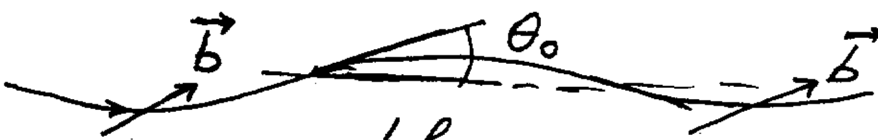
Ya. Derbenev

Report at Fermilab, 2.25/00
NF meeting

# Ionization Cooling on Spiral Orbit in Solenoid

- Field composition:
  - continuos solenoid (or long solenoids) plus
  - low, quasi-resonance dipole field
- Beam track:
  - quasi-resonance spiral not exceeding beam size
  - absorbers with <u>rotating</u> gradient
- Results in 3-dim. cooling
- Dipole field options:
  - a) continuous helix period $\lambda_0$
  - b) one-directed lumped dipoles field, same period

$\vec{B} \longrightarrow$ $\qquad \lambda_c - \lambda_0 \ll \lambda_0 \qquad \lambda_c = 2\pi \frac{pc}{eB}$




$\vec{b} \perp \vec{B}$

$\boxed{\text{orbit angle}} \rightarrow \theta_0 = \frac{b_0 \ell}{B(\lambda_c - \lambda_0)}$

a) $\ell = \lambda_0$

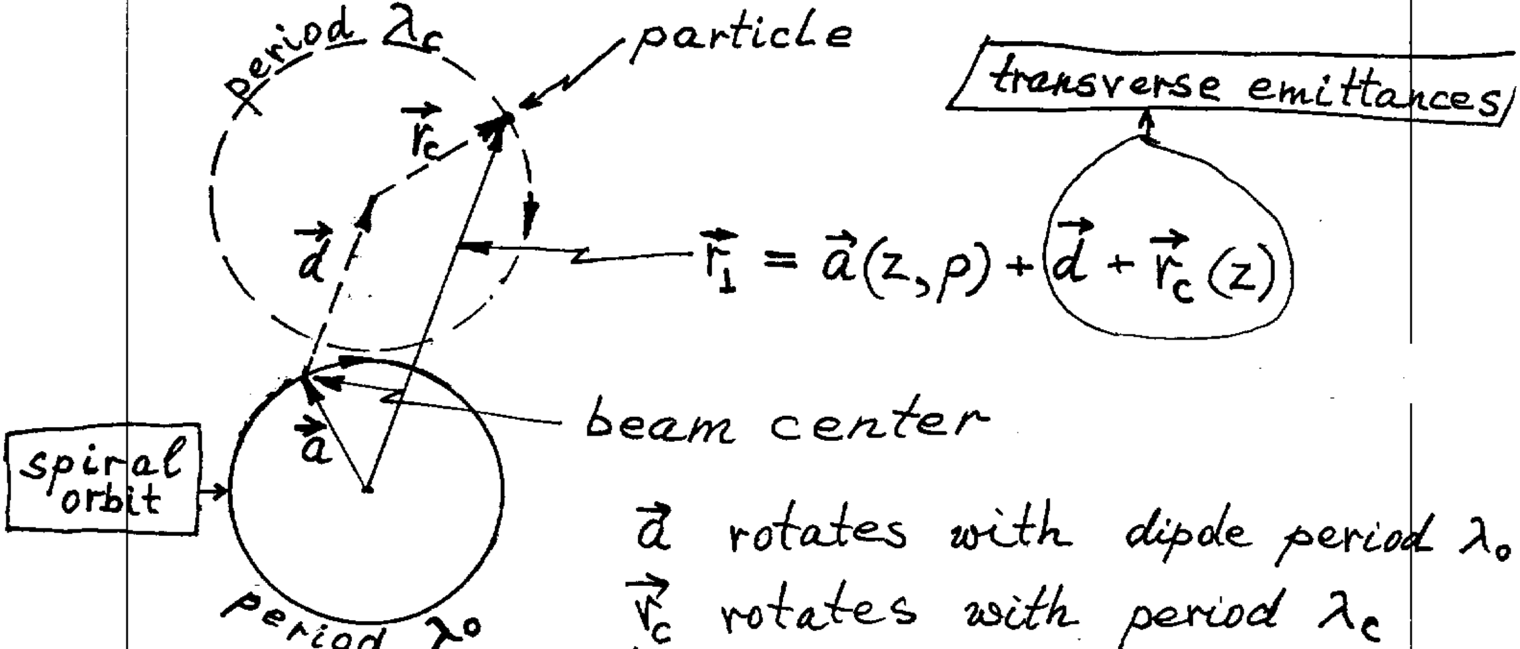
b) $\ell \ll \lambda_0$

<u>Orbit radius:</u> $a = \theta_0 \lambda_0 / 2\pi$

<u>Dispersion:</u> $\vec{D} = p \frac{d\vec{a}}{dp} = -\frac{\lambda_c}{\lambda_c - \lambda_0}\vec{a}$

$$\left| \lambda_c - \lambda_0 \gg \lambda_0 \cdot \left(\theta_0^2 + \theta_c^2 + \frac{\Delta p}{p}\right) \right|$$

## Transverse motion:




$\vec{a}$ rotates with dipole period $\lambda_0$
$\vec{r}_c$ rotates with period $\lambda_c$
$\vec{d}$ is constant (drift coordina

<u>Absorber gradient:</u> $\vec{\nabla} n = \frac{\vec{a}}{a} \nabla n$
(rotates)

$\nabla n = \text{const}$

$$n = n_0 + (\nabla n)\frac{\vec{a}}{a} \cdot \left(\vec{D}\frac{\Delta p}{p} + \vec{d} + \vec{r}_c\right)$$

<u>Friction force:</u> $(\log \approx 15)$

$$\vec{f} = E' \frac{\vec{p}}{p}; \quad -E' = \frac{4\pi n Z e^4}{m_e c^2 \beta^2} \log$$

- *Averaged friction effects:*

$$r_c' = -\frac{1}{2}\Lambda_c r_c \;;$$

$$\vec{d}' = -\frac{1}{2}\Lambda_d \vec{d}\;; \qquad \text{(beam shrinks!)}$$

$$\Delta p' = -\Lambda_\parallel \Delta p \;.$$

Cooling decrements:

$$\Lambda_\parallel = \frac{|\gamma_0'|}{\gamma\beta^2}\left(-\frac{2}{\gamma^2}+\frac{2}{\log}-\frac{\lambda_c}{\lambda_c-\lambda_0}\,\frac{a\nabla n}{n_0}\right)$$

$$\Lambda_d = -\frac{|\gamma_0'|}{\gamma\beta^2}\,\frac{\lambda_c}{\lambda_0}\,\frac{a\nabla n}{n_0}$$

$$\Lambda_c = \frac{|\gamma_0'|}{\gamma\beta^2}\left[2+\frac{\lambda_c^2}{\lambda_0(\lambda_c-\lambda_0)}\,\frac{a\nabla n}{n_0}\right]$$

- Decrement sum:

$$\Lambda_6 = \Lambda_c+\Lambda_\parallel+\Lambda_d = 2\,\frac{|\gamma_0'|}{\gamma}\left(1+\frac{1}{\beta^2\log}\right),$$

as it must be $\left(\Lambda_6 = -\frac{\partial}{\partial\vec{p}}\vec{f}(\vec{p},\vec{r})\right)$

---

Second order effects (negligible...)

- drift tune (slow rotation): $\frac{\lambda_d}{\lambda_c} \sim \frac{1}{\theta_0^2}\,\frac{\lambda_c}{\lambda_c-\lambda_0}$
- decrements shifts:

$$\frac{\Delta\Lambda}{\Lambda} \sim \frac{\lambda_c}{\lambda_d}$$

# Decrements redistribution (options)

1) Conditions for $\Lambda_c = \Lambda_{\parallel} = \Lambda_d = \frac{1}{3}\Lambda_6 = \frac{2}{3}\frac{|\gamma_0'|}{\gamma}$
$(\beta^2 \gg 1/\log)$

$$\left.\begin{aligned} \frac{a\nabla n}{n_0} &= -\frac{2}{3}\beta^2\,\frac{3-2\beta^2}{3-\beta^2} \\ \frac{\lambda_c-\lambda_0}{\lambda_c} &= \frac{\beta^2}{3-\beta^2} \end{aligned}\right\}$$

2) Equalizing between $\Lambda_c$ and $\Lambda_{\parallel}$:

$$\Lambda_c = \Lambda_{\parallel} = \frac{|\gamma_0'|}{\gamma}, \qquad \Lambda_d = -\Lambda_c \frac{a\nabla n}{n_0}$$

at

$$0 < -\frac{a\nabla n}{n_0} = 2\frac{\lambda_0}{\lambda_c}\frac{\lambda_c-\lambda_0}{\lambda_c+\lambda_0}(2-\beta^2) \ll \beta^2$$

3) Total emittance exchange:

$$\Lambda_c \ll \Lambda_{\parallel}, \quad \Lambda_d \ll \Lambda_{\parallel}$$

$$\Lambda_{\parallel} \approx 2\frac{|\gamma_0'|}{\gamma}$$

at

$$0 < -\frac{a\nabla n}{n_0} = 2\,\frac{\lambda_c-\lambda_0}{\lambda_c^2}\lambda_0 \ll 2\beta^2$$

## Important outlines:

- parameter $\frac{a\nabla n}{n_0}$ can always be small
- hence, $\theta_0 = 2\pi a/\lambda_0$ can be quite small, as well.

# Dipole field estimation

assume $a = 6 \Rightarrow \sqrt{\varepsilon \cdot mc^2/eB}$

then

$$\frac{b_0}{B} = \frac{\lambda_c - \lambda_0}{\lambda_0} \cdot 2\pi \frac{\sigma}{\ell} \approx \frac{\lambda_c - \lambda_0}{\lambda_0} \cdot \frac{\lambda_0}{\ell} \theta_c$$

$$\left.\begin{array}{l} \varepsilon = 1.5\ \text{cm} \\ B = 6\ \text{T} \\ \gamma = 2 \end{array}\right\} \begin{array}{l} \rightarrow \theta_c = \frac{1}{\gamma\beta}\sqrt{\varepsilon \cdot \frac{eB}{mc^2}} = 0.25 \\ \lambda_c = 12\ \text{cm} \times 2\pi \simeq 75\ \text{cm} \end{array}$$

assume $\frac{\lambda_c - \lambda_0}{\lambda_0} = 0.25$ ("quasi-resonance")

a) $\ell = \lambda_0$ (continuous helix)

then $\frac{b_0}{B} = 0.0625$; $\boxed{b_0 \approx 4\,\text{kG}}$

b) $\ell = \lambda_0/4$, then $\boxed{b_0 = 1.6\ \text{T}}$

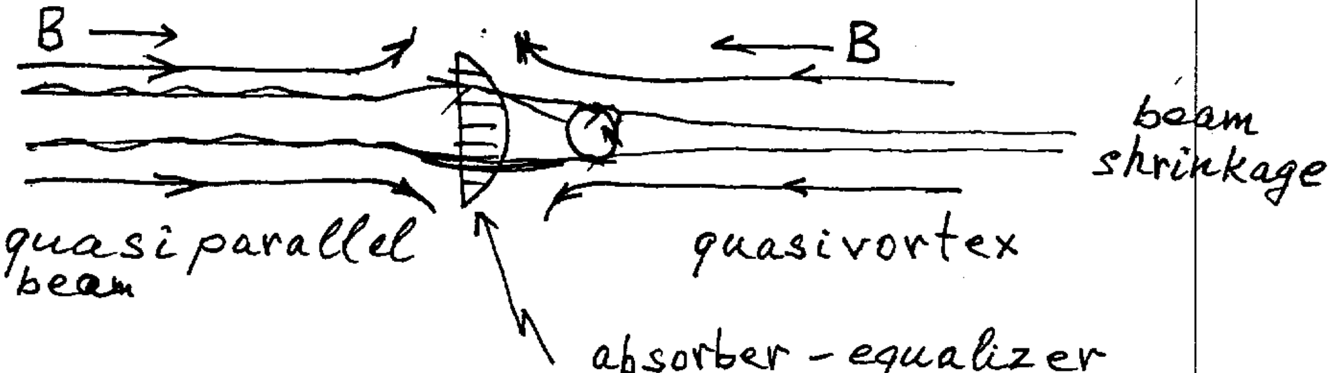
- Required $b_0 \ell$ value reduces with cooling

## Options for NF

- Mini-Cooling with total emittance exchange (i.e. $\Lambda_c \approx 0$, $\Lambda_d \approx 0$):

$$\Delta p = \Delta p_{in} \cdot (\gamma/\gamma_{in})^2$$

- Second cooling stage: $\Lambda_{\parallel} = \Lambda_c$ ; $\Lambda_d \approx 0$
two long solenoids




$$\frac{\Delta\gamma}{\gamma} = \frac{1}{2}(\gamma\beta\theta)^2 = \frac{1}{2}\left(\frac{eB\rho}{mc^2}\right)^2$$

- Final stage: $\Lambda_{\parallel} = \Lambda_c = \Lambda_d$
(or different optimization, to obtain minimum $\mathcal{E}_6$)

- Post-equilibrium stage:
reverse fast emittance exchange

$$\Lambda_{\parallel} < 0 \,, \qquad |\Lambda_{\parallel}| \gg \Lambda_6$$

$$\Lambda_c + \Lambda_d \gg \Lambda_6$$

## Conclusions

- Spiral cooling concept seems efficient and flexible:
  - Continuous solenoid
  - Non-difficult dipole arrangement
  - Stable beam
  - Effective emittance exchange
  - 3-dimensional cooling
  - Mini-Cooling at total emittance exchange seems very important possibility — to make it easier the beam RF capture in both projects, Neutrino Factory and Muon Collider
- Spiral Cooling seems capable to provide the minimum 6-dim emittance (reduction by a factor $\approx 10^6$).