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Design of Transverse Muon-Cooling Channels for a Neutrino Factory

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Ionization cooling is the preferred method for transverse cooling of a muon beam in a neutrino factory. In this paper, we design two different cooling channels, a DF0F0 channel and an alternating solenoid channel, and show that the transverse beam emittance through these cooling channels is cooled down by a factor of 7.7 and 6.5, respectively. The cooling channel design has been carried out by invoking the ICOOL program. Transmission efficiency, transverse beam emittance, growth of longitudinal bunch length, and momentum spread are investigated. It is shown that these two cooling channels provide adequate cooling performance, satisfying the so-called Palmer, Johnson and Keil (PJK) parameter for a neutrino factory. It is also shown that the numerical results agree well with those of analytical formulae.

KEYWORDS: ionization cooling, muon cooling, solenoid focusing, neutrino factory

1. Introduction

A challenge in the design of a neutrino factory is the cooling of a muon beam. When muons are produced from a target, they occupy a large phase-space area. A successful design of a neutrino factory therefore requires that the transverse emittance of the muon beam after the capture, phase rotation, and buncher channels be sufficiently reduced so that the muon beam can be accelerated efficiently in downstream sections. Furthermore, fast cooling is required because of the relatively short lifetime of the muon beam. The lifetime of a muon is given by $\tau_\mu = 2.197 \times 10^{-6} E_\mu/m_\mu$ seconds where E_μ is the energy and m_μ is the rest mass of a muon. A cooling method that can meet such a requirement is ionization cooling.[1–5] In ionization cooling, particles pass through a material medium and lose energy through ionization interactions. The losses are parallel to particle motion and therefore both transverse and longitudinal momenta are lost. Only longitudinal momentum is then restored by acceleration through rf cavities. This results in a reduction of the angular spread of a beam, and therefore leads to a decrease in the transverse beam emittance. However, the random process of multiple Coulomb scattering in material medium leads to an adverse effect, namely the rms beam emittance increases. When an absorber material is placed in a strong focusing field, the heating can be minimized.

The equation for the normalized transverse emittance ϵ_n is given by[3–5]

$$\frac{d\epsilon_n}{dz} = -\frac{1}{\beta^2}\frac{dE_\mu}{dz}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp E_s^2}{2E_\mu m_\mu c^2 L_R}, \tag{1}$$

where z is the path length, E_μ is the muon beam energy, $\beta = v/c$, L_R is the radiation length of the absorber material, $\beta_\perp$ is the betatron function, and E_s is the characteristic scattering energy which is approximately 13.6 MeV. The normalized emittance ϵ_n is related to the conventional unnormalized geometric emittance ϵ by $\epsilon_n = \epsilon\beta_\perp\gamma$, where γ is the conventional relativistic factor. In eq. (1), the first term describes the cooling and the second term represents the heating due to multiple scattering. As the emittance approaches an equilibrium value, the cooling term is balanced by the heating due to multiple scattering. Since the heating is proportional to $\beta_\perp$ and $1/L_R$, small $\beta_\perp$ (i.e., strong focusing) and a material with high L_R (i.e., a low-Z absorber) should be considered in order to maximize the cooling.

Although the theory of ionization cooling is well understood and useful in designing a muon-cooling system, one needs to carry out extensive numerical studies in a large parameter space for the design of cooling channels. In particular, a cooling system in a neutrino factory requires many successive stages of energy loss and re-acceleration by rf cavities, which makes analytical calculation difficult.

Several numerical codes have been written to investigate the cooling process. Among them, the ICOOL[6)] and the DPGeant[7)] programs have been widely used and they use Monte-Carlo techniques to track particles through a cooling system. To obtain strong focusing of a beam in absorber materials, several lattice configurations have been considered by a number of groups. The Lawrence Berkeley National Laboratory (LBNL) group designed a sinusoidal focusing field cooling channel (F0F0) using the ICOOL.[8,9)] The Brookhaven National Laboratory (BNL) group designed a cooling channel with bucked coils also using the ICOOL (super F0F0).[10)] The Fermi National Accelerator Laboratory (FNAL) group designed single and double flip channels using the DPGeant program.[9,11)] In addition to these cooling channels, a biperiodic configuration of the F0F0 lattice (DF0F0) and an alternating solenoid channel have also been designed for a neutrino factory.[12–14)] These groups mainly considered focusing by solenoids, that is, focusing by longitudinal magnetic fields $B_z(z)$. The solenoidal focusing has an advantage of focusing equally in both transverse planes. Solenoids can focus the beam to yield small $\beta_\perp$ at absorber materials, with the absorbers placed inside the magnet, allowing the use of extended absorber lengths.

In this paper, we consider two other cooling channels, a DF0F0 channel and an alternating solenoid channel. The code ICOOL is mainly employed for cooling calculation. In the ICOOL, energy loss is modeled by the Vavilov distribution function and multiple scattering is modeled by the Moliere distribution. We compare results of the ICOOL simulation with the analytical method by Kim and Wang.[15)] Engineering constraint in the design of the cooling channels is also investigated by considering realistic physical parameters. Results of

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these simulations may provide guidelines for optimal cooling systems. Recently, it was shown by front-end simulations of LBNL and FNAL groups that a huge normalized longitudinal emittance of about 70 mm was obtained after buncher channels.[9] Such a large longitudinal emittance requires acceleration section with large acceptance that itself requires high cost or a longitudinal cooling system. On the other hand, a design study of all the components of a neutrino factory by Palmer, Johnson and Keil (PJK)[16] requires that the cooling system result in normalized transverse and longitudinal emittances of about 1500 mm mrad and 30 mm, respectively. Our goal is thus focused to obtain cooling systems that meet the requirements set by the PJK design.

In this paper a DF0F0 channel and an alternating cooling channel are considered for the transverse cooling of a neutrino factory. First, we describe a feature of the designed DF0F0 cooling channel. Then the cooling performance is described by showing variations of transverse and longitudinal emittances through the designed DF0F0 cooling channel. The result of the numerical calculation is compared with that of the analytical formulae. Next, an alternating solenoid-cooling channel is described. Description of the cooling performance then follows.

2. Description of Cooling Channel

In general, a cooling channel consists of a series of identical cells. The length of each cell is chosen to avoid betatron and synchro-betatron resonances. These resonances can lead to large particle loss when the betatron wavelength equals the period of the magnetic field or the synchrotron-oscillation wavelength.

When cooling channels are designed, many different conditions have to be simultaneously satisfied to be an acceptable cooling channel. First, we use liquid hydrogen (LH) as an absorber material since it provides the lowest possible transverse emittance. The thickness of the absorber window is a critical parameter. It must be thick enough to sustain the pressure from the LH and as thin as possible to reduce multiple scattering. The thickness and the radius of the LH absorber and the LH window affect particle losses in the transverse direction. A thicker LH absorber yields more particle loss and therefore it requires a larger rf gradient or a longer channel length per cell. However, thick LH absorber makes it possible to use a thin LH window which can reduce multiple scattering. In our designed cooling channel, a relatively thick LH absorber is considered. The LH is assumed to be contained in a vessel with very thin windows on both ends. The length of the LH is chosen so that the increase in $\beta_\perp$ near the ends of the vessel is not significant. For modeling purposes, the cell is chosen to begin at the center of the absorber region. The total number of cells in the cooling channel is determined by the total ionization loss and the rf phase at which the muon beam is accelerated. An appropriate gap between the LH window and the rf cavity is considered for rf assembly and maintenance.

We also use high field solenoids for focusing since they provide a large angular acceptance as well as simultaneous strong focusing of a beam in both transverse directions. Engineering feasibility in magnets is investigated by considering such parameters as magnetic field, current density, and stress on the conductor. Our design follows the engineering constraint that is shown in a feasibility study report of the neutrino factory.[9] A conservative rule of thumb (based on keeping the hoop stress within manageable limits) for solenoids built from an Nb_3Sn superconductor is given by $BJR < 350$ MPa, where B is the field at the coil, J the area current density, and R the radius of solenoid from the z-axis. Sheets are used to generate magnetic fields. An appropriate interval between sheets is considered for a power supply from klystron. Radius, length, height and current density of sheets are set to generate appropriate magnetic fields.

The space between absorber materials is filled with rf cavities to compensate for beam energy loss in absorber materials. The cavity length is chosen to provide sufficient acceleration to match the energy loss in the LH. We take the pill-box type of the TM010 mode, with $\pi/2$ phase-advance per cell, equipped with thin beryllium windows of 125 μm thickness and five cells per each rf section in the simulation. It is also valuable to investigate different types of rf frequencies for the design of various cooling channels. Rf frequencies ($f_{\rm rf}$) of 250 MHz and 201.25 MHz are used for the DFOFO channel and the alternating solenoid channel, respectively, and an rf gradient of 15 MV/m is used for both cooling channels. Smaller rf gradients may be used if larger rf phase angles are considered. However, large rf phase angles increase nonlinear behaviors of the beam, resulting in more particle losses. Total power deposited on an rf window is proportional to the fourth power of the radius of the rf window. The radius of rf window is critical to the particle losses when the rf window with a small radius is located at the position with large beta function.

While the transverse emittances are cooled down through the cooling channel, the longitudinal emittance of a beam is increased due to straggling and path-length variations. We therefore use the six-dimensional normalized emittance (ϵ_6) as a figure of merit for evaluating the validity of the overall design. When a cooling channel leads to increasing the value of ϵ_6, we terminate the cooling channel.

3. DF0F0 Cooling Channel

As can be seen in the design of a F0F0 cooling channel,[8,9] particle losses in the transverse direction mainly occurred at the midpoint of rf cavities where beta function was a maximum. This requires a large radius of the rf window to reduce the number of particles lost, but this leads to a large power deposited on the rf window. In order to reduce particle loss in the transverse direction, we have reconfigured the F0F0 lattice in such a way that the period of F0F0 magnetic fields is doubled. One cell geometry of our designed cooling channel is shown in Fig. 1. The minimum values of magnetic field B_z occur at the midpoint of the rf cavities and absorbers. Figure 2 shows the magnetic field in one cell of the DF0F0 cooling channel. The lattice is periodic and solenoidal magnetic field sinusoidally varies on axis with a 1.8 m period and 3.8 T amplitude on the z-axis. The beta function in the DFOFO channel is a minimum at the midpoint of rf cavities and therefore large acceptance channels can be obtained. Figure 3 shows the variation of beam size (σ_x) in one cell. We note that the beam size has a minimum at the midpoint of the rf cavities so that particle loss at the position of the rf window can be reduced.

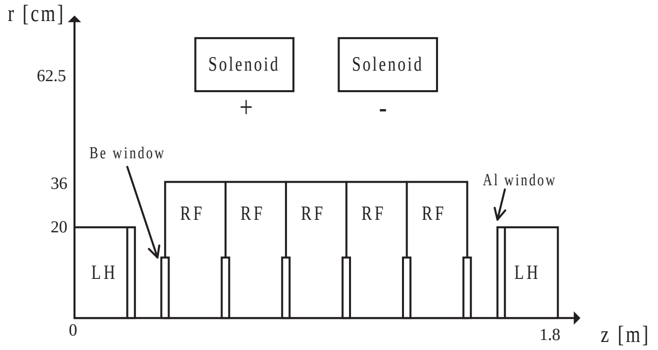

Fig. 1. One cell of the DFOFO solenoid cooling channel.

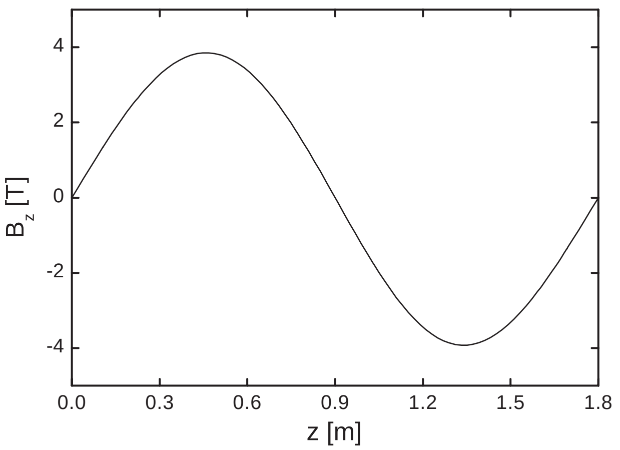

Fig. 2. Magnetic field (B_z) in one cell of the DFOFO cooling channel.

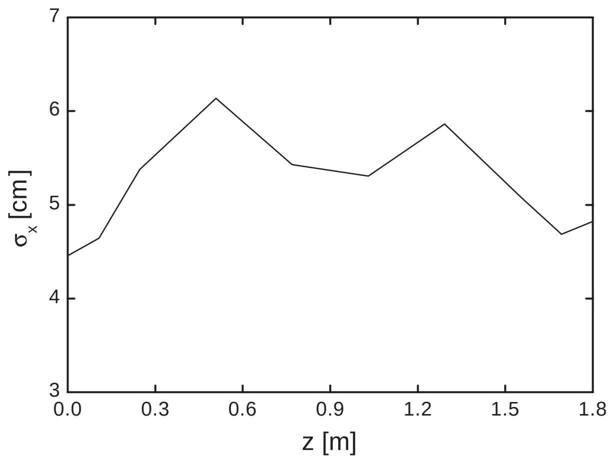

Fig. 3. Beam size σ_x (cm) in one cell of the DFOFO cooling channel.

Table I. Initial and final beam parameters in the 180 m DFOFO channel.

	Unit	Initial	Final	Final/Initial
Number of particles tracked		1000	721	0.72
Normalized transverse emittance (ϵ_{nx})	mm mrad	12212	1669	0.13
Normalized longitudinal emittance (ϵ_{nz})	mm	11.7	23.3	2
Normalized 6-D emittance ($\times 10^{-8}$)	(m rad)^3	253	9.14	0.024
Rms bunch length (σ_z)	cm	9.2 cm	1.2 cm	0.13
Rms beam size (σ_x)	cm	4.4	1.6	0.36
Rms momentum spread	%	13.2	19.7	1.5

Table II. Parameters of designed cooling channels.

	DFOFO channel	Alternating channel
Rf frequency	250 MHz	201.25 MHz
Rf gradient	15 MV/m	15 MV/m
Cell length	1.8 m	2.2 m
Magnetic field on the z-axis	3.8 T	3.4 T
Total channel length	180 m	160 m
Minimum beta function	35 cm	20 cm
Maximum beta function	70 cm	110 cm
LH length/cell	21.4 cm/24 cm/24.4 cm	30 cm
LH window thickness	650 μm/400 μm/300 μm	400 μm
LH window radius	20 cm/16 cm/9 cm	25 cm
Be window thickness	125 μm	125 μm
Be window radius	19 cm/15 cm/13 cm	20 cm

Table III. Parameters of solenoid.

	DFOFO channel	Alternating channel
Length	55 cm	55 cm
Height	15 cm	15 cm
Current density (J)	80 A/mm^2	58 A/mm^2
Radius (R)	62.5 cm	80.5 cm
Magnetic field on solenoid (B)	6.7 T	6 T
BJR	335 MPa	280 MPa

3.1 *Input beam*

The initial beam distribution before entering the cooling channel is generated by a random Gaussian with 12000 mm mrad transverse and 12 mm longitudinal emittances. A beam with a longitudinal momentum of $P_z = 212$ MeV/c with an rms momentum spread of 13.2% is used. The properties of the initial beam used in the simulation are shown in Table I.

3.2 *Parameters of the DFOFO cooling channel*

The designed DF0F0 cooling system is composed of three stage channels that have different thickness of aluminum window on both sides of the LH: first stage channel has 28 cells with window thickness of 650 μm, second stage channel has 25 cells with window thickness of 400 μm, and third stage channel has 47 cells with window thickness of 300 μm. The length of each cell of the cooling channel is 1.8 m. Major parameters of the DF0F0 cooling channel are shown in Table II. For the 3.8 T DF0F0, the relevant values are $B = 7.7$ T at the coil, $J = 80$ A/mm^2, and $R = 0.625$ m, giving $BJR = 335$ MPa. The parameters for the solenoid in the DF0F0 cooling channel are shown in Table III.

3.3 *Cooling performance*

In the designed DF0F0 cooling channel, the transverse emittance in both x and y planes is found to be reduced to 13% of its initial value, while the longitudinal emittance grows by a factor of two. The overall six-dimensional emittance is reduced to 3.6% of its initial value. Figure 4 shows the decrease in transverse normalized emittance as a function of the distance along the channel, while Fig. 5 shows the in-

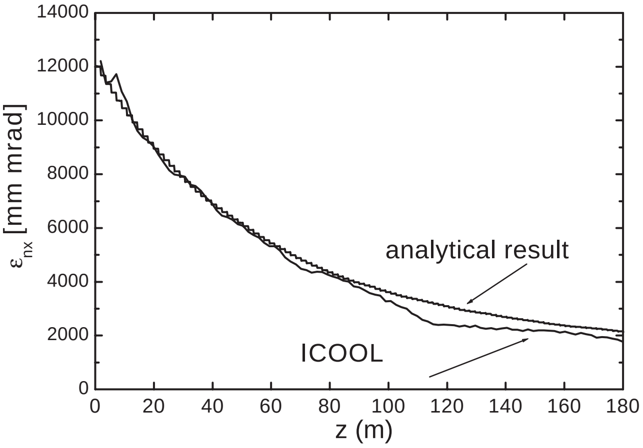

Fig. 4. Transverse emittance (mm mrad) as a function of channel length by ICOOL simulation and analytical formulae.

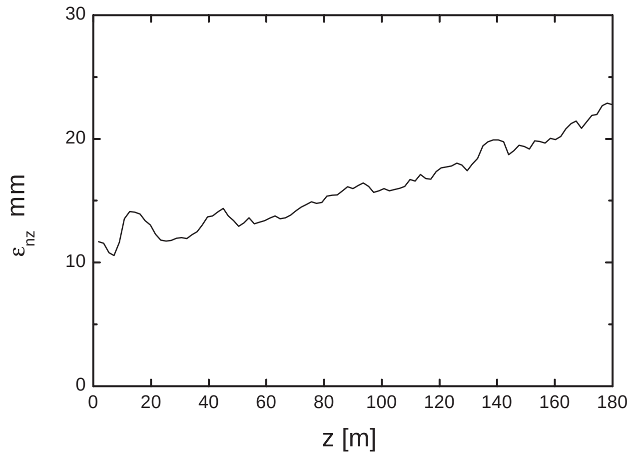

Fig. 5. Longitudinal emittance (mm) as a function of distance along the DF0F0 cooling channel.

Table IV. PJK cooling parameters for a neutrino factory.

	Unit	Initial	Final	Final/Initial
Normalized transverse emittance (ϵ_{nx})	mm mrad	9000	1500	0.17
Normalized longitudinal emittance (ϵ_{nz})	mm	15	30	2
Rms bunch length (σ_z)	cm	9	12	1.33
Rms momentum spread $\Delta P_z/P_z$	%	8	12	1.5

crease in longitudinal normalized emittance in the channels. The cooling channel shows a particle transmission of 72%. Table I also shows the final beam parameters of rms beam size σ_x, rms bunch length σ_z and rms longitudinal momentum spread. The PJK cooling parameters for the neutrino factory are shown in Table IV.

3.4 Comparison with analytical result

The simulation result by ICOOL is also compared with the result of the analytical method.[12] The analytical formula for the reduction of the transverse rms beam emittance is given

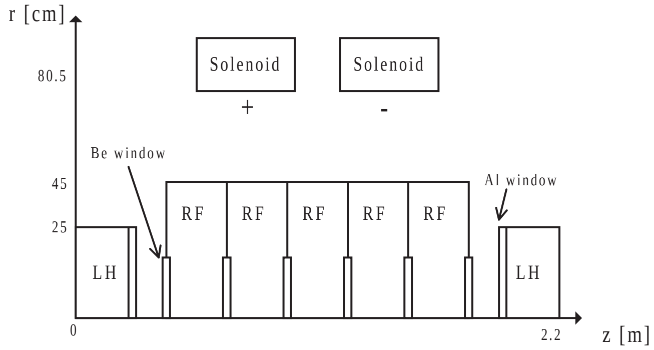

Fig. 6. One cell of the alternating solenoid cooling channel. In this case, the magnetic field of the adjacent cell has a mirror symmetric configuration.

by

$$\epsilon(z) = \frac{e^{-\zeta_-(z)}}{2}[\epsilon^0 + L^0 + \mathcal{D}^-(z)] + \frac{e^{-\zeta_+(z)}}{2}[\epsilon^0 - L^0 + \mathcal{D}^+(z)], \tag{2}$$

where

$$\zeta_\pm(z) = \int_0^s d\bar{z}\eta(\bar{z})[1 \pm \kappa(\bar{z})\beta(\bar{z})] \tag{3}$$

$$\mathcal{D}^\pm(z) = \int_0^z d\bar{z}e^{\zeta_\pm(\bar{z})}\beta(\bar{z})\xi(\bar{z}). \tag{4}$$

In the above equation, η is the scaled parameter specifying the energy loss per unit length in the absorber material, and $\xi(z)$ is the angular excitation due to the stochastic kick arising from multiple scattering. The parameters ϵ^0, β, and L^0 are initial transverse emittance, betatron function and angular momentum, respectively. They are explicitly given by

$$\eta = \frac{1}{p_s v}\frac{dE}{dz} = \frac{1}{p_z}\frac{dp}{dz} \tag{5}$$

$$\xi(z) = \left(\frac{13.6\,\text{MeV}}{pv}\right)^2 \frac{1}{L_{\text{rad}}}, \tag{6}$$

where L_{rad} is the radiation length of the material. For the designed DF0F0 cooling channel, eq. (2) is plotted together with the simulation result by ICOOL in Fig. 4. It is seen that they are in good agreement.

4. Alternating Solenoid Cooling Channel

As another option for the cooling structure of the neutrino factory, an alternating solenoid cooling channel is also considered. In this channel, a lattice of solenoids with alternating direction is used to achieve the required focusing. Figure 6 shows the configuration of the alternating solenoid cooling channel with a cell length of 2.2 m. The cooling channel with 201.25 MHz rf cavities has a maximum magnetic field of 3.4 T on the longitudinal axis and 6 T on the solenoid with a current density of 58 A/mm^2. The minimum beta function is 20 cm and the maximum beta function is 110 cm. The magnetic field peaks with a magnitude of 3.4 T near the center of the absorbers. The field falls to zero and alternates in direction at the center of the rf cavities. Figure 7 shows the magnetic field (B_z) in one cell of alternating solenoid along the longitudinal axis. The maximum and minimum values of

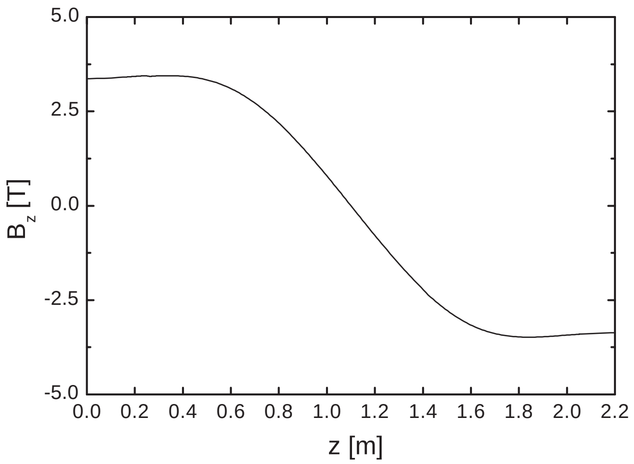

Fig. 7. Magnetic field (B_z) in one cell of the alternating cooling channel.

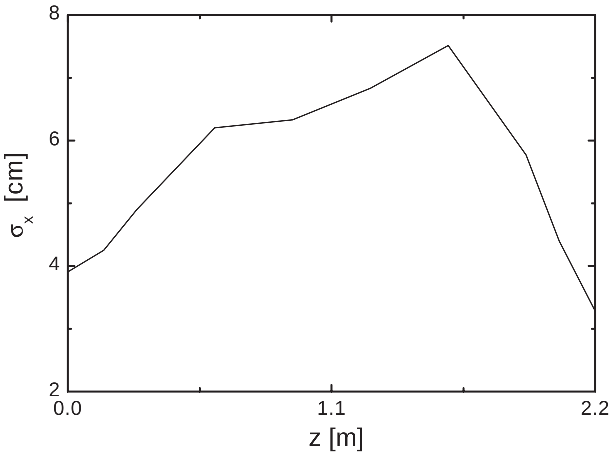

Fig. 8. Beam size σ_x (cm) in one cell of the alternating cooling channel.

magnetic field B_z occur near the midpoint of absorber and rf cavities, respectively. In the alternating channel, the maximum value of the beta function occurs at the midpoint of the rf cavities. The minimum value occurs near the midpoint of absorbers. Accordingly, more particles are lost than in the DFOFO cooling channel. The reason is that particle loss is mainly generated at the position of the rf window where the beam size is large. In Fig. 8, the beam size (σ_x) is shown as a function of the distance in one cooling cell.

4.1 Input beam

The initial beam distribution is generated by a random Gaussian with 11000 mm-mrad transverse and 16 mm longitudinal emittances. A beam with a longitudinal momentum of $P_z = 214$ MeV/c with an rms momentum spread of 9% is used. The properties of the initial beam used in the simulation are shown in Table I.

4.2 Parameters of the alternating solenoid cooling channel

The designed cooling channels have 73 cells with 3.4 T magnetic field B_z. Cell length of all cooling channels is 2.2 m. The parameters of the alternating cooling channel are shown in Table I. For the 3.4 T alternating solenoid channel, the relevant values are $B = 6$ T at the coil, $J = 58$ A/mm^2, and $R = 0.805$, giving $BJR = 280$ MPa. The parameters of

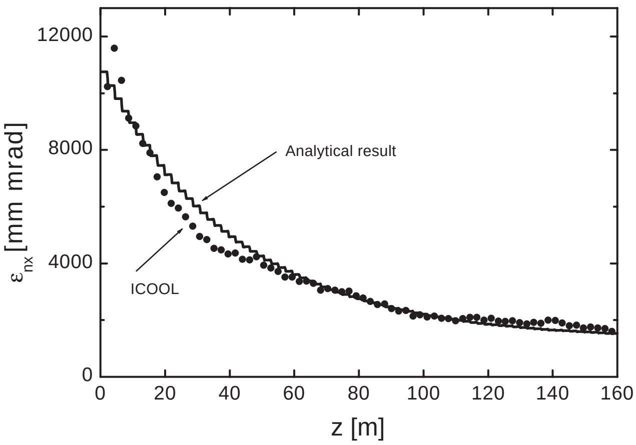

Fig. 9. Transverse emittances ϵ_x (mm mrad) as a function of channel length by ICOOL simulation and analytical formulae. Analytical result is shown by a step function.

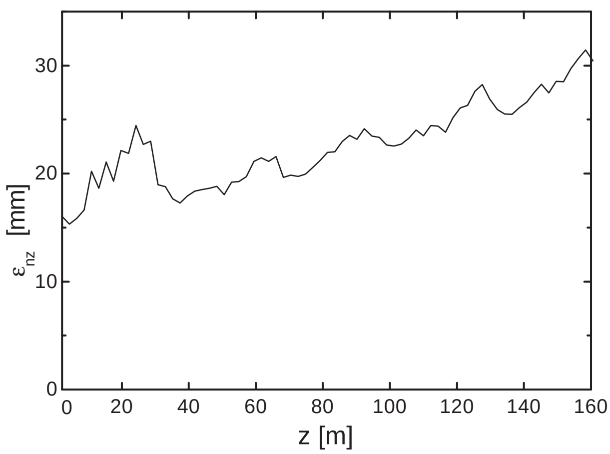

Fig. 10. Longitudinal emittance (mm) as a function of distance along the alternating cooling channel.

Table V. Initial and final beam parameters in 160 m alternating solenoid channel.

	Unit	Initial	Final	Final/Initial
Particles tracked		1000	515	0.51
Normalized transverse emittance (ϵ_{nx})	mm mrad	10227	1585	0.15
Normalized longitudinal emittance (ϵ_{nz})	mm	16	30	1.87
Normalized 6-D emittance ($\times 10^{-8}$)	(m rad)^3	197	27	0.13
Rms bunch length (σ_z)	cm	10.2	14.4	1.4
Rms beam size (σ_x)	cm	3.9	1.1	0.28
Rms momentum spread $\Delta P_z/P_z$	%	9	25	2.7

the solenoid in the alternating cooling channel are shown in Table III.

4.3 Cooling performance

The transverse emittance in both the x and y planes is reduced to 15% of its initial value, while the longitudinal emittance grows by a factor of two. The overall normalized six-dimensional emittance is reduced to 13% of its initial value.

Figure 9 shows the decrease in normalized transverse emittance as a function of the distance along the channel. It shows that transverse emittance decreases to 1585 mm mrad. Figure 10 shows the increase in normalized longitudinal emittance in the channel. Table V also shows the final beam parameters of rms beam size σ_x, rms bunch length σ_z and rms longitudinal momentum spread.

4.3.1 Comparison with analytical result

The simulation result by ICOOL is compared with that of the analytical method. The results show good agreements, as described in Fig. 9.

5. Conclusion

In this paper, we have shown two ionization cooling channels based on different design principles. A DF0F0 channel and an alternating solenoid cooling channels are designed and their cooling performance is calculated with the help of the ICOOL program. It is shown that cooling parameters of the PJK design for the neutrino factory can be achieved by these cooling channels. The purpose of our study is to obtain efficient transverse cooling channels that satisfy engineering constraint. Detailed simulation studies have been performed to obtain the optimal parameters for each cooling channel. This study will be valuable in selecting the best design based on cooling performance, engineering constraint and cost. It is also shown that the simulation results agree well with those of analytical method. The DF0F0 cooling channel shows less particle loss than the alternating channel.

Acknowledgement

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