Phase rotation of muons by induction linac

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Abstract

A possibility to use an induction linac for phase rotation of muons after target and decay channel is considered. Requited magnet field of the cores, stored energy and power are estimated. Optimal operation mode of the linac is discussed. It is shown by simulation that the induction linac is capable to provide the phase rotation without additional low-frequency RF system

1 Introduction

In the present scenario, an induction linac is used to phase rotate the muon beam after leaving a drift section of approximately 100 to 150 meters [1]. This drift section is described in more detail in the part of the overall document where the different solenoid channels are described. It is only used to develop a correlation in the longitudinal phase space of the muon beam. For given energy spread of the beam the drift velocity of the individual muons are different and after a given distance the lower energy particles are behind the higher energy particles. Therefore the position in the beam is correlated to a certain energy, which enables phase rotation, e.g. application of a time dependent voltage to reduce the energy spread, accepting that the bunch is longer afterwards. It is proposed to do the phase rotation with an induction linac, because the muon bunch length is of the order of 160 nsec (50 meters) or more. This is a very typical pulse length for an induction linac.

2 The induction linac

The features of the induction linac which is required in order to phase rotate the beam are described in more detail below. A few very special requirements though are specific to this application and will be mentioned here.

- High gradient is desirable in order to provide the phase rotation as soon as possible because of the finite decay time of the muons.
- A voltage swing from $-V_{max}$ to $+V_{max}$ is necessary (without flat top operation) in order to make optimum use of the applicable voltage (stored energy in the cores) for the phase rotation.
- A large aperture beam channel for the muon beam because of the large emittance beam coming from the source. At least 30 cm free aperture for the beam is required.
- SC magnets (1.25-3.0 T) integrated into the induction linac to focus the muon beam through the linac are necessary.
- Multiple pulse per cycle (e.g. 4 pulses within 2 μsec) are desirable.

Multiple bunch operation from the proton source, and sending these bunches directly to the target, will require multiple pulses from the induction linac per booster cycle. Apart from the general question of feasibility of an induction linac for this type of application, this is the most critical issue which has to be resolved as soon as possible. Certainly it would be desirable to have the capability of multiple bunch operation, at least in a later stage. If only single bunch / single pulse operation is possible, a specific solution has to be found in the proton driver to generate and accelerate a single bunch or before the target to recombine bunches before they hit the target. From the technical point of view, complexity and cost of this solution has to be compared with multiple pulsing of the induction linac.

Section of the induction linac is schematically represented on Fig.1. To estimate its most important parameters, it is assumed further that magnet permiability of the cores $\mu = const$, and $r_1 = 30$ cm, $r_2 = 50$ cm are taken for numerical estimations and simulation.

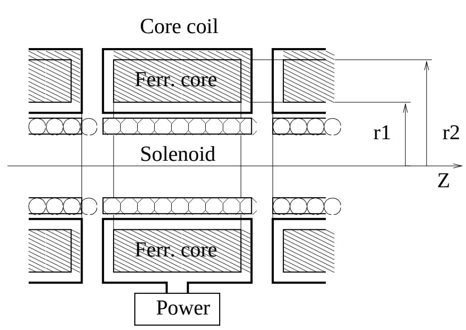

Figure 1: Schematic of an induction linac section.

2.1 Magnet field of the cores.

Let $B(r)$ is magnet induction in a core of the induction linac, E - induced electric field, μ - magnet permiability. At $\mu = const$

$$E = \frac{1}{c}\int_{r_1}^{r_2} \dot{B}(r)dr = \frac{\dot{B}_1 r_1}{c}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\dot{B}_1 r_1}{c}\ln\frac{r_1}{r_2}, \tag{1}$$

where $B_1 = B(r_1)$. It is necessary to accelerate slow muons and decelerate fast ones. But 'fast' part of the beam is relatively short and can be neglected at estimation. In such an approximation $E > 0$ and

$$\dot{B}_1 \propto E \propto t, \qquad 0 < t < T, \tag{2}$$

where T is the pulse duration. Required magnet field is minimal at

$$B_1 = B_{max}\left(\frac{2t^2}{T^2} - 1\right), \qquad \dot{B}_1 = B_{max}\frac{4t}{T^2} \tag{3}$$

It is seen that $|B_1| \le B_{max}$ at $0 \le t \le T$.

Production $cT = L$ has to be about the beam length. Therefore:

$$E = \frac{4B_{max} r_1 t}{LT}\ln\frac{r_2}{r_1}, \qquad B_{max} = \frac{E_{max}L}{4r_1\ln(r_2/r_1)} \tag{4}$$

It gives $B_{max} = 0.33$ T at $E_{max} = 1$ MV/m $= 1/300$ T, $r_1 = 30$ cm, $r_2 = 50$ cm, $L = 60$ m. A shortening of the beam lets to reduce magnet field:

$$B_{max} = 0.065 \text{ T} \quad \text{at} \quad L = 12 \text{ m}.$$

2.2 Stored energy and power.

Stored energy of magnet field per unit of length of the induction linac is:

$$W' = \int\int \frac{B^2(r)}{8\pi\mu} r dr d\phi = \int_{r_1}^{r_2} \frac{B_1^2 r_1^2}{4\mu r^2} r dr = \frac{B_1^2 r_1^2}{4\mu}\ln\frac{r_2}{r_1} \tag{5}$$

(CGSE system used: $[r]$ = cm, $[B]$ = Oe, $[W']$ = egr/cm = 10^{-5} J/m). Corresponding power is:

$$P' = \dot{W}' = \frac{B_1 \dot{B}_1 r_1^2}{2\mu}\ln\frac{r_2}{r_1} = \frac{cB_1 E r_1}{2\mu} \tag{6}$$

It gives at $E_{max} = 1$ MeV/m, $\quad r_1 = 30$ cm, $\quad \mu = 1000$

$$P'_{max} = \frac{500 \text{ MW/m at} B_1 = 0.330 \text{ T } (L = 60 \text{ m})}{100 \text{ MW/m at} B_1 = 0.065 \text{ T } (L = 12 \text{ m})}$$

This power is reactive and convertible, in principle. Estimation of active power requires an additional consideration (resistivity of coils, hysteresis, eddy current should be taken into account).

2.3 Winding: voltage and current.

Any induction linac is essentially transformer where a beam plays is 1-coil secondary winding. If primary winding is 1-coil, too, and active power is enough small its required voltage is:

$$V' = E, \qquad V'_{max} = 1\ \text{MV/m} \tag{7}$$

Corresponding current of the winding is:

$$J = \frac{P'}{V'}, \qquad J_{max} = \frac{\overline{500\ \text{A at}\ L\ = 60\ \text{m}}}{\underline{100\ \text{A at}\ L\ = 12\ \text{m}}} \tag{8}$$

3 Phase rotation

Let us consider muon production system including target, decay/longitudinal drift channel ($L_{d/d}$)and induction linac for phase rotation, as is shown on Fig.2. By [1], low-frequency RF in the beginning of the channel is used for phase rotation, too, but in this paper a system without RF is discussed. Two versions are considered: long channel $L_{d/d} = 200$ m and short one $L_{d/d} = 50$ m. Length of the linac is assumed as 100 m, accelerating gradient up to 1 MeV/m in both cases. Transverse focusing is fulfilled by a continuous solenoid $B = 1.25$ T, $R = 28$ cm both in the decay/drift channel and the linac. Following paper [2], transverse distribution of pions generated in the target by 16 GeV protons is simulated as Gaussian with r.m.s. parameters $\sigma_x = \sigma_y = 8$ cm, $\sigma_{p_x} = \sigma_{p_y} = 15$ MeV/c. Basic energy distribution is taken from [2], too, but it is represented as a linear function in any partial range which really can be captured by accelerating field of the linac.

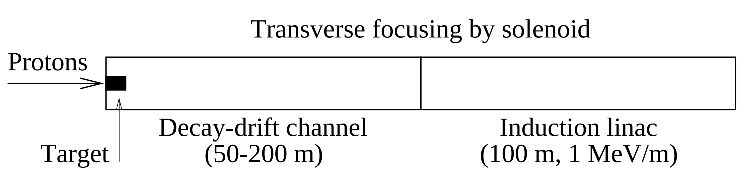

Figure 2: Layout of target – decay/drift cannel – induction linac.

3.1 Long channel: $L_{d/d} = 200$ m.

For optimal phase rotation, the linac has to create accelerating field $E(z,t)$ traveling together with muon beam, and in any cross section of the linac should be:

$$E(z,t) \propto U_{ave}(z,t) - U_{ref}$$

where U_{ave} is average energy of muons in considered section, and arbitrary constant U_{ref} is a reference energy of muons. Actually a pilot 'muon' beam was used for calculation of U_{ave}: transverse emittance and length of the beam are 0 in the beginning of the decay/drift channel, no decay.

Figs. 3a-b represent required accelerating gradient and magnet induction of cores at $r = r_1$ in 6 cross sections of the linac from its beginning through 20 m. Only 'working' parts of the linac pulses are shown here. 'Relaxing' part can be arbitrary but must finish before the next portion muons arrives at the linac what is very important at multi-bunch operation.

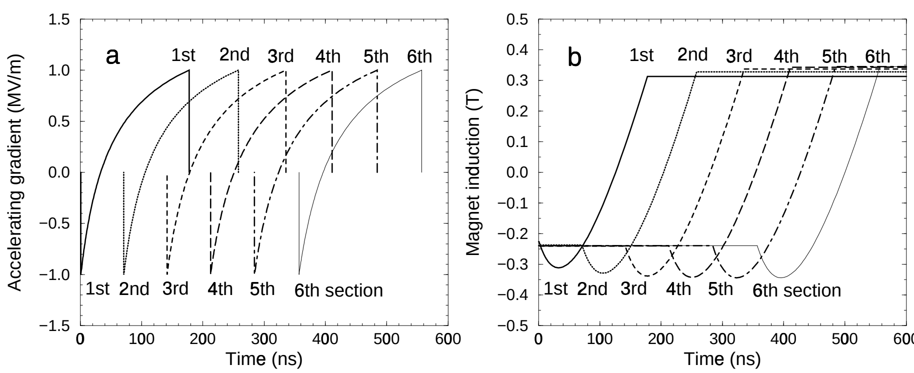

Figure 3: Time dependence of required accelerating gradient (a) and magnet induction in cores of the linac (b)

Fig.4 represents results of simulation with given accelerating gradient. Only pions with energy 220-470 MeV after target are considered because the others give rise to muons which cannot be accepted by the linac. Transverse emittance of pions $\varepsilon = 0$, and initial length of pion bunch $\sigma_{cT} = 10$ cm in this simulation (see below about dependence on the emittance and bunch length).

Almost all pions decay in 200 m channel. Muon energy after decay is about 0.79 of pion energy in average, but an additional spread appears by the decay. As a result, muon energy is distributed in the interval 135-460 MeV whereas the linac can capture and rotate muons in the interval 170-370 MeV at $U_{ref} = 270$ MeV (total energy).

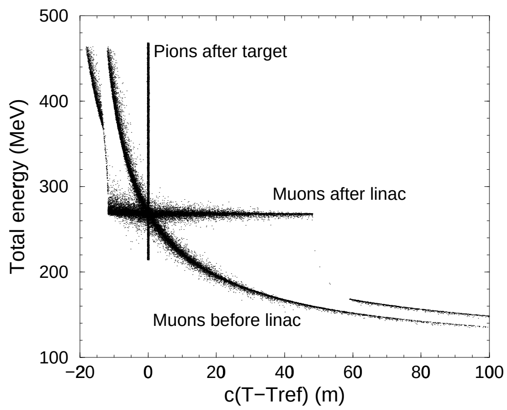

Figure 4: Longitudinal phase space of pions and muons

3.2 Short channel: $L_{d/d} = 50$ m

Figs.5-6 represent similar results for short channel. In this case pions with energy 280-530 MeV are considered. 91% of them decay in the channel and almost all others in the linac. Muon beam is 5 times shorter in comparison with 'long' version, but energy spread after phase rotation is about 3 times more what is bad for further adiabatic capture. Nevertheless such a regime might be used to facilitate operation conditions of the induction linac.

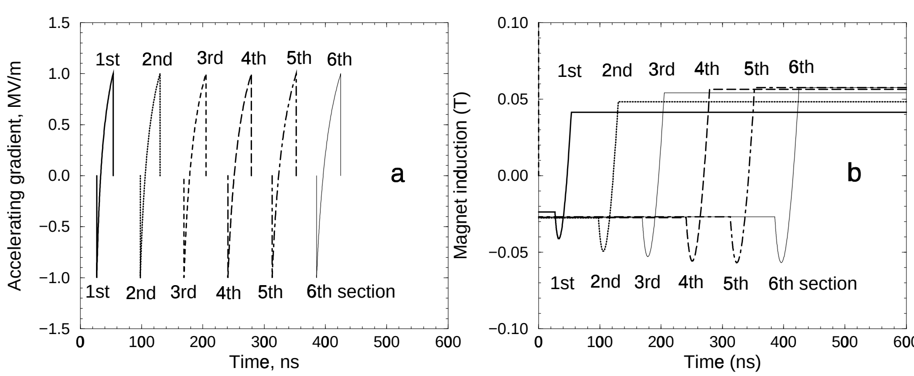

Figure 5: Time dependence of required accelerating gradient (a) and magnet induction in cores of the linac (b)

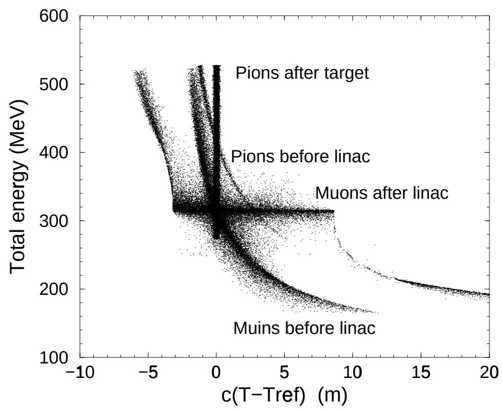

Figure 6: Longitudinal phase space of pions and muons

3.3 Dependence on the length of proton bunch and transverse emittance

Lengthening of proton bunch as well as increase of transverse emittance of accepted pion bunch lead to increase of energy spread of muon beam. It is illustrated by Fig.7 where longitudinal phase space of muons after phase rotation at zero and nonzero sizes of incident pion bunch are compared. In both cases 200

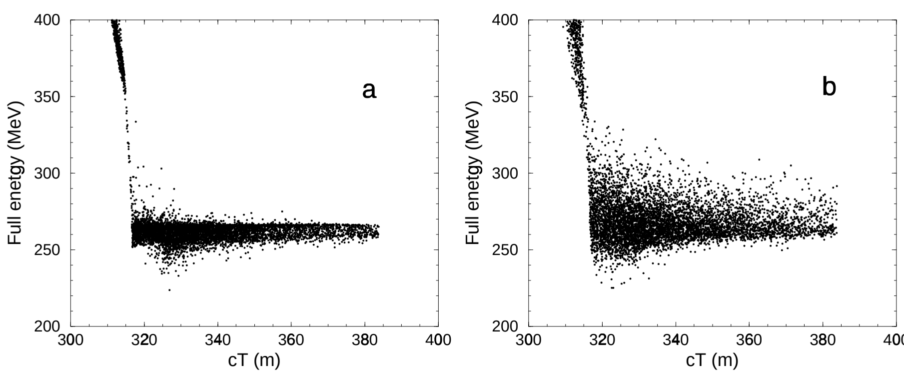

Figure 7: Part of longitudinal phase space of muons after phase rotation in dependence on size of pion beam: (a) $\sigma_x = \sigma_y = \sigma_{p_x} = \sigma_{p_y} = \sigma_{cT} = 0$; (b) $\sigma_x = \sigma_y = 8$ cm, $\sigma_{p_x} = \sigma_{p_y} = 15$ MeV/c, $\sigma_{cT} = 1$ m.

meters decay/drift channel is considered and energy interval of pions (220-400) MeV is cut out.

Transmission of the channel (μ/π - ratio) is about 0.97 independently on proton bunch length. But only narrow energy interval can be adiabatically captured into RF-bucket after phase rotation. The following Table gives transmission in dependence on bunch length and transverse size of pion beam with the following cut on muon energy:

$$253 \text{ MeV } < E_\mu < \text{ 283 MeV}$$

(it is assumed that $\sigma_y = \sigma_x$ and $\sigma_{p_x} = \sigma_{p_y} = \sigma_x \times$ 1.875 MeV/c/cm.)

Table 1. Transmission with energy cut in dependence on σ_{cT} and σ_x.

σ_x (cm)\ \ σ_{cT} (m)	0	0.1	0.3	1	3	10
0	0.777	0.774	0.767	0.725	0.571	0.320
4	0.772	0.776	0.765	0.719	0.562	0.323
6	0.750	0.741	0.736	0.703	0.567	0.322
8	0.688	0.671	0.679	0.647	0.517	0.307

It is seen that decrease of transmission is not very strong at least at $\sigma_{cT} < 1$ m.

There is no problems in this system with transverse motion as follows from Fig.8 where phase space of pions in the beginning of the channel and muons after phase rotation are shown. R.m.s. sizes of pions are 8.0 cm$\times$ 15 MeV/c and muons 8.9 cm*times* 17 MeV/c. A small growth is explained by transverse kick at decay of pions.

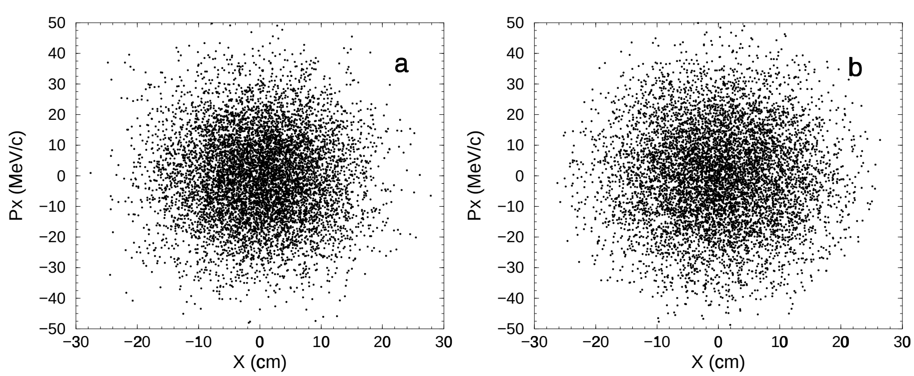

Figure 8: Transverse phase space of pions (a) and muons (b).

4 Conclusion

It is shown that induction linac is suitable, in principle, for phase rotation of muons. In principle two different scenarios are of interest. Either the induction linac is used almost directly behind the target which requires about 100 MV or more in order to phase rotate the beam or it is used later on, after the so called mini cooling, where the transverse emittance is already reduced (3x in both planes). In the latter case the energy spread is only ±50 MeV or so, because it has partially been removed with low frequency RF cavities beforehand [1]. For both cases the same general criteria for optimization apply. The factor of 3 in transverse emittance in combination with the focusing field could make a difference in required aperture. The average total momentum of the muon beam is round 300 MeV/c in the first case, while it is more like 187 MeV/c in the second case. For the same acceptance required in the channel, the magnetic field on axis is probably a factor of two (0.7 from transverse emittance and 0.6 from average momentum) lower in the second case than in the first one, and the larger total energy spread means a longer linac

If the induction linac is used almost directly behind the target, with a generally higher average momentum of the muon beam, the linac length could be longer, while if used later on, having a smaller average energy in the muon beam, the length has to be shorter because of the shorter decay length. The difference in γ is approximately 70 % which means that the difference in total length for the same decay loss can be about the same. Comparing the different requirements for total voltage used in the phase rotation, the required average gradient is the approximately the same for both scenarios. Therefore the study does not have to distinguish between these two cases. In general the design should ask for a cost(=length) optimized induction linac versus decay loss in the beam, assuming that ±100 MV total voltage is required. From this linac we can scale two a slightly shorter or longer linac for whatever the final layout will look like.

REFERENCES

1. R. B. Palmer. "Neutrino Factory Draft Parameters". www-mucool.fnal.gov/mcnotes/muc0046.ps. 1999.
2. R. J. Weggel and N. V. Mokhov. "Pions Yield vs. Geometry of Target and $\tilde{2}$0 T Pulse Solenoid for a Muon Collider Experiment". Proc. of the 1999 PAC, v.5, p.3047. NY, 1999,