and 0.79ns, respectively. So, depending on the frequency used for the single bunch RF capture, bunches beyond the intended design may be useful and contribute muons to the single coalesced bunch. The gross characteristics of this drift section are collected Table 4.

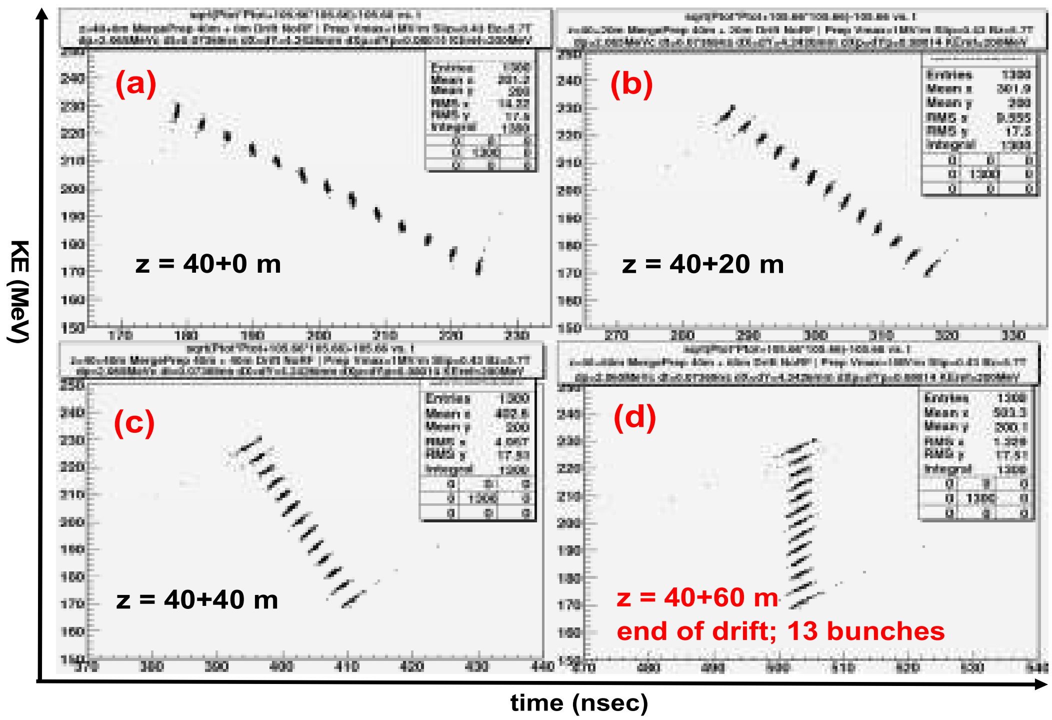

Figure 5: Drifting of bunches to alignment in time. The 13 bunches exit the linear energy-time correlation creation section at z = 40 m and enters the drift region in (a). The bunches are drifted 20m, 40m, and 60m in the drift section in (b), (c), and (d) respectively and are aligned at 60 m, as calculated.

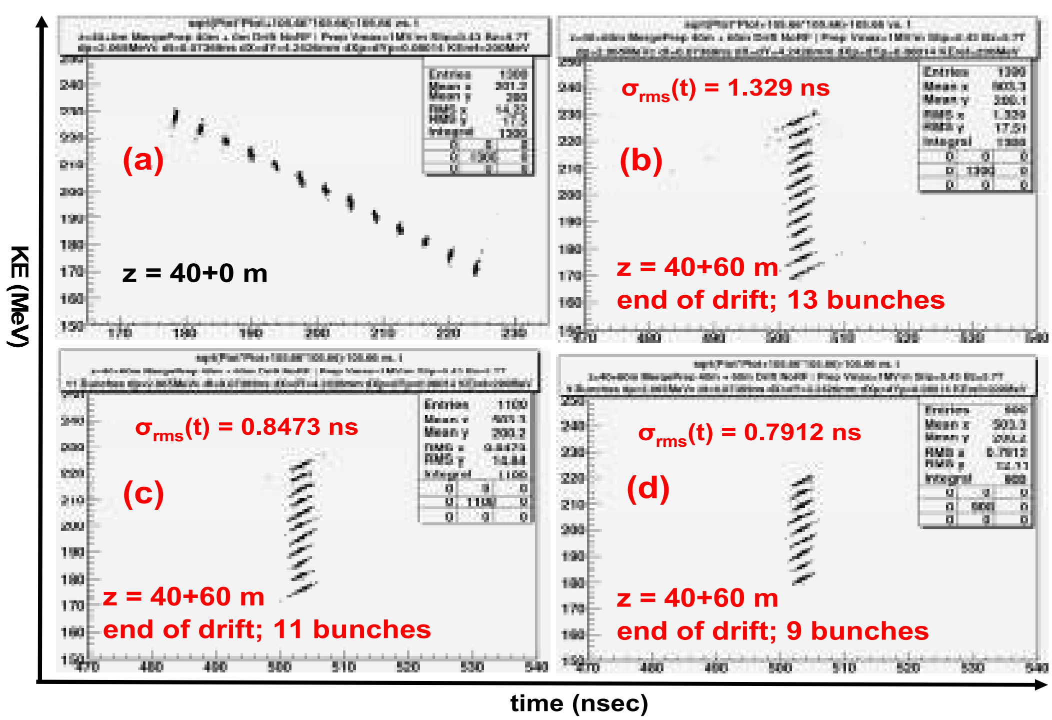

Figure 6: Reduction of time spread for bunches closer to reference bunch. At end of drift, $\sigma_{rms}(t)$ for 13 bunches, 11 bunches, and 9 bunches are 1.33ns, 0.85ns, and 0.79ns as shown in (b), (c), and (d), respectively. The beginning of the drift section at z = 40 m is shown in (a) for reference.

Table 4: High level features of the drift section that aligns bunches

Drift Section Feature(s)
Length is 60m
No RF

2.4 Capture into a Single RF Bucket

Once the multiple bunches are time aligned, RF is needed to hold them together in a single bunch. The amount of RF is somewhat arbitrary from the viewpoint of a broader design, since there is no design of the second HCC for which the design of this RF capture can target. This study utilized a voltage gradient of 10 MV/m at 200 MHz in vacuum and the longitudinal dynamics 5 m into the RF capture are shown in Figure 7 and Figure 8, where muons are considered captured in the RF bucket at this distance, as evidenced by their lost multi-bunch history. The rate of capture of the original 9, 11, and 13 bunches are 99.7%(897/900), 98.4%(1082/1100), and 94.2%(1225/1300), where each bunch began the coalescing channel with emittances equivalent to that exiting an HCC. Muon decays are ignored, which would be ~8% for the 105m (horizontally) long channel in the current design, which is also missing the section that provides the initial equal acceleration to all bunches. The gross characteristics of this RF capture section are collected in Table 5.

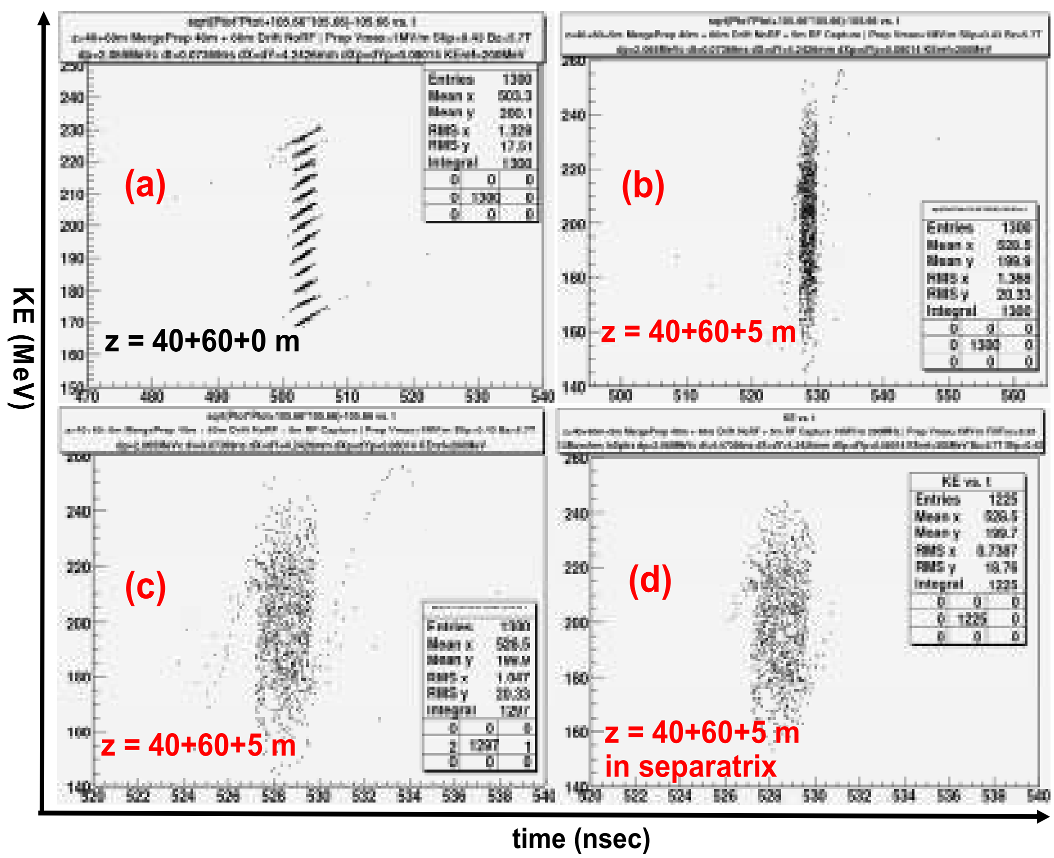

Figure 7: RF capture into a single bunch from 13 initial bunches. Bunches at the end of the drift section at z = 40(E-t correlation build-up) + 60(drift) = 100 m and start of RF capture are shown in (a). Longitudinal dynamics of muons at 5 m into RF capture is shown in (b) using same time scale as (a). Zoomed in views are shown in (c) and (d), where only muons in the separatrix are shown in (d).

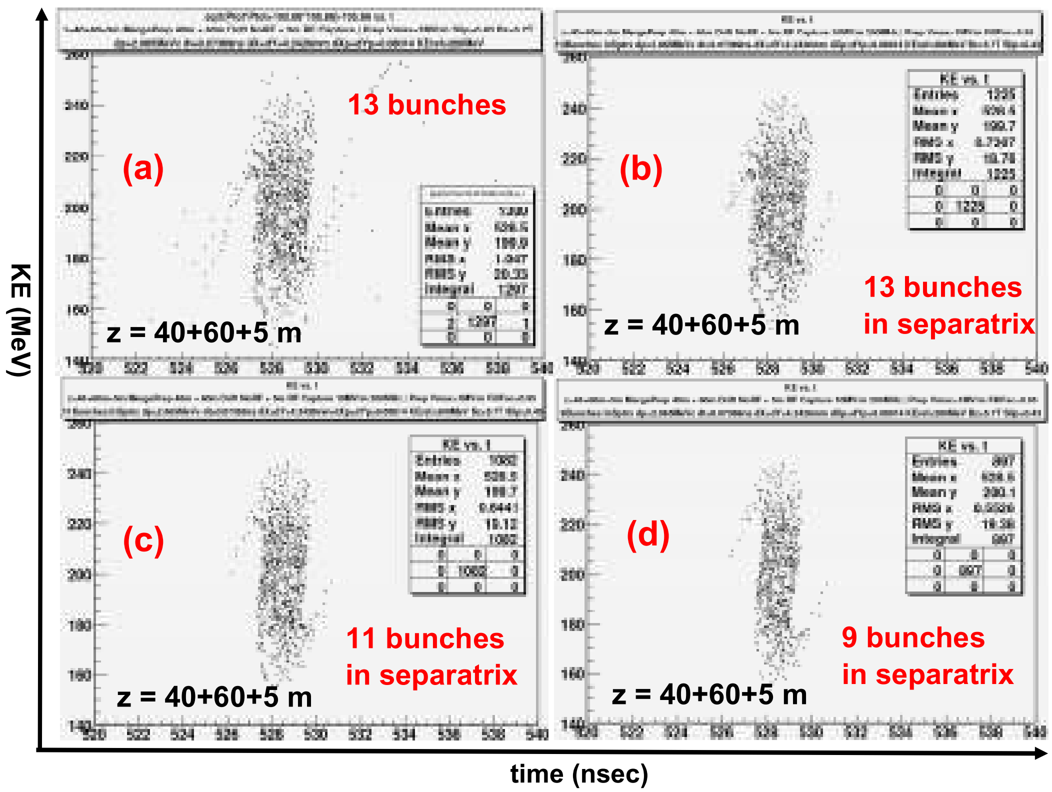

Figure 8: RF capture into a single bunch at 5 m into the section. Result of RF capture into a single bunch from 13 original bunches is shown in (a) and the subset of those in the separatrix is shown in (b). Muons in the separatrix from originally 11 and 9 bunches are shown in (c) and (d), respectively.

Table 5: High level features of single RF bucket capture section

Single RF Bucket Capture Section Features
Length is 5 m
V'_{max} = 10 MV/m (~arbitrary, see text)
Frequency = 200 MHz

To estimate the need for matching into a downstream single bunch HCC, as well as to provide guidance for that HCC's acceptance in its future design, we compared the emittance of the bunch in the RF capture section to the emittance of muons that are accepted in HCCs with documented designs [2,3]. The emittance values are rms values, so if emittance values of the single bunch are equal to muons accepted by an HCC, then the HCC can be considered to be matched to the single bunch emittance. In this arbitrary comparison, the single bunch has transverse emittance much smaller than those of either HCC, so the bunch will easily fit in those HCCs, as shown in Figure 9. However, Figure 10 shows that the longitudinal emittance of the single bunch is larger than (marginally for 9 bunches) the 200 MHz HCC, so some longitudinal cooling may be needed if that HCC were to be used as well as the extra bunches that go beyond the design (more than 9 bunches). Such would not be the case, if the 325 MHz design is used. The difference in emittances between the 200 MHz and 325 MHz designs are primarily due to the larger voltage gradient used for the 325 MHz configuration.

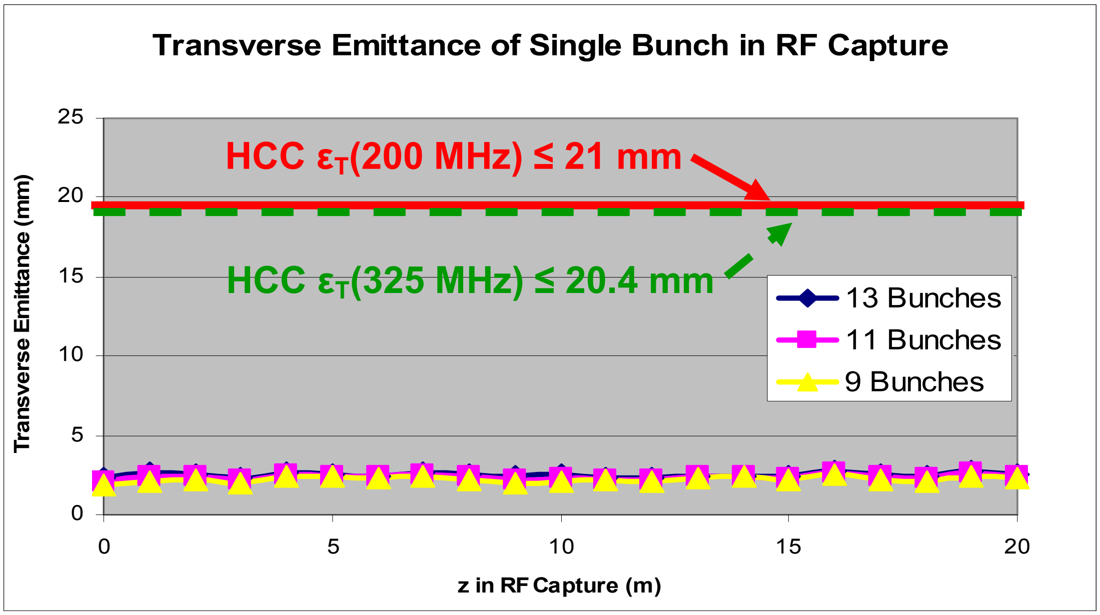

Figure 9: Transverse emittance of single bunch in RF capture section for the single bunch formed from 13, 11, and 9 initial bunches. Bunches are considered captured into a single bunch at 5m into RF capture as multi-bunch history becomes lost. Arbitrary comparison is made against HCC's with documented designs for 200 MHz [2] and 325 MHz [3]. Comparison is arbitrary, since downstream HCC is yet to be designed to match the single bunch emittance.

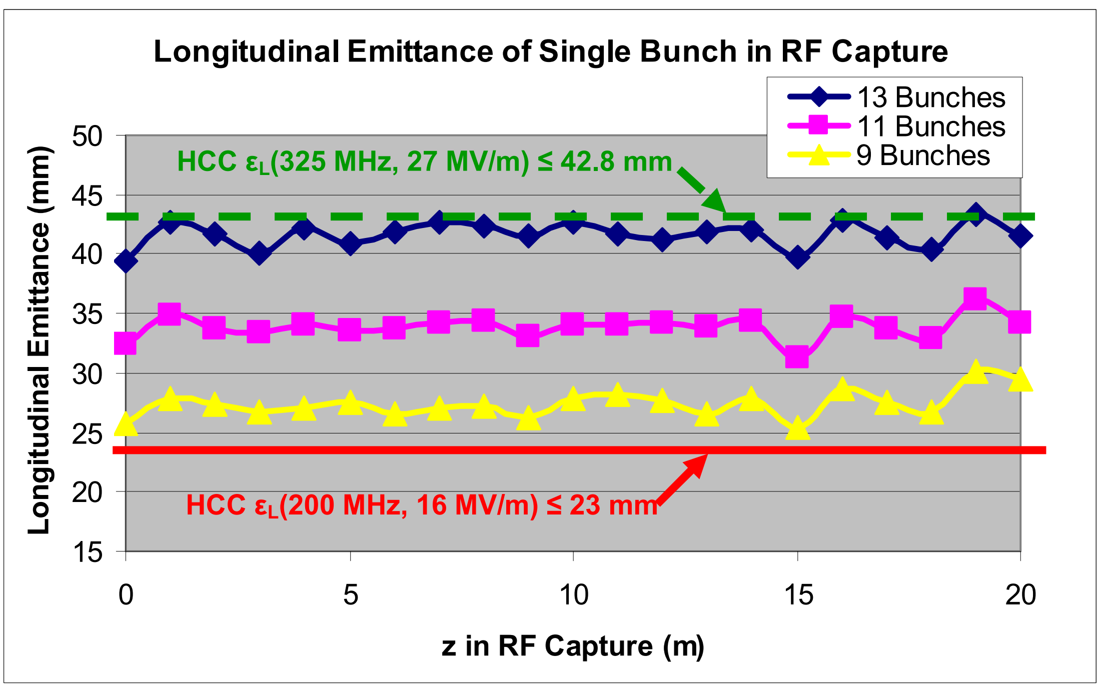

Figure 10: Longitudinal emittance of single bunch in RF capture section for single bunch formed from 13, 11, and 9 initial bunches. Bunches are considered captured into a single bunch at 5m into RF capture as multi-bunch history becomes lost. Arbitrary comparison is made against HCC's with documented designs for 200 MHz [2] and 325 MHz [3]. Comparison is arbitrary, since downstream HCC is yet to be designed to match the single bunch emittance.

We note that the subsequent section will decelerate and may longitudinally cool the single bunch, so incorporating (H_2) gas and radial wedge absorbers in this RF capture section would not only create larger RF buckets via higher RF gradients from incorporation of gas, but the enhanced longitudinal cooling afforded by the wedges should increase the RF capture rate and provide a head start for the possibly needed downstream longitudinal cooling.

2.5 Deceleration and Longitudinal Cooling

As mentioned above, the desired operating energy and acceptance of the second single bunch HCC is not yet known, so the amount of deceleration and longitudinal cooling is not known. The interplay between the bunch coalescing subsystem and this HCC is likely to be an iterative process and the results from this initial study provide a starting point. Based on these results, we can anticipate the following needs to be expected of the section after the RF capture and before the second HCC:

- The single muon bunch will likely need to be decelerated to the appropriate energy of the second HCC.
- The transverse emittance, ε_T, is likely to be smaller than the second HCC's acceptance, so emittance exchange can be utilized if necessary to reduce longitudinal emittance, ε_L.

A design that addresses the above issues is to implement radial wedges and phase the RF to zero crossing at the single bunch center, which realizes the following benefits:

- Deceleration is accomplished by material, not RF, so the bucket size remains large.
- Emittance exchange shrinks ε_L at expense of growing ε_T, where growth of ε_T is tolerable.

As stated earlier, the amount of deceleration and longitudinal cooling desired will be the result of a future iterative process involving the design of the second HCC.

3 Simplification of the Helical Bunch Coalescing Subsystem

The most complicated section in the bunch coalescing subsystem is the portion that creates the linear energy-time correlation via a large number of RF cavities, each having its own frequency. Although the RF voltage gradient was a modest 1MV/m, each cavity had a short length of 10 cm. Using a fill factor of approximately 100% requires 400 cavities to cover the 40 m longitudinal length in creating the linear energy-time correlation. The study here investigates the compromise in performance by using longer RF cavities (25 cm long), less fill factors (~50% and ~25%), and larger gradients (2.2 MV/m and 4.4 MV/m) to compensate for the lower fill factors. The original configuration and the two simplified versions have their parameters shown in Table 6. Longitudinal dynamics at the end of the energy-time creation in the original, ~50% fill factor, and ~25% fill factor configurations are shown in Figure 11. These results are visually identical, but to quantify any differences, the simulations are carried further and end at 5 m into the RF capture, with efficiency being calculated by muons inside the separatrix, as was performed earlier and shown in Figure 7(d) and Figure 8 for the original configuration. Figure 12 shows the efficiency of bunch coalescing for the different configurations to create the linear energy-time correlation as well as for the different number of bunches. The configurations with the reduced fill factor (~50%

and ~25%) performed equally well and were consistently ~4% less efficient compared to the original configuration that consists of 400 cavities.

Table 6: Parameters in original and simplified configurations of section creating linear energy-time correlation

	Configuration		
	Original	~50% Fill Factor	~25% Fill Factor
RF Field Fill Factor (%)	95%	49%	24.5%
Cavity Length (cm)	10	25	25
Number of Cavities	400	80	40
V'max (MV/m)	1.0	2.2	4.4

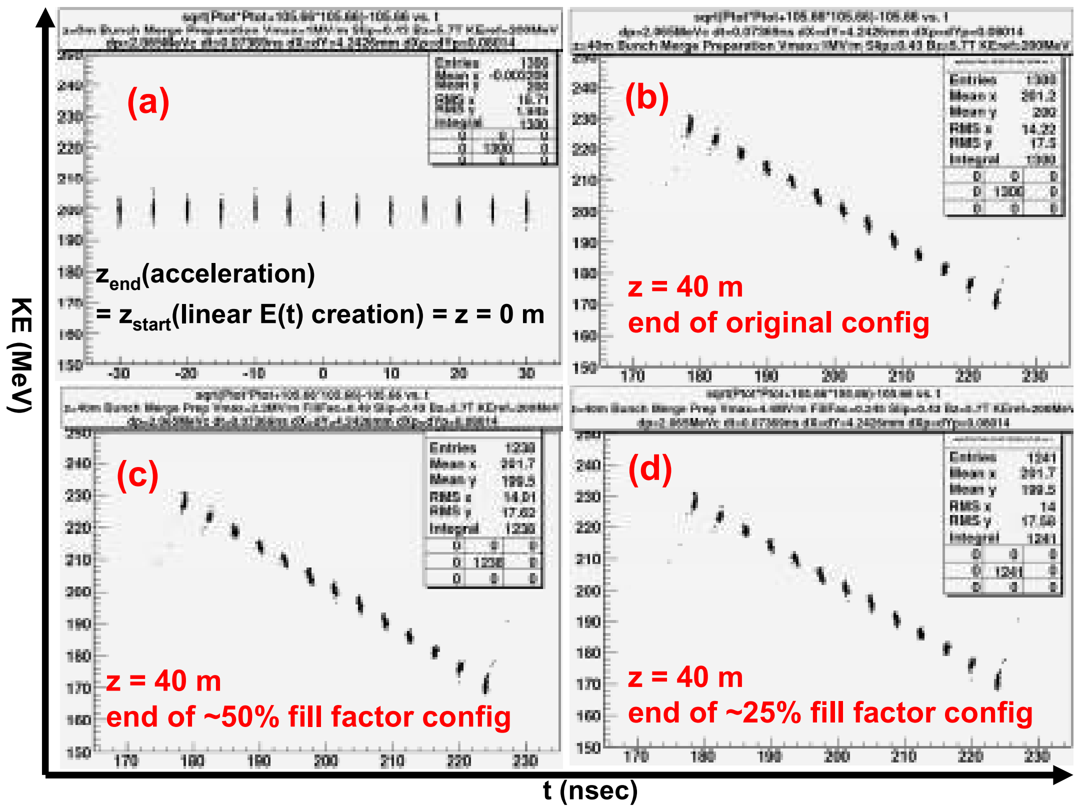

Figure 11: Comparison of original to simplified schemes at end of linear energy-time correlation creation. The string of bunches at end of the initial acceleration to 200 MeV is the common starting distribution for all configurations and are shown in (a). Bunch strings at end of original, ~50% fill factor, and ~25% fill factor configurations are shown in (b), (c), and (d), respectively.

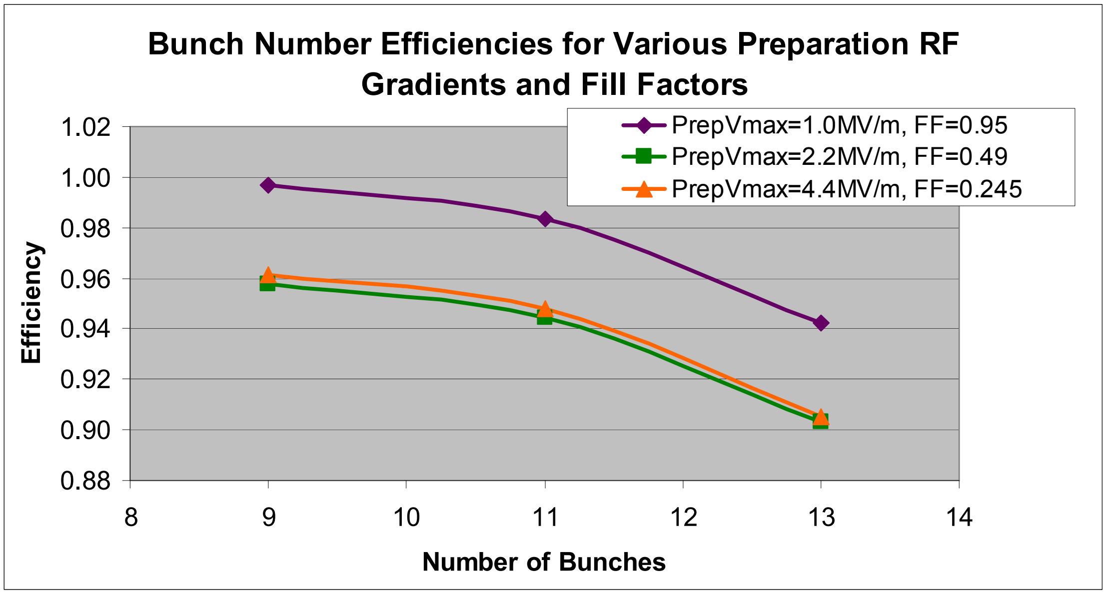

Figure 12: Efficiency of bunch coalescing for different configurations to create the linear energy-time correlation and different number of bunches, as calculated by muons at 5 m into RF capture that are in the single bunch separatrix.

We finally considered the possible emittance dilution due to simplifications of channels with a lower fill factor that use longer RF cavities with larger gradients. Figure 13 shows the transverse emittance for the original and simplified configurations for 13, 11, and 9 bunches that are merged in the RF capture section. Recall that bunches are considered captured into a single bunch at 5m in the RF capture, as their multi-bunch history becomes lost as was shown in Figure 7. As expected, there are effectively no differences in ε_T due to modifications that affect the longitudinal dynamics. However, differences are observed for the longitudinal emittance, as shown in Figure 14. The dilution is most prevalent for 9 bunches and is reduced with increasing number of bunches. At 13 bunches, the expected behavior is even reversed (we expect lower ε_L for the original configuration), but the discrepancies are within the ~3% statistical error of this preliminary study. In all cases, the single RF bunch would fit the acceptance of the 325 MHz HCC, although as stated above, these comparisons are arbitrary and the results are primarily meant to provide guidance for the design of a future HCC that cools the single bunch of muons.

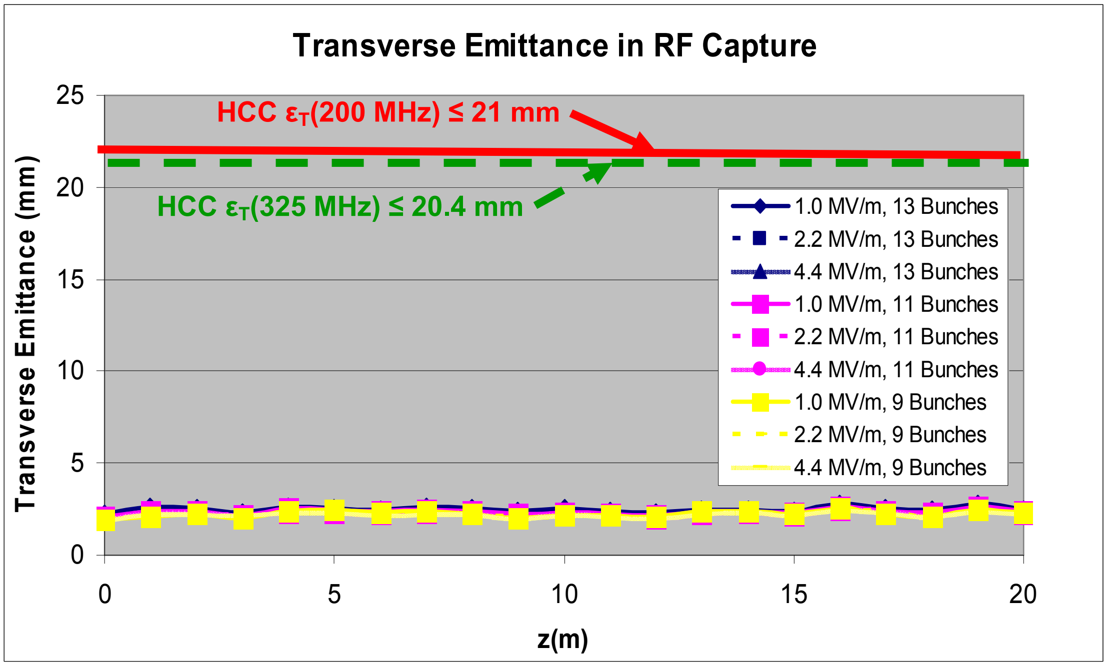

Figure 13: Transverse emittance in RF capture utilizing original and simplified schemes to create linear energy-time correlation. Bunches are considered captured into a single bunch at 5 m into RF capture as multi-bunch history becomes lost. Arbitrary comparison is made against HCCs with documented designs for 200 MHz [2] and 325 MHz [3] for sake of guidance in design of a future HCC.

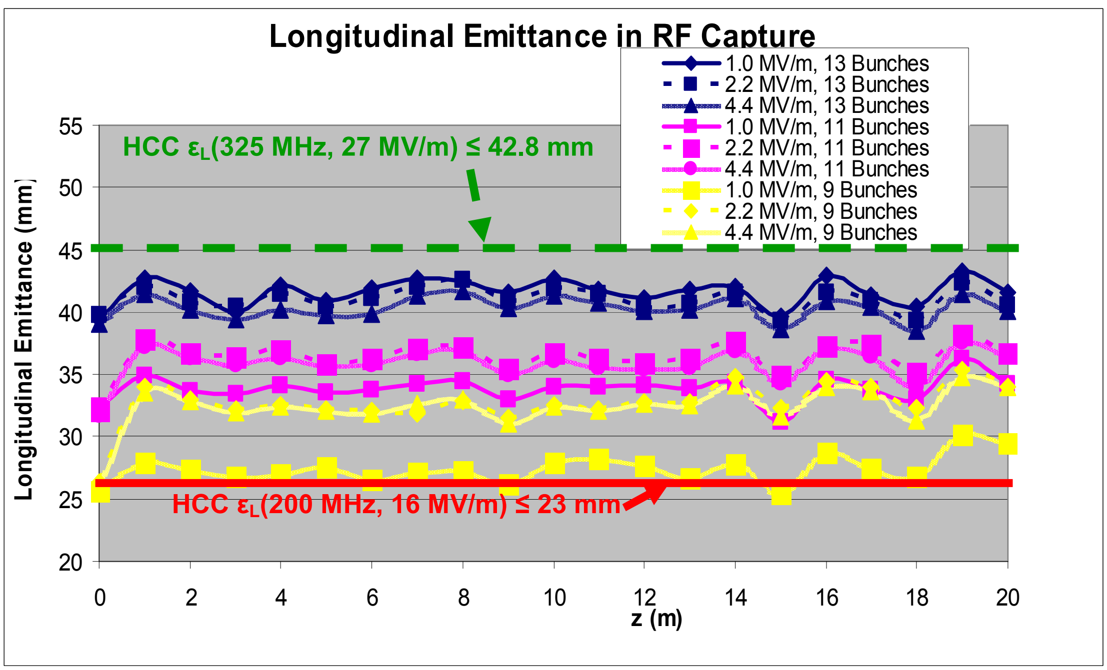

Figure 14: Longitudinal emittance in RF capture utilizing original and simplified schemes to create linear energy-time correlation. Bunches are considered captured into a single bunch at 5 m into RF capture as multi-bunch history becomes lost. Arbitrary comparison is made against HCCs with documented designs for 200 MHz [2] and 325 MHz [3] for sake of guidance in design of a future HCC.

4 Summary and Future

A bunch coalescing subsystem in a helical channel has been designed that makes minimal modifications to the magnetic fields of an existing HCC design that cool muons. Alternate designs have been developed to investigate the tradeoffs between simplifications in reducing the number of RF cavities versus performance. All key components of the bunch coalescing subsystem have been simulated in 3-D. Excluding the initial acceleration of all bunches to the desired operating energy (not simulated), the coalescing subsystem that is designed to merge 9 bunches has a horizontal length of ~105m and is able to achieve efficiencies of 99.7%, 98.4%, and 94.2% for 9, 11, and 13 bunches, respectively, where each bunch has emittances expected at the end of a HCC. Designs to merge more bunches will be longer, but should achieve comparable efficiencies. The simplified designs incorporating fill factors for RF cavities of ~25% and ~50% obtained efficiencies of 96%, 94-95%, and 90-91% for 9, 11, and 13 bunches, respectively. The efficiencies do not include decay losses, which would be ~8% for muons with kinetic energy of 200 MeV. Following the RF capture into a single bunch, a series of radial wedges in the helical channel may be needed to reduce the longitudinal emittance as well as reduce the operating energy to that of the second HCC. The amount of longitudinal cooling and energy reduction for the single bunch is dependent on the acceptance of the downstream HCC, which is yet to be designed.

The interplay between the RF capture and the single bunch HCC may begin by designing an HCC that would accept muon bunches with rms emittance values suggested in Figure 13 and Figure 14. If such a design is not possible (muon beam is too hot longitudinally for HCC), the most feasible design could be considered and radial wedges can then be added into the channel to both cool the muons longitudinally (via emittance exchange afforded by allowable growth of transverse emittance) and reduce the operating energy to that of the newly designed second HCC. Such a process aims to create the shortest and most efficient integrated design that transforms a string of hot muon bunches into a single cooled bunch that is ready for downstream extreme cooling (emittance exchange), acceleration, and collision with its particle counterpart at the energy frontier.

5 Acknowledgments

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