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# An Alternating Solenoid Focused Ionization Cooling Ring

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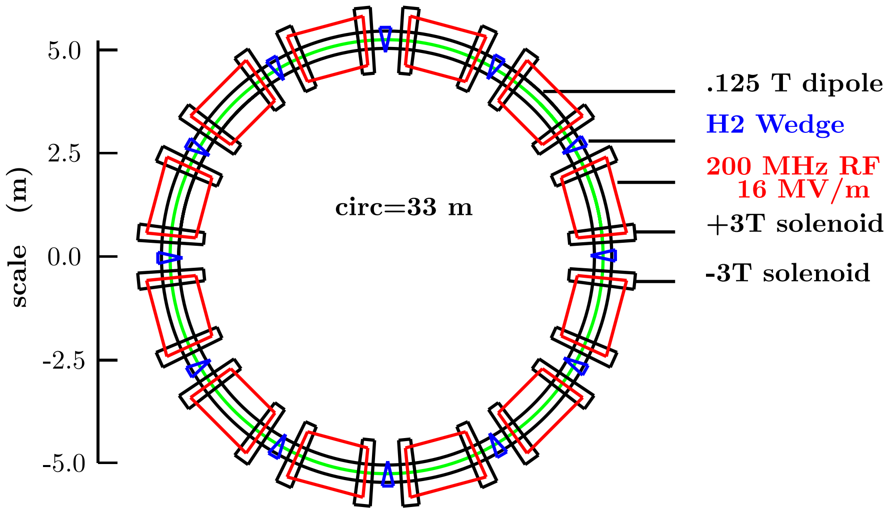
## Abstract

This note describes the design of an ionization cooling ring that uses an alternating polarity solenoid lattice. The ring is approximately 33 m in circumference and has 11 cells. Each cell has two opposing focusing solenoids placed either side of a hydrogen wedge absorber. The solenoid coils are located outside pillbox rf cavities. Bending is provided by an overall dipole field of 0.125 T. The simulated "merit factor" (≈the increase in 6D phase space density) is approximately 50.

# 1 Introduction




The above figure shows the cooling ring drawn approximately to scale.

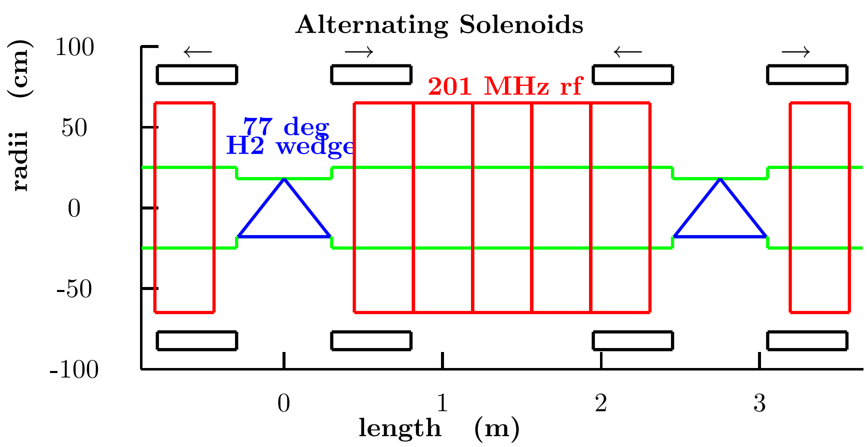
Early muon collider studies[1] had assumed that transverse cooling and emittance exchange would be done in alternating stages, the former being straight channels with rf and absorbers; the latter using bent solenoids and wedges. A serious problem with this approach was found to be the matching between the two types of lattices.

A solenoid focused cooling ring was proposed[2] that overcame much of this matching problem by rapidly alternating the two functions. Alternate cells contained 1) transverse cooling in a long solenoid containing acceleration and a single hydrogen absorber; and 2) emittance exchange in a cell containing two bend magnets, two opposed solenoids, and a LiH wedge. Matching between them has currently been achieved only with hard-edged magnetic fields. Each pair of cells is long enough that at least one half integer betatron resonance is present within the momentum acceptance. This may be reducing this ring's transmission.

Later[3] quadrupole focused rings took the process a step further, using a single cell type in which a wedge absorber cooled in both longitudinal and transverse phase space. These designs eased the lattice design, but introduced some transverse emittance growth from energy straggling in the absorber, since it was in a dispersive location. However, the weaker focusing in such quadrupole lattices limit the amount of cooling and the momentum acceptance.

The ring discussed here follows the quadrupole design in employing a single cell for both functions, but by using solenoids has larger angular and momentum acceptances. The cell includes dispersion, acceleration, and energy loss in a single thick hydrogen wedge. It is appreciated that the present lattice has dispersion at the rf cavities, which introduces additional synchrotron-betatron mixing. It is also appreciated that performing the transverse cooling with an absorber where there is dispersion introduces additional emittance growth. But it was hoped that these disadvantages would be compensated by the greater acceptance from the use of a single repeating cell with no integer or half integer betatron resonances in the momentum acceptance.

# 2 Design

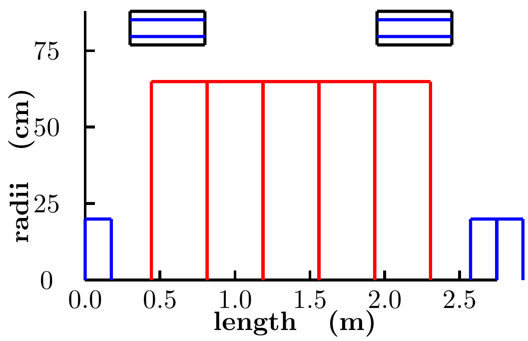


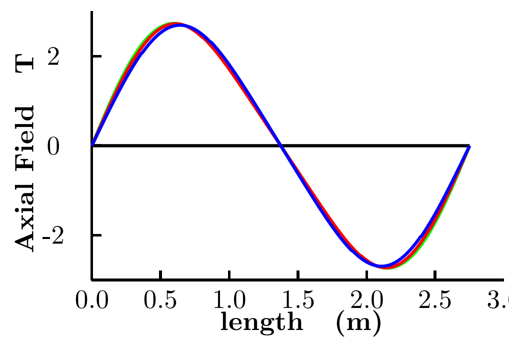

The basic ring is made up of 11 identical 2.75 m long cells. This symmetry will have to be broken for injection and extraction, but the intention is to make such changes as small as possible. Two cells of the lattice are illustrated above without the bend.

## 2.1 Transverse Focusing Lattice

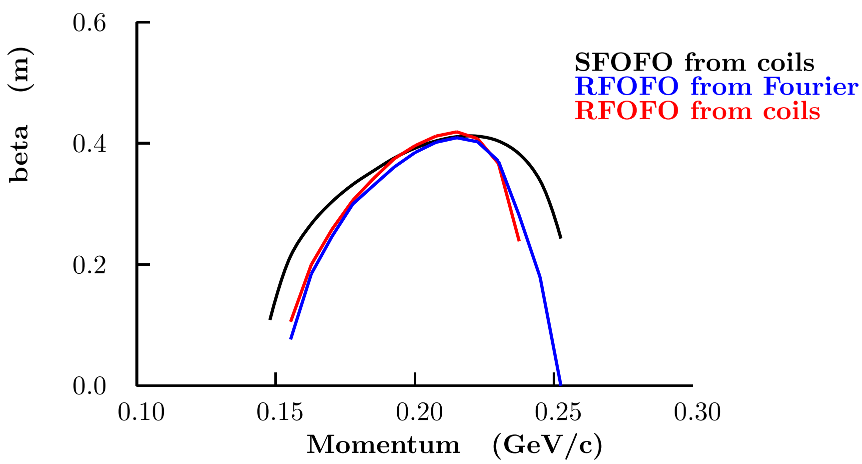
The type of lattice used has been described as a Reverse FOFO (RFOFO)[4] to distinguish it from the Super FOFO (SFOFO) used in the Second Feasibility Study of a Muon based Neutrino Factory [5], hereafter referred to as “Study 2”. The following table and figure shows the coil locations and current densities, together with the axial magnetic fields in the focusing lattice. The red line in the magnetic field plot shows the axial fields calculated from these coils in the absence of the bending field. The blue line gives the axial fields as used in the simulations with the bending field. The latter are generated by a truncated Fourier decomposition of the fields from a bent solenoid. The actual coils to generate the axial fields, in the presence of the bending fields, would have to be slightly different from those given. But since the 3D fields used are consistent with Maxwell’s equations, there is no question but that suitable coil positions can be found. It does mean, however, that a full 3D field calculation will have to be done before a fully realistic design is defined.

| len1 | dl | rad | dr | | I/A |
|---|---|---|---|---|---|
| m | m | m | m | | A/mm$^2$ |
| 0.300 | 0.500 | 0.770 | 0.110 | 2 | 95.27 |
| 1.950 | 0.500 | 0.770 | 0.110 | 2 | -95.27 |

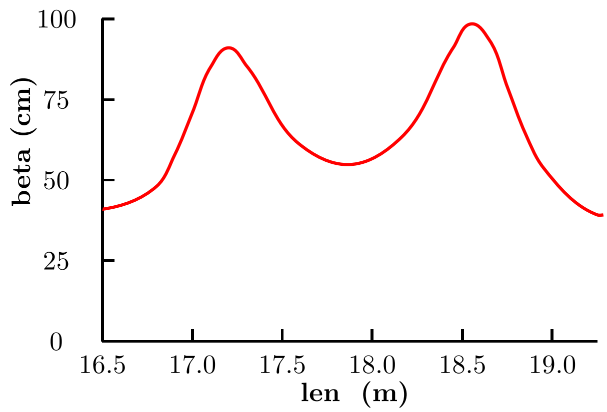



The amp turns per cell are 10.48 (MA), and the amp turns times the cable length is 54.3 (MA m) per cell. These are larger than those for the Study 2 SFOFO lattice, and the momentum acceptance is somewhat less (see fig. below) than with an SFOFO, but the RFOFO was chosen because, unlike in the SFOFO case, all cells are strictly identical, and the presence of an integer betatron resonance within the momentum acceptance is eliminated.




The above figure shows the transverse beta function at the center of the absorber as a function of momentum. These calculations were done without any bending field. Again the red curve is for the coil block derived fields, and the blue curve for the Fourier approximation. The black curve gives the beta functions for an SFOFO lattice which is seen to give a somewhat wider acceptance.

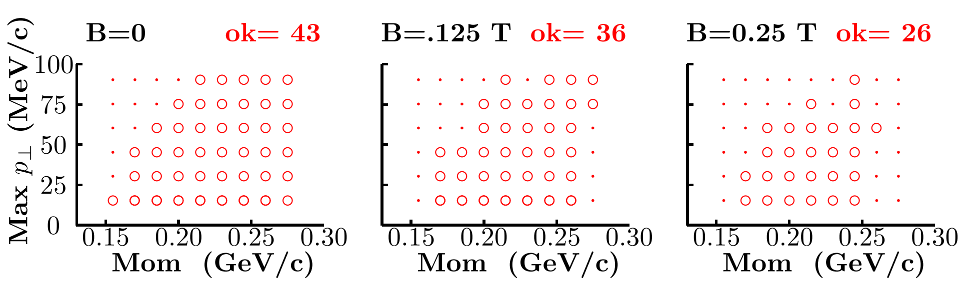



The above figure shows the beta function at the central momentum as a function of distance along the cell.

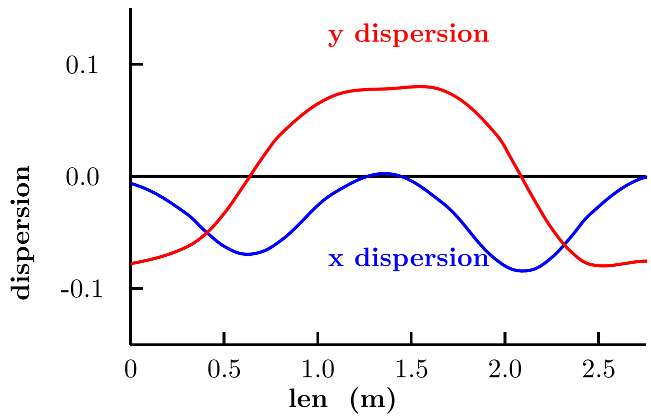
## 2.2 Bending Fields

Dispersion is provided by applying a 0.125 T transverse bending field everywhere along the reference orbit. Radially the field increases as $B \propto r^n$ where the index $n$ is equal to 0.5. With this condition, the focusing effect of the bend is made equal in x and y.

In the following figure we examine the dynamic acceptance of the lattice, with no rf or absorber, for three different bending fields: 0.0 T, 0.125 T, and 0.25 T. Using ICOOL, particles are injected into the lattice at 9 different momenta and 6 different angles. In the plots, particles that were lost are marked by dots, and those that survive 275 m are plotted as circles.




Out of a maximum of 54 particles, the numbers transmitted for the three fields (0, .125, and .25 T) are 43, 36, and 26. clearly, the acceptance is reduced as the bending field is increased. We may thus expect that it is best to use the least bending field consistent with adequate emittance exchange.




Dispersion in x and y vs. distance along the lattice is shown above. The slight left-right asymmetry is probably due to the energy asymmetry (see figure below). Note that the dispersion at the absorber of -7 cm (z=0 or 2.75 m) is in the y direction, perpendicular to the bend. This is a result of the Larmor rotation generated by the axial fields. The dispersion at the rf is of the opposite sign, and still mostly in the y direction.

This uniform bending field is not intended as a practical or even desirable arrangement. It is used here only for its ease of application. Other distributions may be either more practical or desirable.

## 2.3 Absorber

The wedge parameters are given below. No windows are included in this simulation. The wedge has a zero thickness at one side and 57.2 cm at the other.

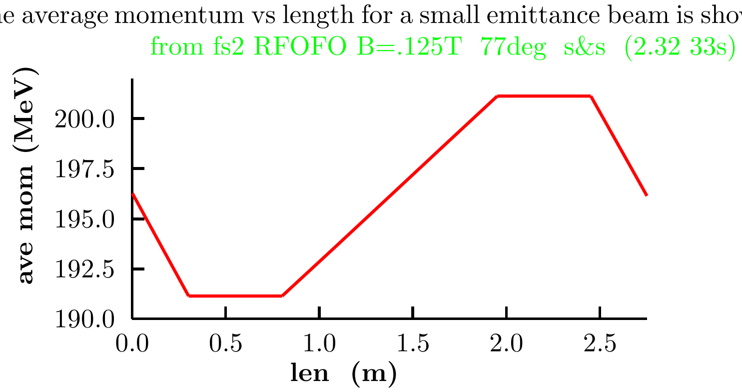
| | | |
|---|---|---|
| Material | | H2 |
| Windows | | none |
| Radius | cm | 18 |
| Central thickness | cm | 28.6 |
| Min thickness | cm | 0 |
| Angle | deg | 76.93 |

## 2.4 RF

RF parameters are given below. It is seen that no windows are included and that the aperture of 25 cm radius is somewhat larger than the windows in Study 2 (21 cm). The gradients are similar to those in Study 2. The phase given is that used in the simulation and is relative to an unrealistic reference particle that is not accelerated or decelerated by the field. The phase with respect to the bunch center will be somewhat more than this.

| | | |
|---|---|---|
| Cavities per cell | | 4 |
| Lengths | cm | 28.75 |
| Apertures | cm | 25 |
| Windows | | none |
| Frequency | MHz | 201.25 |
| Gradient | MV/m | 16 |
| Phase rel to fixed ref | deg | 33 |

The average momentum vs length for a small emittance beam is shown below




# 3 ICOOL Simulations

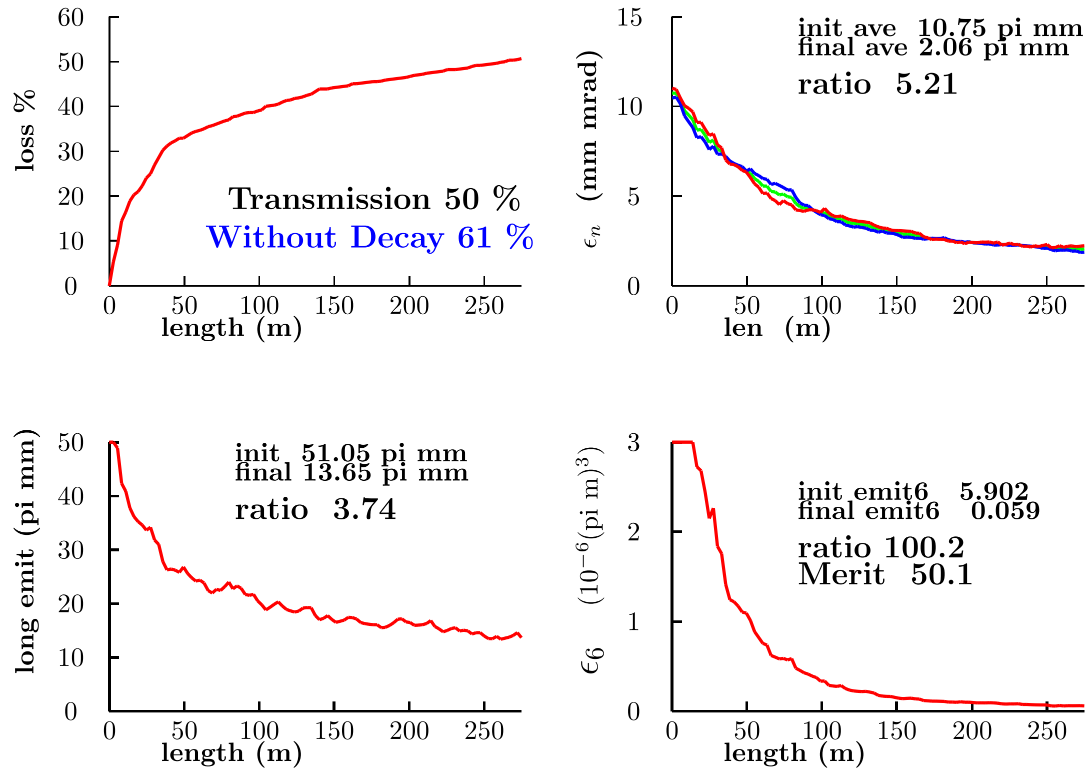
The following ICOOL simulations use version 2.32 and the Fourier representations of axial and transverse magnetic fields. The rf is represented as fields in perfect pillbox cavities. The input tracks are taken from a Feasibility Study 2[5] simulation, using distributions from just before transverse cooling. No attempt was made to match the ring dispersion or slight differences in the transverse beta functions.

The use of Study 2 simulated distributions is intended to allow a more realistic estimate of the ring's performance, but it must be understood that the actual bunch train from Study 2 could not be injected into this ring, whose circumference is less than the bunch train length.

## 3.1 This Ring

The following plots show the transmission, transverse emittance (in x, y and their average), longitudinal emittance, and 6 dimensional emittance vs. length in the ring.

RFOFO B=.125T 2 wedge 76deg s&s (2.32 fs33b)



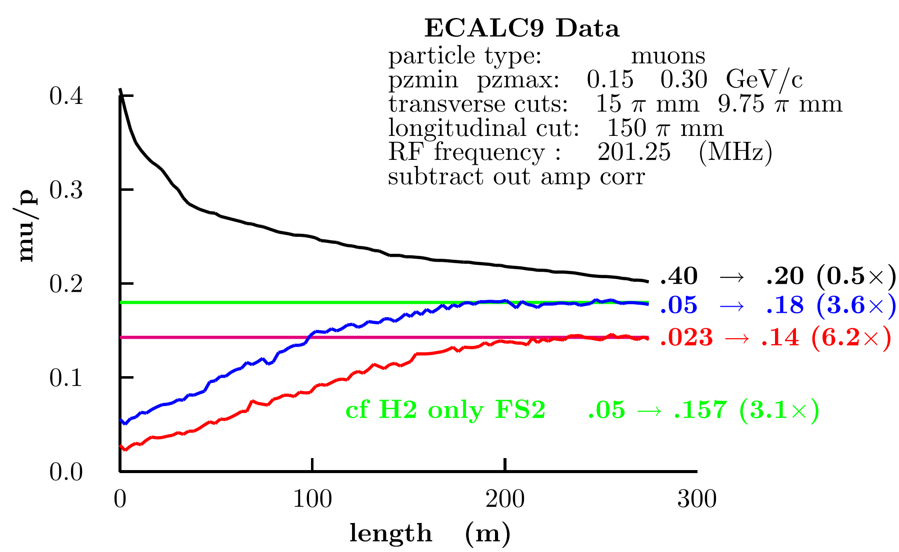

The following table gives initial, final and ratios of beam parameters.

| | | Initial | final | init/final |
|---|---|---|---|---|
| Trans. without decay | % | 100 | 61 | |
| Transmission | % | 100 | 50 | |
| rms x,y | cm | 4.9 | 2.0 | 2.45 |
| rms dp/p | % | 12 | 7 | 1.7 |
| rms ct | cm | 23 | 11 | 2.1 |
| emit trans | $\pi$ mm | 10.75 | 2.06 | 5.21 |
| emit long | $\pi$ mm | 51.1 | 13.7 | 3.74 |
| emit 6D | $10^{-6}(\pi\ m)^3$ | 5.9 | 0.059 | 100.2 |
| Merit Factor | | | | 50.1 |

It is seen that initially, the x emittance falls more rapidly than the y. This is expected because it is the y emittance that is exchanged with the longitudinal emittance. What is less expected is the subsequent convergence of the two emittances followed by a reversal of their positions. Clearly there is some mixing between the x and y amplitudes. This might not have been expected in a solenoid focused ring with net axial field equal to zero, and $n = .5$ field gradient. Indeed, in the absence of material and re-acceleration, no mixing is observed. The mixing, which is desirable, arises because the particle energies are consistently lower in one sign of the axial field, than the other.

After a distance of 275 m (≈9 turns), the 6 dimensional emittance has fallen by a factor of 100 with a transmission of 50 % (61% without decay). A "merit factor" has been defined as the ratio of initial to final 6 dimensional emittance multiplied by the transmission. For Gaussian distributions, this would equal the increase in central 6D phase space density. For this example, the merit factor is 50. The same factor for the Study 2 cooling lattice, with no windows, is 13 ($\epsilon_\perp$ : 1/4.25, $\epsilon_\parallel$ : 1/1.49, trans=49%); it is about 3 for the quadrupole lattice with decay[3], and about 38 for a similar Balbekov[2] ring.

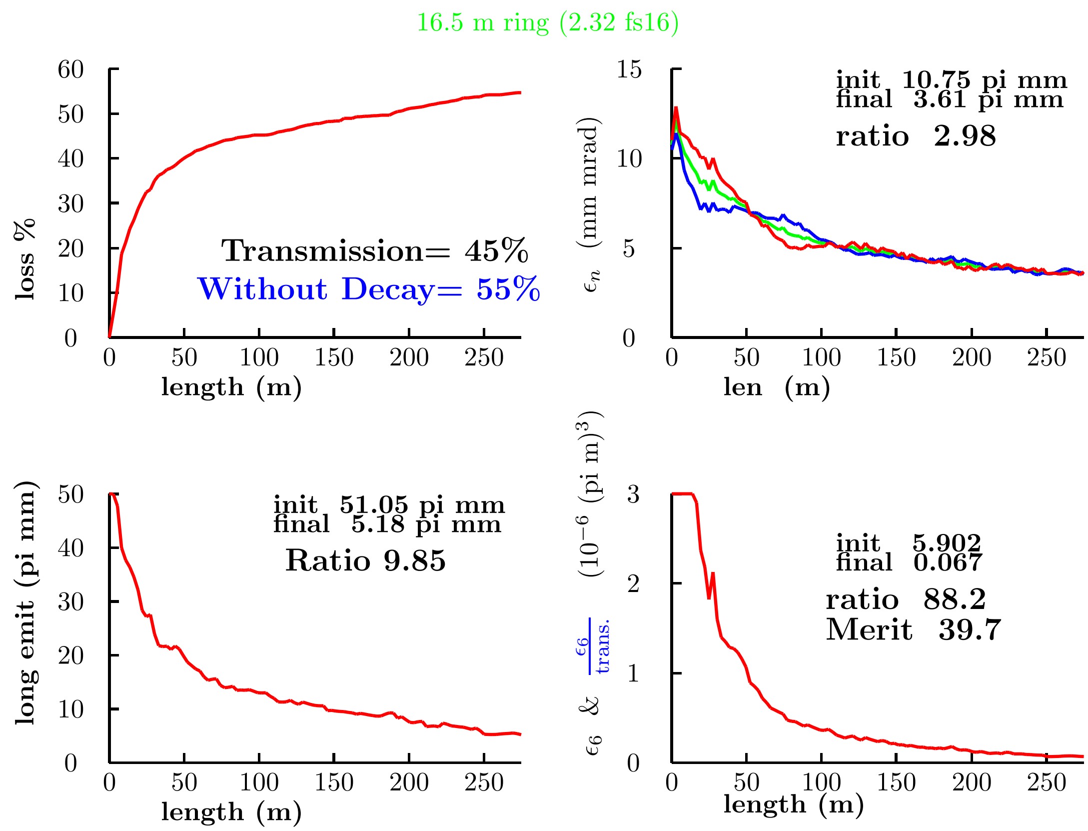
If the ring is to be used in a neutrino factory, then a better criterion to use might be the number of particles accepted in the predefined 6 dimensional acceptance of the following acceleration. The following plot shows this criterion for two different transverse acceptances (15 and 9.75 pi mm), both requiring the same longitudinal acceptance to be 150 pi mm. The gain into 15 pi mm is 3.6, somewhat better than that of the linear cooling channel of Study 2 (3.1). Some of this improvement is coming from the emittance exchange, but some performance is lost because the focusing beta function in the ring cannot be tapered down as the emittance falls, as is done in the Study 2 case.




It must be remembered, however, that this ring could not be used, as is, to replace the Study 2 cooling channel because the bunch train in this case is far too long to fit in the ring. If, instead, a spiral 3D cooling channel were used to replace the linear Study 2 channel, then an even greater performance gain could be expected if the spiral were also tapered.

### 3.2 c.f. A 16.5 m ring

For comparison, we have also simulated a case with twice the bending field (0.25 T) and thus ring circumference of only 16.5 m. Such a ring would require an even shorter initial bunch train, and is not expected to have as good a dynamic aperture. Plots of performance are shown below.




It is seen that the greater dispersion allow a greater reduction in longitudinal emittance at the price of less transverse cooling. The overall merit factor is not quite as good.

## 4 Comparison with other rings

The following table gives a summary of parameters of this and two other ring designs: Balbekov's 4-sided solenoid focused ring[2], and a recent design of a quadrupole ring that was presented at a meeting in UCLA, which has much in common with that published in the Snowmass proceedings[3].

| | | This Ref | | Balbekov | | Quadrupole | |
|---|---|---|---|---|---|---|---|
| Circumference | m | 33 | | 37 | | 165 | |
| Ref momentum | MeV/c | 200 | | 227 | | 500 | |
| beta at absorber | cm | 40 | | 27 | | 25 | |
| turns | | 0 | 9 | 0 | 15 | 0 | 8 |
| Trans. without decay | % | 100 | 61 | 100 | 71 | 100 | 67 |
| Trans. + decay | % | 100 | 50 | 100 | 48 | 100 | 44 |
| rms dp/p | % | 12 | 7 | 14 | 9 | 5.2 | 2.9 |
| emit trans | $\pi$ mm | 10.75 | 2.06 | 12 | 2.1 | 2.5×2.6 | 2.7×1.1 |
| emit long | $\pi$ mm | 51.1 | 13.7 | 15 | 6.3 | 29.2 | 10.3 |
| emit 6D | $10^{-6}(\pi\ m)^3$ | 5.9 | 0.059 | 2.16 | 0.028 | 0.19 | 0.030 |
| **Merit Factor** | | | **50.1** | | **38** | | **3** |

It must be remembered that there are many different assumptions used in the three examples, and they were designed to perform somewhat different tasks. The differing merit factors should not be interpreted too strictly. Nevertheless, it does appear that it is harder to achieve good performance with quadrupole focusing than with solenoids.

# 5 Conclusion

- This simulation uses Maxwellian fields to the 5th order in distance from the central orbit, and all higher order aberrations are included. The quadrupole ring studies[3] have not yet included higher order aberrations, and Balbekov's studies include non Maxwellian hard ends to the magnetic fields. This may thus be the most realistic ring simulation yet performed.

- Using the "merit factor" as a criterion, the performance of this alternating solenoid cooling ring is somewhat better than the Study 2 linear cooling channel, is significantly better than that of the quadrupole focused ring[3];and similar to, or possibly a little better than, that of Balbekov's solenoid focused ring.

Thus this approach seems very attractive, but it is still far from fully realistic, and much work needs to be done:

- Realistic windows must be included, and the use of LiH studied as an alternative.

- More realistic bending fields should be considered. Generating such fields by tilting the focus magnets should be investigated.

- Injection and extraction must be included. Some modifications to the lattice will be inevitable, but should be kept as small as possible to avoid increasing losses.

- The ring parameters need adaption to its possible use. In order to inject into such a small ring, we must either generate a suitably short pulse train or accept a single long bunch. In the former case, it will be necessary to raise the ring energy to accept the larger momentum spread. In the second case, the rf frequency will have to be lowered.
- Eventually, the ring should be engineered and costed, but it is too soon to attempt that now.

# References


[1] "Muon Muon Collider: A Feasibility Study", BNL-52503;Fermi Lab-Conf.-96/092; LBNL-38946.

[2] V. Balbekov; "Solenoid Based Ring Coolers", Fermilab MUC-NOTE-COOL_THEORY-0232; www-mucool.fnal.gov/mcnotes/public/ps/muc0232/muc0232.ps.gz; Feb. 2002.

[3] H. Kirk, D. Cline, Y. Fukui, and A. Garren; "Progress towards a Muon Ring Cooler"; Proc. Snowmass Workshop, July 2001; BNL-68735.

[4] Eun-San Kim, Charles Kim, Greg Penn, Andrew M. Sessler and Jonathan S. Wurtele; "LBNL progress report on simulation and theoretical studies of muon ionization cooling"; Fermilab MUC-NOTE-COOL_THEORY-0036; July 1999. www-mucool.fnal.gov/mcnotes/public/ps/muc0036/muc0036.ps.gz

[5] S. Ozaki, R. Palmer, M.S. Zisman, J. Gallardo; "Feasibility Study-II of a Muon Based Neutrino Source"; BNL 52623; June 2001.