Bent Solenoids as Beam Transport Elements

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Bent solenoids can transmit charged particle beams while providing momentum dispersion. While less familiar than quadrupole and dipole systems, bent solenoids can produce superficially simple transport lines and large acceptance spectrometers for use at low energies. Design issues such as drift compensation and coupling sections between straight and bent solenoids are identified, and aberrations such as sheers produced by perpendicular error fields are discussed. Examples are considered which provide the basis for the design of emittance exchange elements for the cooling system of a muon collider.

I. INTRODUCTION

Solenoidal focusing systems have been used in many low energy beam applications for many years [1]. High energy optical systems which will use solenoids with bends are also being considered, for transport and cooling of muons in a muon collider [2] [3], transport of muons for the study of muon decays [4] [5], charge separation [6], and electron cooling of high energy proton and antiproton beams [7].

Although quadrupole focusing systems are useful at high energy, solenoids tend to be used more for low energy beams, since the focusing power, K, of solenoids goes like $K \sim 1/f \sim p^{-2}$, where f and p are the focal length of the solenoid section and momentum of the particles. Straight solenoids also have the advantage that they can have large apertures, and are homogeneous and simple.

Since bent solenoids can produce dispersion, these components can do many of the things normally associated with magnetic spectrometers, such as charge separation, magnetic analysis, and longitudinal to transverse emittance exchange. On the other hand, bent solenoids are generally less familiar than dipoles and quads since their optics involve cross field drifts, error fields, the effects of straight to curved transitions and coil dimensions, orientations and external fields.

The mechanics of particle motion in essentially circular solenoids has been covered in Ref [1], however fields produced by physical coils in a geometry with a variable radius bend have a number of additional problems which have not been discussed in the literature. This paper reviews the optics of bent solenoids, developing algorithms to evaluate aberrations and describing methods to minimize these problems. Since the effects can at first seem nonintuitive, they are described systematically, with ray tracing examples using the code GPT [8]. An outline for the design of practical systems is presented. The focus of this note is the optimization of compact high dispersion systems for longitudinal emittance exchange and momentum measurements for experimental muon cooling systems [9]. Examples are shown which give the magnitude of effects in realistic cases.

II. VARIABLES

One of the basic assumptions generally made in high energy beam optics is that **B** fields are perpendicular to the direction of motion. Low energy transport lines use solenoids either as short lenses or confining fields, but as the energy increases, maintaining a field parallel to the beam direction is inefficient. Cases where the beam and the fields are roughly parallel have been worked out in detail in plasma physics examples [10].

To a first approximation, the design of a straight solenoid requires only that coils be placed in a line, with the current circulating around the desired volume equal to $i = B/\mu_0$, where i is the current per unit length, B the desired field and μ_0 is the permeability constant.

The Larmor frequency, radius and wavelength for motion in a solenoid are

$$f_L = qB/2\pi\gamma m_0$$

$$r_L = v_\perp/2\pi f_L = \gamma m_0 \beta_\perp c/qB = (B\rho)_\perp/B$$

$$\lambda_L = \beta c/f_L = 2\pi\gamma m_0 \beta c/qB = 2\pi B\rho/B$$

where c is the speed of light, f_L is the Larmor frequency, m_0 is the rest mass of the muon, and q its charge. The magnetic rigidity, $B\rho$ in Tm, is equal to $p/0.3$, where the momentum is in GeV/c. The Larmor oscillations are somewhat similar to betatron oscillations with the beta function equal to $B\rho/B$. This paper assumes the magnet dimensions are at least a few λ_L.

For a bent solenoid the same relations must still apply. One nevertheless needs to be concerned with the details of the cross field drift, which will tend to force particles perpendicular to the plane of the bend. In addition, accurate calculation of fields and orbits in the transition between the straight and curved sections is important since the mismatch in the orbits in the two sections will tend to excite Larmor oscillations around what would be the equilibrium orbit in the next section. Thus, the basic parameters of a bent solenoid are determined by: 1) the dimensions of the system (B field, coil and bend radius, R, and bend angle Θ), 2) the method used to couple the bent section to a straight solenoid, and 3) the vertical field and the method of providing it, as shown in Figure 1.

There are a number of additional complexities. In a straight solenoid the field lines are homogeneously distributed, but in a bend. Thus the field lines are distributed according to $1/R$, where R is the radius of the bend, thus the transition must require some field line redistribution in the bend plane, and, except in special cases, perpendicular to it. There are a number of ways of specifying the field in the bend, depending on whether: 1) the field is determined along some specified orbit, 2) the overall magnetic flux $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$ is constant through the bend (giving no fringe field at the coupler), 3) or some other constraint. In general the desirability of eliminating external fields may have more weight than other constraints. The dependence of $\Phi(r/R)$ on the aspect ratio is shown in Figure 2 below, for constant magnetic induction on axis.

A plot of the radial field in the straight section before the bend shows the fields from the bend sections penetrating into the straight section, as shown in Figure 3. These fields could produce tracking errors with beam diagnostics located in the straights.

Compensating fields perpendicular to the bend plane can be used to minimize cross field drifts, however there are a number of ways of applying these fields and the beam optics depend strongly on the particular geometry, producing rotation around the center of the beam and sheers in the bend plane and perpendicular to the bend plane of the beams.

III. COUPLING SECTIONS AND EMITTANCE GROWTH

The simplest example is a series of coils of radius r in a straight line, where a sudden transition is made to an arc of radius R. This geometry is shown in Figure 1, together with the path of a particle tracked through it, both in two dimensions and three dimensions. The particle orbit is shown with the bend straightened out for simplicity. The vertical drift can be caused by two effects: centrifugal drift,

$$\mathbf{v}_C = \frac{\gamma m_0 v_\parallel^2}{qB^2} \frac{\mathbf{R} \times \mathbf{B}}{R^2},$$

which depends on the components of the beam parallel to the magnetic field, and grad B drift,

$$\mathbf{v}_{\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\mathbf{B} \times \nabla B}{B^2},$$

which depends on components of the beam velocity perpendicular to the magnetic field [10]. The total drift is the sum

$$\mathbf{v}_{drift} = \mathbf{v}_C + \mathbf{v}_{\nabla B} = \frac{\gamma m_0}{q} \frac{\mathbf{R} \times \mathbf{B}}{R^2 B^2} (v_\parallel^2 + 0.5 v_\perp^2).$$

In high energy beam transport the perpendicular components of the velocity tend to be small, $< 10\%$ of the parallel components, so the grad B drift is usually a much smaller effect than the curvature drift, i.e. $< 0.5\%$. This is shown in Figures 4 and 5 for particles with $x' = 0$ and 0.05. The dominant effect is thus centrifugal drift. Note that the drift angle is $\phi = v_\perp / v_\parallel = B\rho / B_\parallel R$. In Figure 5, the effects of cusp orbits and drifts have been cancelled by the methods outlined below to emphasize the effects of perpendicular momentum.

A. Simple Bends, $L = n\lambda_L$

An examination of the drift orbit in Figure 4 will show that a particle coming into a bent section effectively begins to execute a smooth vertical drift motion around a point radially offset from the initial major radius, plus a Larmor oscillation around the point of vertical drift. The sum of the two motions gives a cusp motion like the point of a circle rolling on a line with zero instantaneous motion at the entrance point, but rapid vertical motion at large R. The radius of the circle, as well as the offset of the orbit is equal to $r = \gamma m_0 v_{drift}/qB = (B\rho)_{drift}/B$. In fact there can be some additional vertical drift not coupled into Larmor oscillations due to the smoothness of the coupling, as described in Section IIIB, below.

The emittance of a beam is affected by the length of the bent section, since a nonintegral number of cusps will result in a residual perpendicular velocity which will result in Larmor oscillations. On the other hand if the bend is exactly $L = \lambda_L$ long, there should be no transverse motion at the end of the cusp and it should be possible to exit the bend and couple directly to a straight section with minimal emittance growth [9]. Since the Larmor length is momentum dependent, such a bend will be optimized for only one momentum. Bends of $L = n\lambda_L$, for small integers n, should also have zero emittance growth, however for even smaller momentum acceptances.

Two effects can cause emittance growth when a straight and bent solenoid meet. One is due to the radial offset of the magnetic field lines in the bend, and is a function only of coil geometries, and the other is due to the centrifugal offset of the beam around the bend. Both can excite oscillations leading to emittance growth and both can be effectively eliminated by a correct coil design, at least for one momentum. The centrifugal offset is eliminated by using compensating fields which eliminate vertical drift, and the geometrical offset can be eliminated by using couplers which are tuned to minimize the oscillations excited by the radial field offset. In cases studied here, the centrifugal effects are larger than thosed caused by radial offset of the field.

B. Adiabatic Couplers

If space and cost were not a consideration, the solution to most of the problems associated with bent solenoids would be to have an adiabatic transition between bend and straight sections. Figure 6 shows the effects of an adiabatic coupling section.

Adiabatic couplers can transmit a very wide momentum spectrum, however the length of the coupler would be $n\lambda_L$, with n large, for the highest momentum transmitted.

C. $L = \lambda_L/2$ Couplers

In general, coupling between straight and bent solenoids is easiest when not only the bending angle changes smoothly with distance but the first and second derivatives are also smooth. An exception to this is a special case where the cusp motion itself can be used to couple to bends of arbitrary radius.

Using the picture described in Section III A, it is possible to couple a straight section to a bent section with minimal residual oscillation, by using half a cusp oscillation between the straight and bent sections. Thus one can use a bend section of arbitrary length and dispersion with compact coupling. An example of this is shown in Figure 7.

This method, while practical in special cases, may be difficult to optimize, since coils of finite radii tend to introduce end effects which complicate the geometry of the transition region and can result in some residual oscillation. The effect is due to the fact that the magnetic field lines do not follow the curvature of the coils unless the coil radius is very small. This effect is present in all coupler geometries. The example shown in the figure was not entirely optimized and shows the effects of such a slight mismatch.

D. Smooth $L = \lambda_L/2$ Couplers

It has been shown from tracking results that not only does $d\Theta/ds$ have to be continuous to avoid exciting Larmor oscillations and offsets, but the derivatives also have to be continuous and smooth. This can be accomplished by a number of options, but the smooth bend profile used by Fernow in ICOOL [11] has been found useful because it is simple and has smooth derivatives. To some extent this method is a combination of adiabatic and $\lambda_L/2$ coupling. The expression for a bend starting at s_1 and ending at s_2 with coupling length σ, is

$$\frac{d\Theta}{ds} = 0.5 \; [\; \tanh(\frac{s - s_1}{\sigma}) - \tanh(\frac{s - s_2}{\sigma}) \;].$$

The best measure of the usefulness of this curve is given by the amplitude of osillations excited at the end of a bend. Plots of the radius of these oscillations are a realistic measure of the magnitude of emittance growth using this method. Such a plot is shown in Figure 8, which plots the radius of the Larmor oscillation excited as a function of the length σ

over which the transition is made. It is seen that the minimum at $\sigma \sim \lambda_L/2$ is roughly consistent with the arguments presented in Section III C.

IV. DRIFT COMPENSATION

In order to eliminate the effect of the perpendicular drift, it is possible to introduce a vertical field equal to $B_v = B\rho/R$, where $B\rho$ is the rigidity of the beam and R is the radius of the bend. There are a number of ways of providing this field using external field coils: tilting the solenoidal coils to produce a transverse component, external coils, or a combination of external coils and tilted coils.

The drift velocity in a strong $\mathbf{B}$ field is given by, $\mathbf{v}_{drift} = (\mathbf{F} \times \mathbf{B})/qB^2$, for a force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B}_\perp)$. Using the vector identity $(\mathbf{S} \times \mathbf{T}) \times \mathbf{U} = \mathbf{U}(\mathbf{S} \cdot \mathbf{T}) - \mathbf{T}(\mathbf{S} \cdot \mathbf{U})$ with $(\mathbf{v} \cdot \mathbf{B}_\perp) \sim 0$ allows one to see that that the drift velocity is in the direction of the perpendicular field component producing the drift, $\mathbf{v}_{drift} = v_\| \mathbf{B}_\perp / B_\|$, which is equivalent to saying particles follow field lines.

The simplest geometry for producing a uniform vertical field over a small circular region would be to use external coils carrying a longitudinal current whose density is proportional to the cosine of the angle from the horizontal. This geometry has wide application in the design of superconducting bending magnets. We have considered using external fields of this type, however the overall configuration of a system with bends of varying radii, including the return coils at the ends of the arcs and the effects on the beam from these return coils, is difficult to design and optimize.

A. Tipped and Turned Coils

It is possible to tilt the solenoidal coil elements to provide the majority of the required vertical component without external coils, sas shown in Figure 9. Note that the tilted coils produce a longitudinal component of the current which is proportional to $\cos\theta$, as described above, and the tilting the coils by an amount proportional to the local bend radius provides a natural transition between the straight and curved regions. The vertical bend field which can compensate the centrifugal drift is most easily provided by introducing a tip angle θ which produces a vertical component of the solenoidal field $B_v(x,y)$ for a particle moving down the center of the solenoid. This angle can be adjusted using tracking to give zero net vertical drift through a bend as shown in Figure 10. Tracking has given a value of

$$\theta \sim \frac{6.7\, p_{[GeV/c]}}{R_{[m]}\, B_{[T]}}$$

for local radii and fields of a few m and T. Since the tilt angle depends on momentum, this option tends fixes the central momentum of the bend. Horizontal drifts are smaller and the yaw angle, η, required for the coils to be turned is a small fraction of the tilt angle, perhaps $\eta \sim 0.04\ \theta$. Note that the field produced by tipped solenoidal elements is, in principle, fairly close to the $1/R$ dependence that would be desirable for transmission of a beam over a large horizontal aperture.

In the case shown in Figure 10, a number of details of the optics are visible. A particle enters the magnet on axis and is deflected towards smaller R, since it tends to follow the inward motion of field lines going around the bend. A slight upward then downward motion is also seen in the coupling section, due to a slight mismatch between the required vertical field and that produced by tipping the coils. This effect could be presumeably eliminated by trimming the longitudinal distribution of the vertical field by trimming tilt angles of the magnets, (i.e. using an slightly increased value of σ for the tilt angle distribution. At the exit of the magnet the field lines return to their initial distribution, and the same slight mismatch produces a downward then upward motion. A reference orbit can exit with the same (x, y) coordinates it entered the bend.

B. Dispersion

A bent solenoid will produce a vertical dispersion due to the momentum dependence of the vertical drift angle. This vertical dispersion will be modified by the vertical drift introduced by the compensating field.

The vertical drift angle, including the centrifugal term, will then be

$$\phi = v_\perp / v_\| = B\rho / B_\| R \ - B_\perp / B_\|,$$

and the dispersion is due only to the first term,

$$D = \int B\rho/B_{\|} R \, ds \sim \Theta B\rho/B,$$

where Θ is the total bend angle. Figure 11 shows the results of tracking a particle 1.5 times the nominal momentum shown in Figure 10. The longer Larmor length produces some residual perpendicular velocity as described in section III D.

C. Sheers

Although particles can be transmitted through the system without emittance growth, translation or deflection, the motion of off axis particles is complex. The primary aberration associated with off axis orbits is a linear sheer between the entrance and exit of a bend. Tracking results from an array of particles introduced into a bend with circular coils tipped and turned to give no axial deflection are shown in Figure 10. These tracks show that there is an offset between the entrance and exit positions and, for a bend angle of $\Theta = \pi/2$ and a 5 T field, this offset has the form

$$\Delta x = 0.11\ y, \quad \mathit{and} \quad \Delta y = 0.047\ x,$$

assuming that the central ray $(x, y) = (0, 0)$ shows no deflection. A preliminary analysis of the functional dependence has shown that the offsets are primarily linear.

These sheers result from perpendicular field components due to the tip and turn of the coils. For tipped circular coils and a circular bend, there is a horizontal component of the vertical field that depends on vertical position, resulting in a curvature of the vertical magnetic field lines. This curvature causes the horizontal drift/sheer. The vertical drift/sheer is caused by the difference between the vertical field produced and the $1/R$ field required. These components can be modified somewhat by external coils.

Ideal beam optics requires a vertical field whose intensity is proportional to $1/R$, with no horizontal components in the volume occupied by the beam. Unfortunately this requirement violates Ampere's Law. Integrating $\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$, where $I = 0$ is the inclosed current, would require nonzero horizontal fields at the top and bottom if the values of $\mathbf{B}$ at two radial positions were different, as shown in Fig. 12. Thus, the sum of the magnitudes of the horizontal and vertical sheers produced is an invariant for a given bend geometry, dependent on the aspect ratio of the bend, r/R, end effects, and the particular geometries of how currents are returned. Accurate evaluation of aberrations must rely on calculation directly from the exact coil geometries since these effects are caused by geometry of the coils.

D. Optimizing External Fields

For circular bends, the field can be provided entirely by the tipped and turned coils or external coils can be used either as correctors to provide additional field uniformity and flexibility. For the cases described here, the length of the bent solenoids tends to be small multiples of the $2 \times \lambda_L/2$ probably required for the two coupling sections.

External coils seem to provide the most flexibility, however the fields required would tend to be strong. Using tipped and turned coils may provide less of a magnetic perturbation on the beam, but it seems desirable to have some correction capability to minimize sheers and provide some momentum tuning capability. This implies that some capability of generating fields from external coils is desirable.

V. APPROXIMATIONS

The processes outlined above imply that the primary method used to attack the problems of bent solenoids should be to use coil positions to derive fields and then track particles through fields. While precise, this method can be very time consuming, and the relevance of tracking simpler cases inevitably arises. The problem with approximations is that the phenomena described here tend to be due to the strong longitudinal field, which is not present in most particle tracking environments, and the specific positions of coils and the way these coils are used to make transitions between bent and straight solenoids. Since these problems tend to be associated only with this class of coil geometry, the effects are likely to have nonintuitive results. Thus it seems desirable to avoid approximations until they have been carefully calibrated against precise calculations.

VI. EMITTANCE EXCHANGE SECTION DESIGN

Emittance exchange from longitudinal to transverse requires that the beam be dispersed in momentum so that wedges can then preferentially degrade the high energy beams. In general this operation will take place in more than one bend. The particular example described in Figure 13, below, proposed by Palmer and Fernow [11], uses short smooth couplers with $L = \lambda/2$ to attempt the maximum possible dispersion with the minimum emittance growth at the transitions. Tracking [2] has shown that transverse emittance is not significantly increased by the bends.

A. Magnet Issues

The primary magnet constraint seems to be the ratio of dispersion/cost, since a very wide range of parameters will satisfy the optical constraints. It is assumed that the coils are spaced densely enough so that coil ripple will be negligible. However, the coils can be quite large and bend radii can be small, giving aspect ratios r/R limited primarily by coil engineering. Dispersion is limited by the fact that bends greater than $\Theta = \pi$ are difficult. Smooth $\lambda_L/2$ couplers seem to provide the best combination of small emittance growth and minimum stored energy (cost), and tipped coils seem to be the simplest way of providing a compensating field, if desired. With no compensating field both charge states could be accommodated, however the volume and cost would be increased. A variety of coil shapes could be used and these would weakly affect the optics.

B. Wedge Constraints

Emittance growth must also be minimized in the energy loss process. Wedges must be distributed along the beam over a length $> \lambda_L/2$. Since the centroid of a Larmor oscillation will be moved an amount $\Delta p/p\ r_L$ by a single absorber, if $\chi_{\mathbf{i}} = \Delta p/p\ \mathbf{r}_{\mathbf{L},\mathbf{i}}$ is a vector whose amplitude is the magnitude of the absorber, and whose phase is the phase of the Larmor oscillation, there should be minimal emittance growth if the vector sum $\Sigma\chi_{\mathbf{i}}$ is minimized, or zero over the length of the absorber.

C. Timing

A particle which moves around a bend on a larger radius will fall behind another on a smaller radius by an amount $\delta s = \Delta R\Theta$, where the bend angle is Θ and the difference in radii is ΔR. The time interval generated will be $\delta t = \delta s/\beta c$. Since the magnetic flux lines are continuous through a series of bends it is a small additional constraint that the total number of right and left, up and down bends is equal, which will cancel this effect. An additional complexity is time spread due to the difference in momenta. It is assumed that linac sections can cancel or compensate for this using sections which have a synchrotron phase advance of π radians, which will slow down fast particles and speed up slow ones.

D. Momentum Acceptance

The momentum acceptance of the bend system will be limited by the magnet aperture and the tolerable emittance increase from mismatches which become somewhat more difficult at higher momenta, due to the longer larmor length. Designing large acceptance, high dispersion, systems seems to be straightforward.

VII. CONCLUSIONS

Although superficially simple, optimization of bent solenoid systems can be difficult because of their nonlinear behavior and the lack of symmetry. This makes the use of approximate solutions and simplifying assumptions difficult, in what, for most, can be a nonintuitive environment.

While the basic beam optics of bent solenoids is fairly simple, depending on the dimensions of the coils and the bends, it is important to optimize coupling sections between straights and bends, use appropriate fields for drift compensation, and understand aberrations that may result from the completed system. Tracking programs which use coil positions and currents as input give the most complete and reliable solutions to these problems, and their input provides a natural interface with those who would actually construct such a coil system.

VIII. ACKNOWLEDGEMENTS

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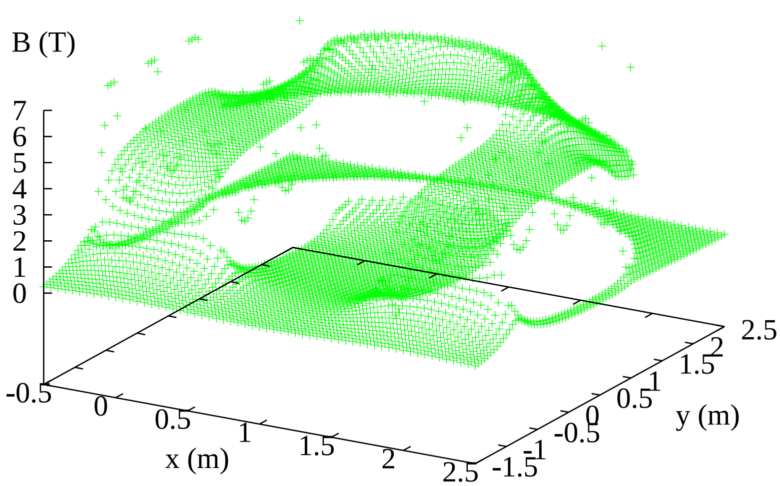

FIG. 1. A map of $\mid B(x,y) \mid$ over a bend which has two straight sections and no coupling sections.

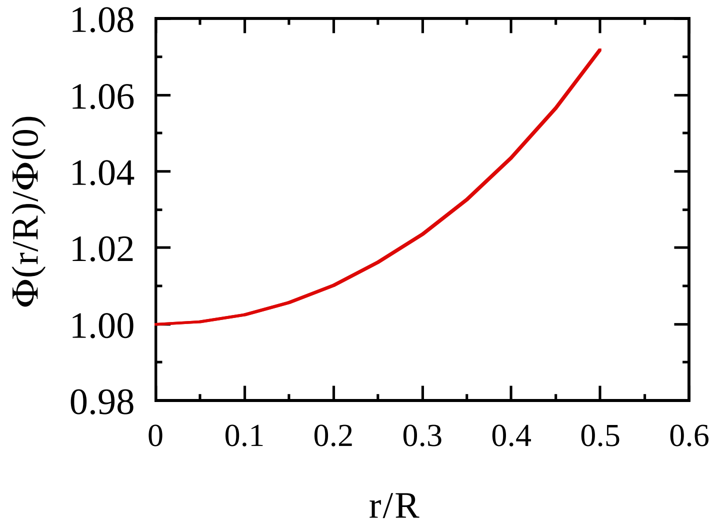

FIG. 2. Dependence of Φ on aspect ratio r/R, for constant axial field.

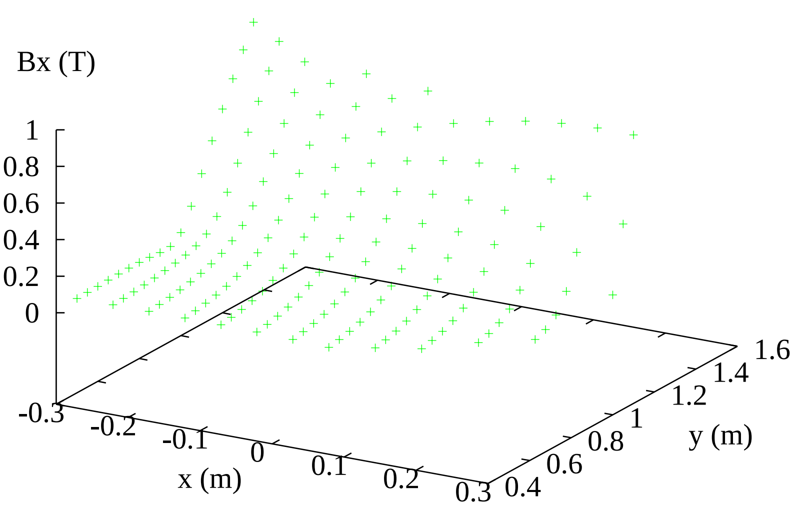

FIG. 3. Variation of $B_x(x, y)$, just before the bend.

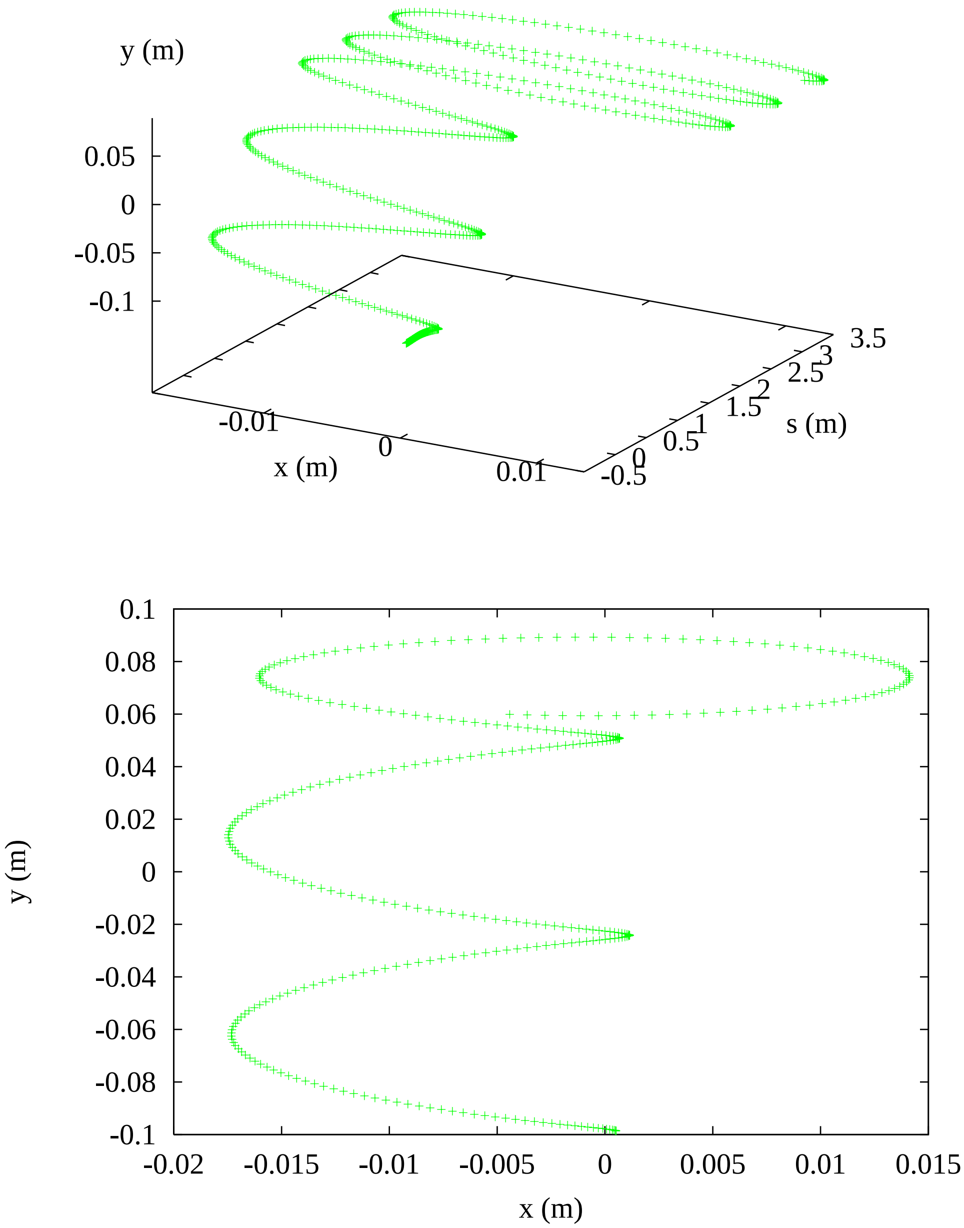

FIG. 4. vertical drift with cusp motion as a function of the length along the beamline s.

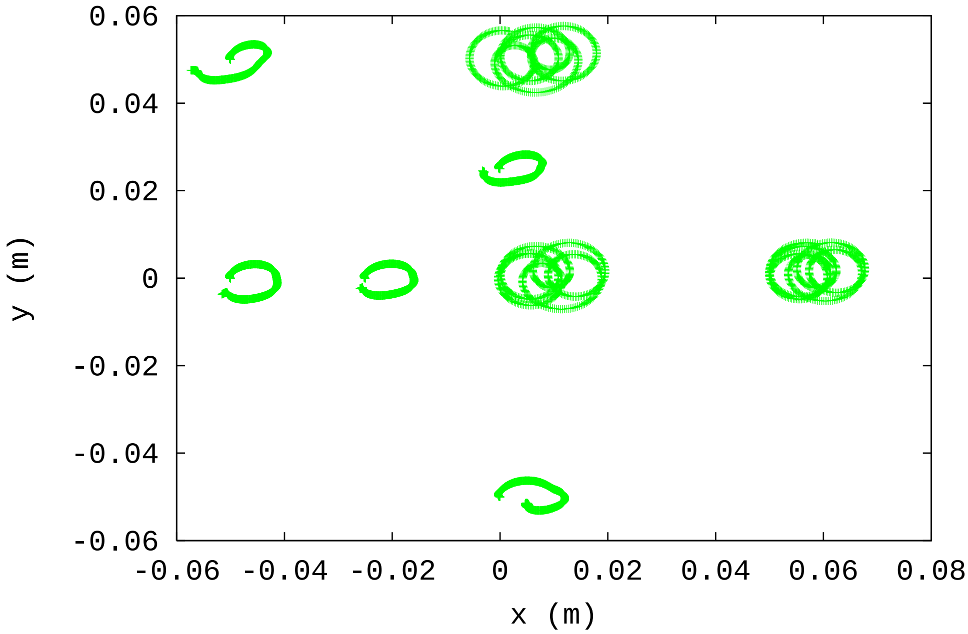

FIG. 5. Effects of perpendicular momentum (cusp effects have been cancelled). The particles at (0, 0) and (0,5 cm) and (5 cm, 0) have an initial $x' = 0.05$, which makes negligible change in their subsequent orbits.

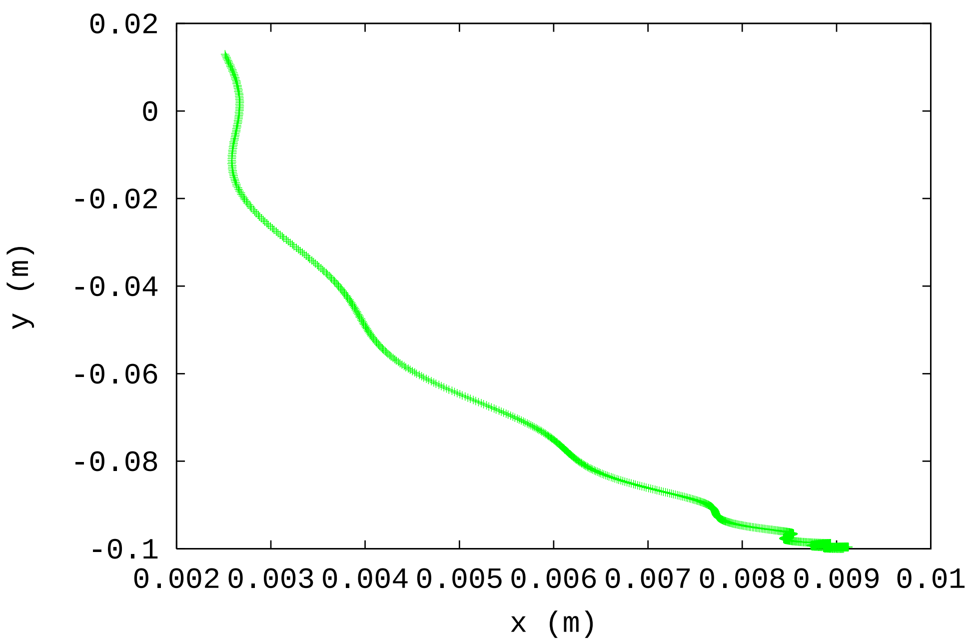

FIG. 6. An adiabatic coupler, the particle starts at y = -0.1 m.

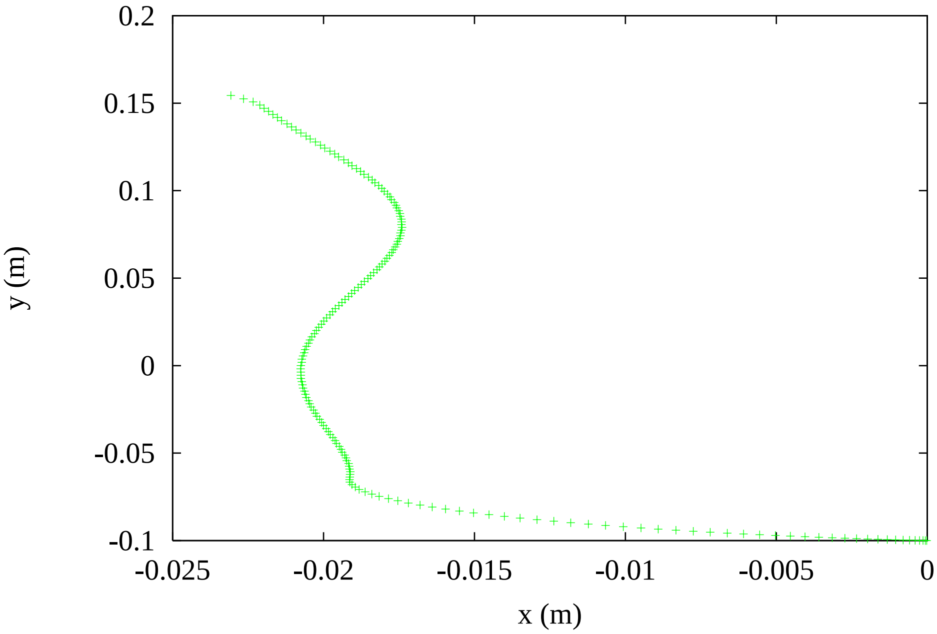

FIG. 7. An example of an $L = \lambda/2$ coupler with $R_{coupler} \sim 2R_{bend}$, showing some mismatch. In this case the coil radius was 0.15 m and the particle exited the solenoid.

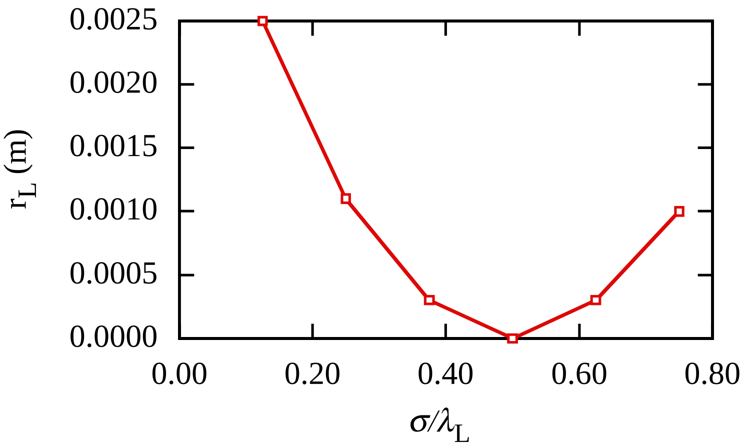

FIG. 8. Emittance growth vs coupling length in smooth couplers.

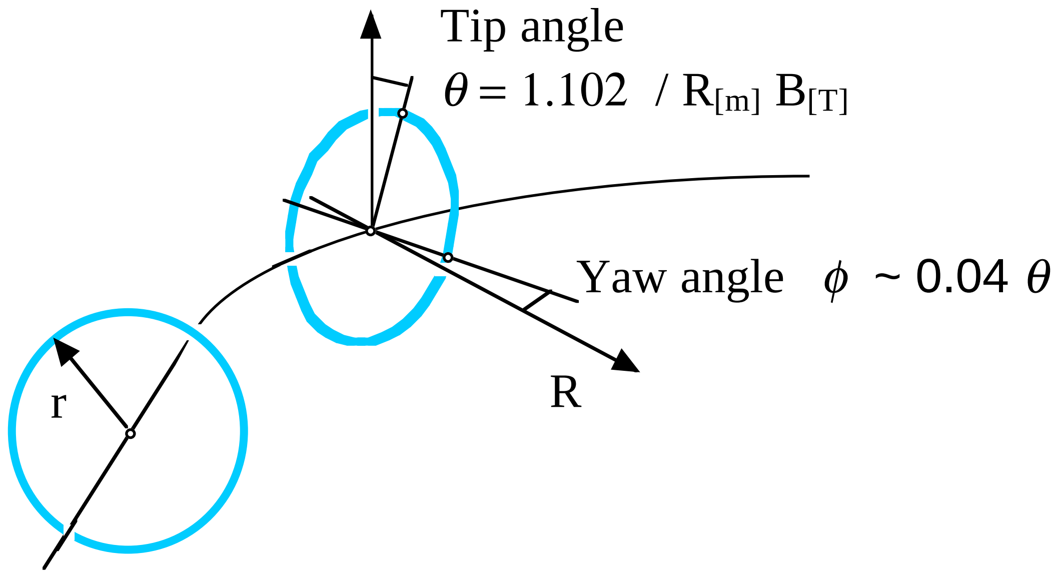

FIG. 9. tipped and turned coils for a momentum $p = 165$ MeV/c.

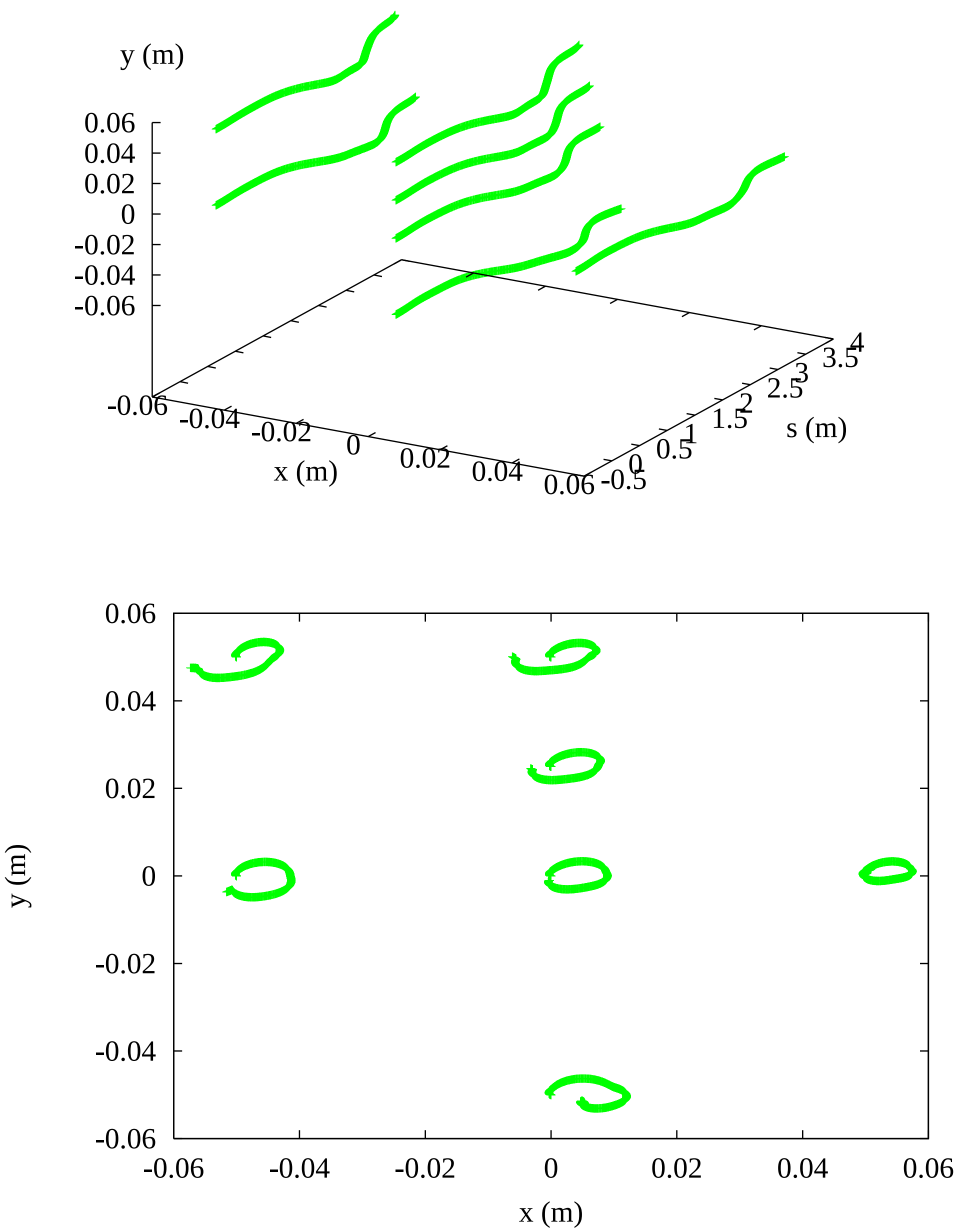

FIG. 10. Tracking through a bend with tipped and turned coils. The tracks show sheer from perpendicular field components.

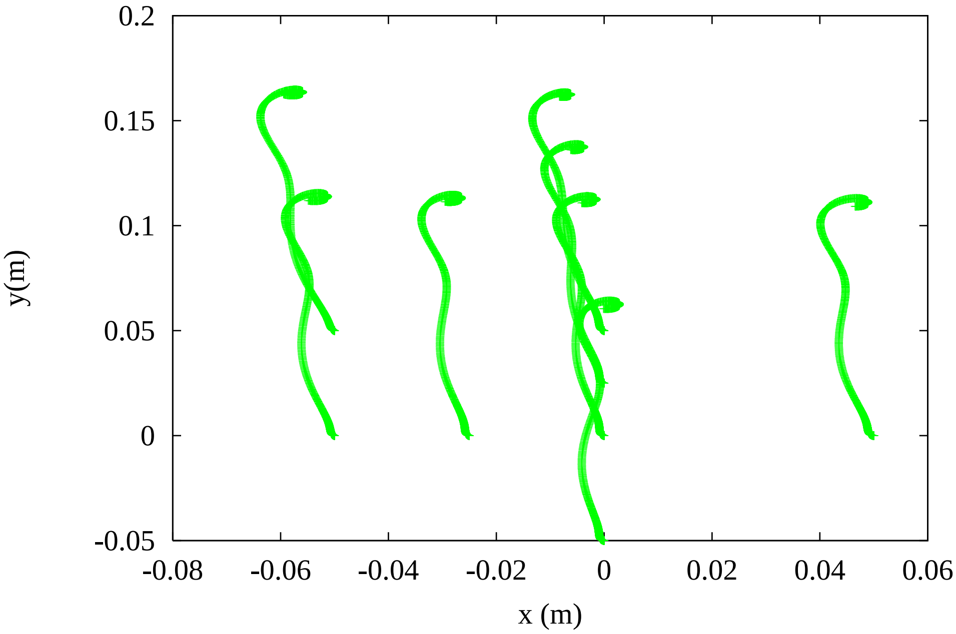

FIG. 11. Tracking a momentum 1.5 times that shown in Figure 10 shows some mismatch.

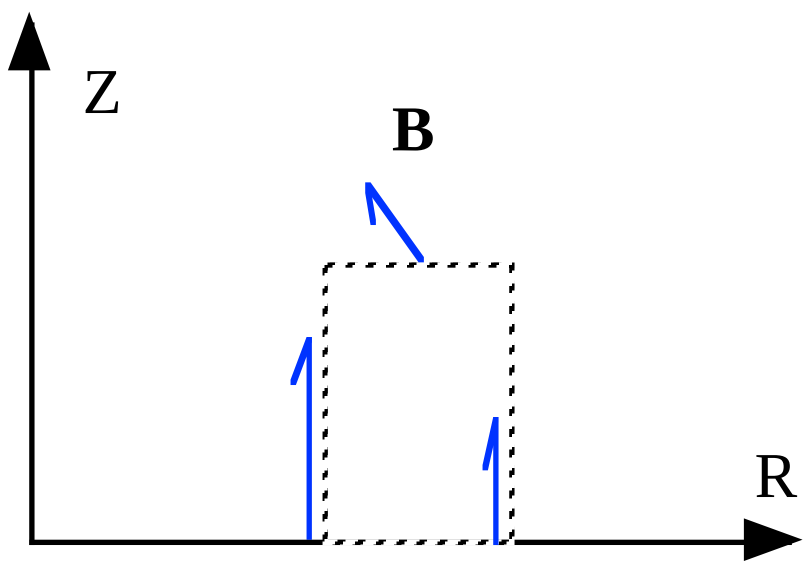

FIG. 12. The integral of $\int \mathbf{B} \cdot d\mathbf{l}$ around the dotted line must be zero if no current is carried.

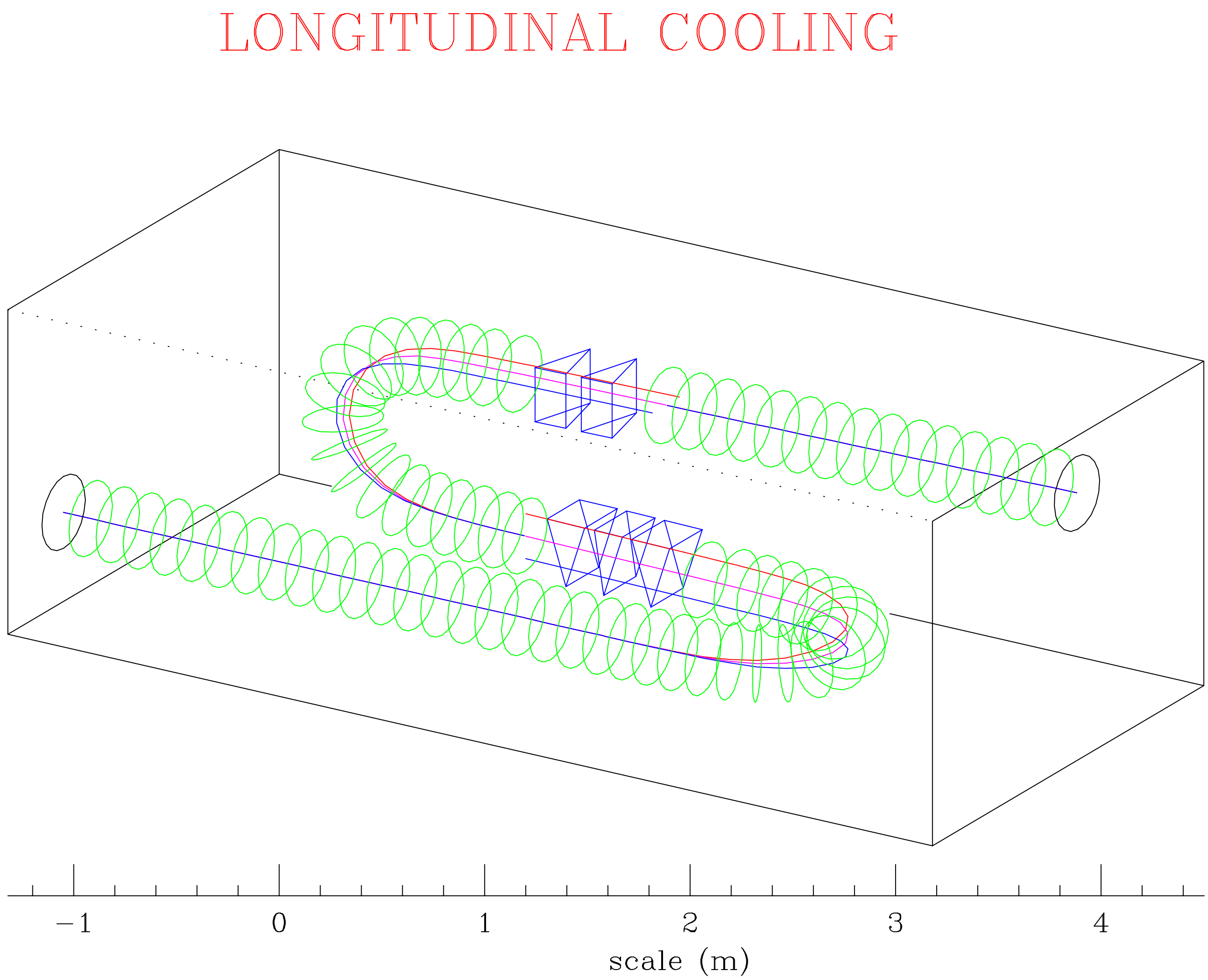

FIG. 13. An emittance exchange section.

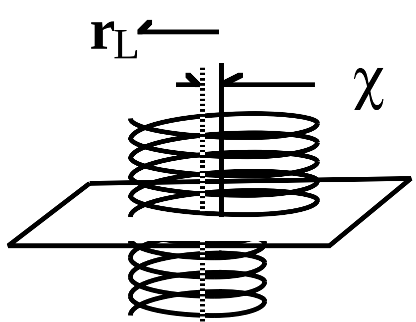

FIG. 14. Translation of centroid in absorber.