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Calculation of RFOFO fields using the off-axis expansion in ICOOL

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We examine the off-axis field components for the RFOFO cooling ring. The ring consists of 12 cells, each containing two solenoids. Each solenoid is modeled here as a current sheet. We check the accuracy of the off-axis field expansions used in ICOOL by comparing with the fields computed directly using an independent sum-of-sheets code. For the complicated fields present in the RFOFO ring the 4th order off-axis expansions used in ICOOL reproduce the true field values to ~0.2 gauss. This is probably sufficient for the simulations to reproduce any nonlinear effects introduced by the real field. Typical fractional errors for 4th order on a 10 cm test circle are ~2 10^{-4} for B_X and B_Y and ~1 10^{-5} for B_S.

1. Introduction

The RFOFO cooling ring was conceived by Bob Palmer and represents a promising approach to achieving 6D cooling for a neutrino factory or muon collider [1,2]. In a previous note [3] we described an analytic method of calculating the fields on the system axis (SA) of a ring of tipped solenoids. We found that the on-axis field components from the tipped solenoids are very non-uniform. Fig. 1 shows the on-axis fields for one cell of the RFOFO ring. Note that we are using a positive tilt orientation of the solenoids to produce the vertical bending field.

Off-axis fields for a similar, but not identical, RFOFO model have been presented previously by Valeri Balbekov [4]. The fields found here agree qualitatively with the results of his paper.

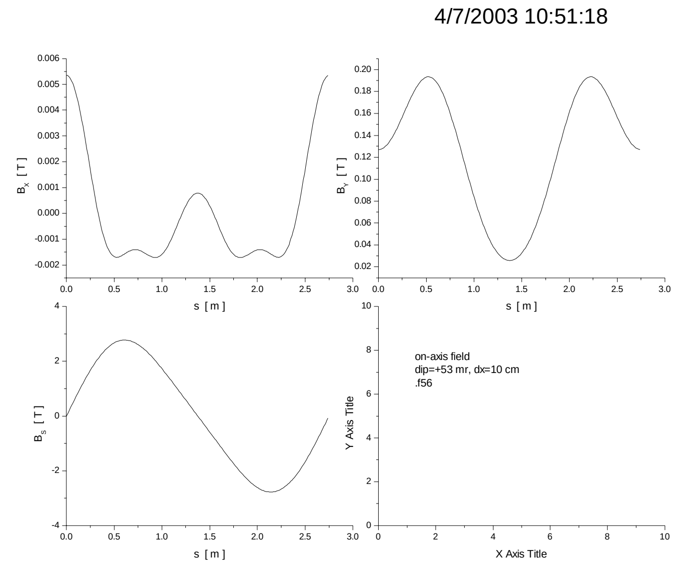


Figure 1. On-axis fields for one cell of the RFOFO ring.

We use the sum-of-sheets method here to find the field components for points away from the SA. Steve Bracker has demonstrated [5] that the off-axis fields determined by this method satisfy Maxwell's equations to high accuracy and we find similar agreement. Knowing the true values of the fields gives us the opportunity to check the accuracy of the off-axis field expansions used in ICOOL for an extremely non-trivial example.

2. Calculation of the multipoles

The multipoles of the RFOFO field were determined by first computing the field components along a radial (x) grid line using the sum-of-sheets code [3]. In the midplane (y=0) the vertical component of the magnetic field can be expanded in the series

$$B_Y(x,0,s) = b_0(s) + b_1(s)x + b_2(s)x^2 + b_3(s)x^3 + \dots \qquad (1)$$

We computed the actual value of B_Y at 41 points equally spaced radially over ±20 cm from the system axis (SA). For an n^{th} order fit Eq. 1 was terminated with the b_n term. The points were fit to the resulting polynomial to determine the normal multipoles b_n at that *s* location. Using the corresponding B_X components it is possible to determine the skew multipoles from the series

$$B_X(x,0,s) = a_0(s) + (a_1(s) - \tfrac{1}{2}b_S' \;)x + a_2(s)x^2 + (a_3 + \tfrac{1}{16}b_S'''(s))x^3 + \dots \qquad (2)$$

Primes refer to derivatives with respect to *s*.

Note that the odd, skew multipoles are mixed with the radial component of the solenoid field in this expansion [6]. The actual values of the field components in the midplane at a test location located ~20 cm axially from the beginning of the cell are shown in Fig. 2.

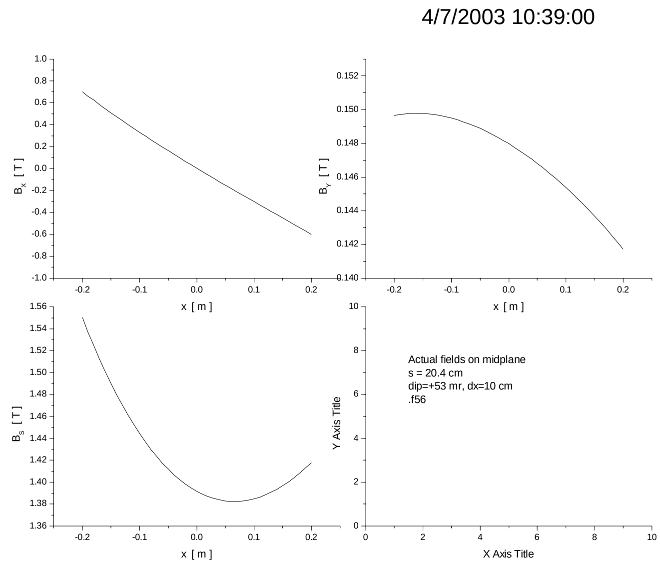


Figure 2. The actual values of the field components as a function of x at an axial location 20.4 cm from the beginning of the cell.

For the odd, skew harmonics the fit coefficient is a combination of the skew multipole and a derivative of the solenoidal field. To extract the actual skew multipole we first constructed a Fourier series for b_S. The derivatives could then be determined accurately from the Fourier coefficients. Fig. 3 shows the derivatives used for this analysis.

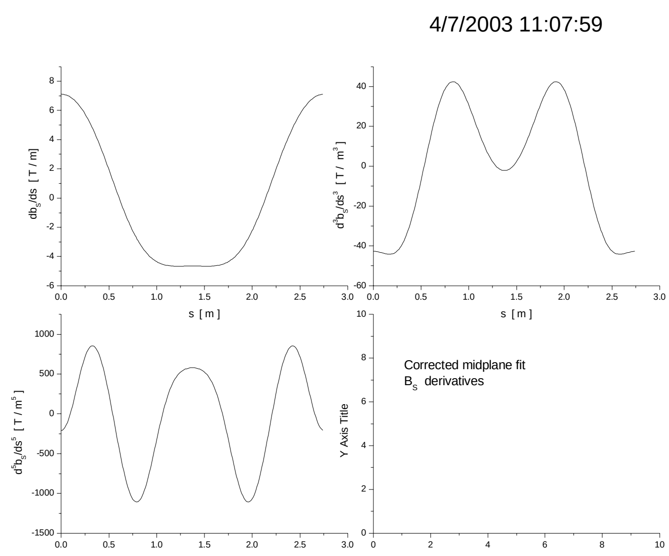


Figure 3. Odd derivatives of b_S used for extracting the odd, skew multipoles.

Once all the multipoles were determined they were then fit to Fourier series as a function of *s* for use in ICOOL. Figures 4-6 shows the originally calculated multipoles (top) together with the multipoles reconstructed in ICOOL from their Fourier series (bottom). A Fourier series with 9 terms is able to reproduce the original multipoles very accurately.

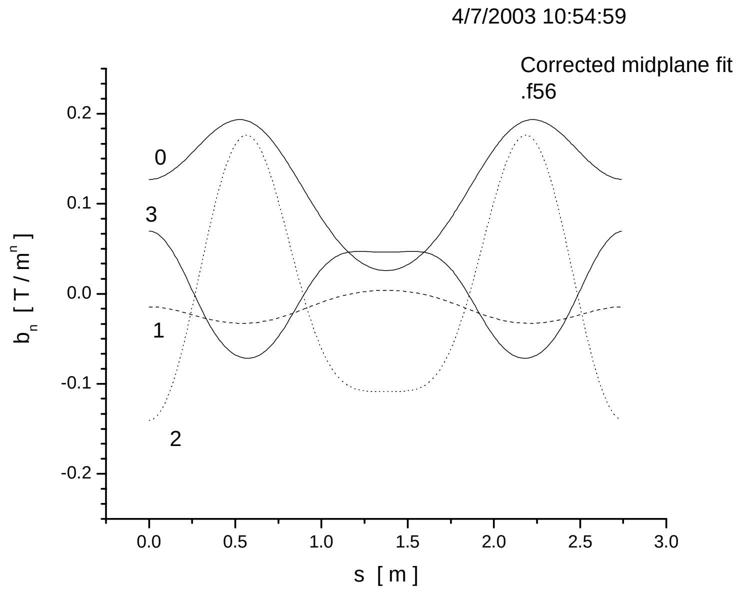
The off-axis field expansions depend in principle on the curvature factor *h(s)* in the *x-s* plane, the curvature factor *g(s)* in the *y-s* plane, together with their derivatives with respect to *s*. However, the multipoles derived above were computed using the constant values

$$h = \frac{1}{\rho_o}$$

$$g = 0$$

where ρ_o is the geometrical radius of curvature of the SA. We use these same curvature values when evaluating the off-axis field expansions.

A complication in the definition of the curvature factors arises when, for example, q is positive and b_0 is negative or q is negative and b_0 is positive. In this case in order for the particle to be confined to the same circle the direction of circulation must be reversed. Then, if the direction of the y axis is preserved, the direction of the x axis is reversed. This introduces a conflict with the coordinate system used in the field definition, which assumed that the y axis was always up and the x axis was always radially outwards. To correct this problem ICOOL has been modified to reverse the directions of h and g and their derivatives in the field calculations when this situation arises.



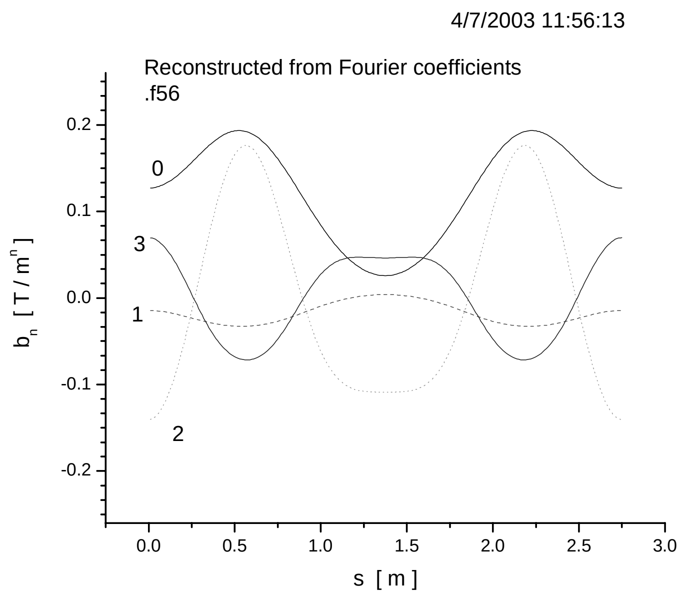
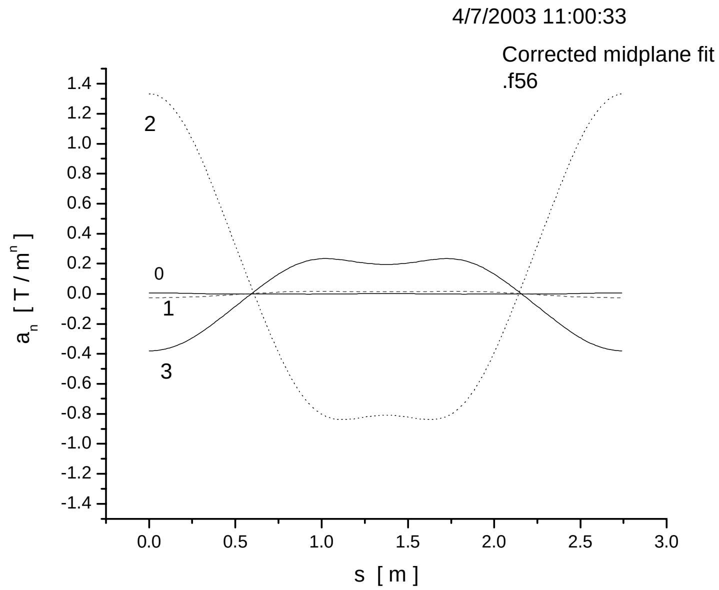


Figure 4. Normal transverse multipoles as a function of axial distance along the cell. Fitted values (top); values reconstructed from the Fourier series in ICOOL (bottom).



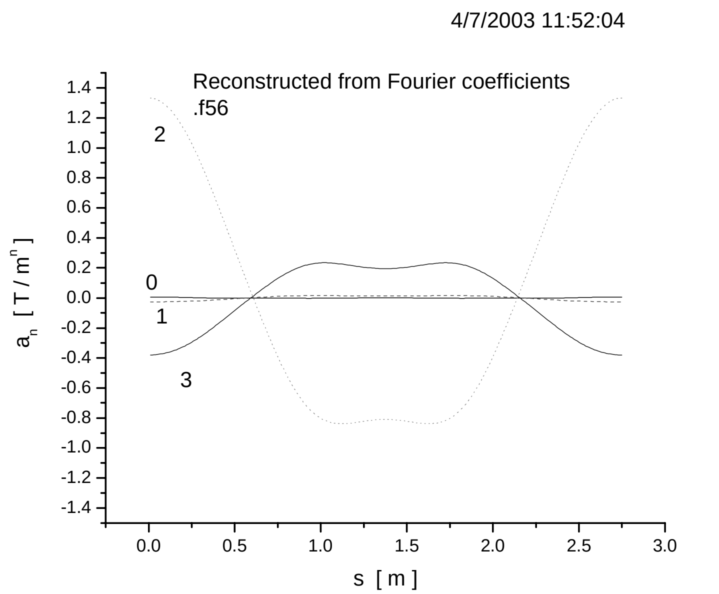
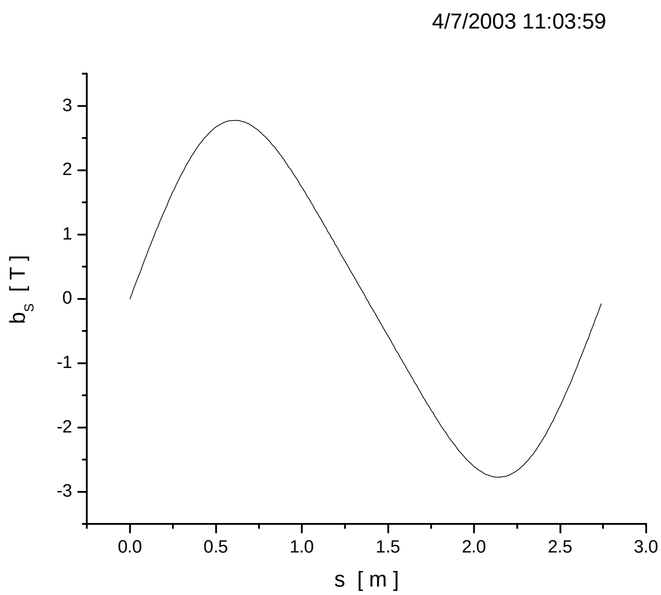


Figure 5. Skew transverse multipoles as a function of axial distance along the cell. Fitted values (top); values reconstructed from the Fourier series in ICOOL (bottom).



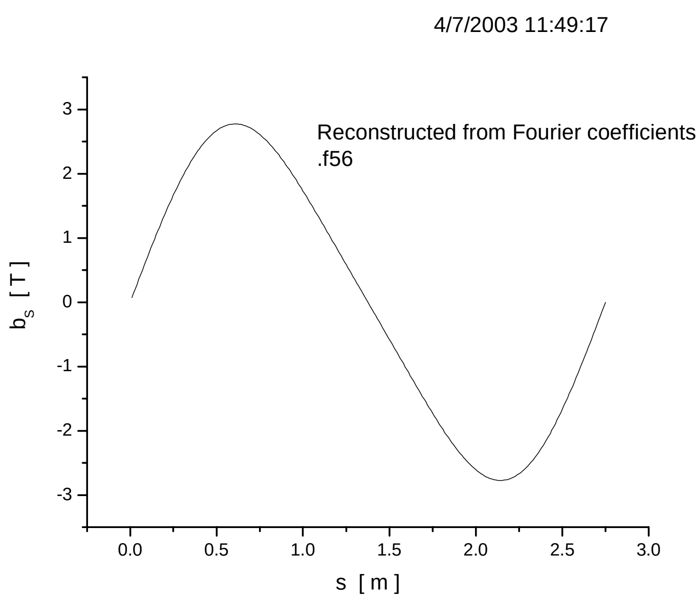


Figure 6. Solenoidal multipole as a function of axial distance along the cell. Fitted values (top); values reconstructed from the Fourier series in ICOOL (bottom).

The order of the off-axis expansion is a parameter in ICOOL. The order corresponds to the highest power of the transverse coordinate that appears in the expressions for the field components. Currently expansions up to 5th order can be used.[1] Orders 1-4 include all terms that depend on the curvature in the *y-s* plane (i.e. if *g* is not zero, there is bending out of the midplane).[2] The 5th order expansion presently only contains terms for bending in the midplane (i.e. *g*=0).

The accuracy of the ICOOL expansions for the transverse field components in the midplane increases steadily with increasing order. Fig. 7 shows the error field as a function of *x* for the 4th order expansion. We show the errors out to ±20 cm, since the tail of the RFOFO particle distribution is expected to extend out that far. The error in B_Y ~5 10^{-3} gauss is very small on the midplane for all orders. The maximum error in B_X is larger, ~5 10^{-2} gauss for 4th order.

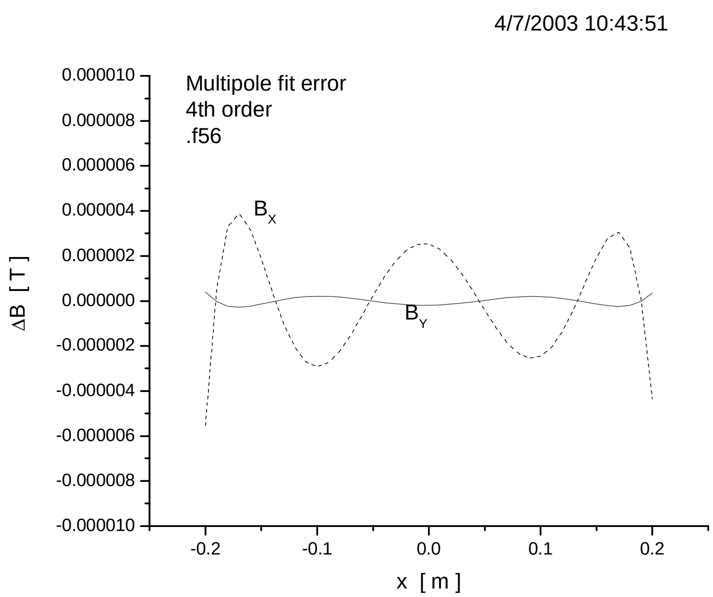


Figure 7. 4th order error in transverse field components as a function of *x*.

[1] These updated routines are available in version 2.50 of ICOOL.
[2] A 3rd order expansion without the terms proportional to a_0 and g has been given by Wang & Teng [6].

3. Accuracy of the expansions off the midplane

We check the accuracy of the off-axis field expansions at a set of points on a circle with radius R shown in Fig. 8. The centers of the 24 current sheets used in the RFOFO model lie along the coil placement circle[3] (CPC). The SA is displaced 10 cm radially inwards from the CPC.

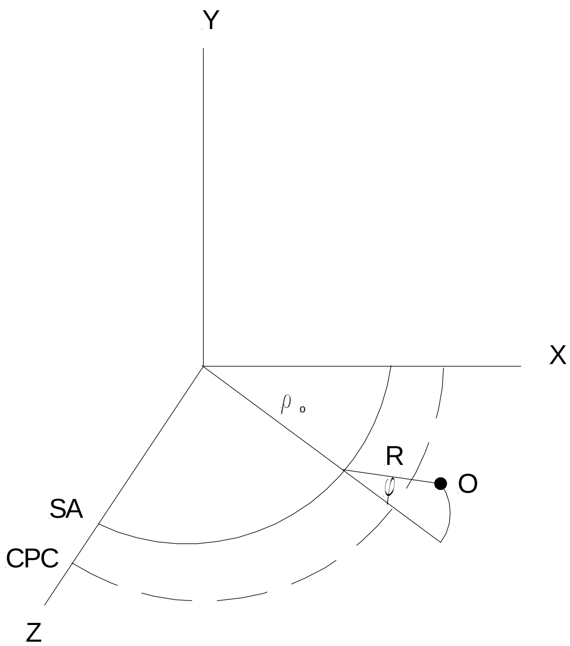


Figure 8. Location of observation points used in the check of the field accuracy.

The actual values of the field components along the test arc are shown in Fig. 9.

[3] This designation comes from Steve Bracker.

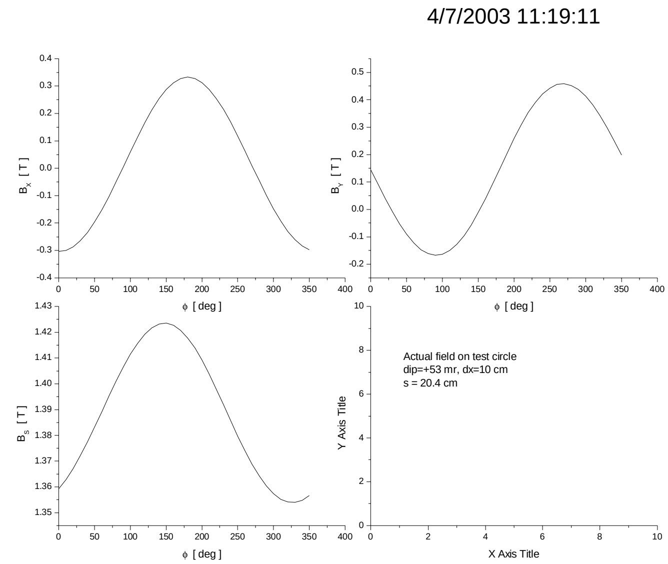


Figure 9. Actual values of the field components along the test arc.

The errors in reproducing the field components for the order 2-4 expansions are shown in Figs. 10-12.

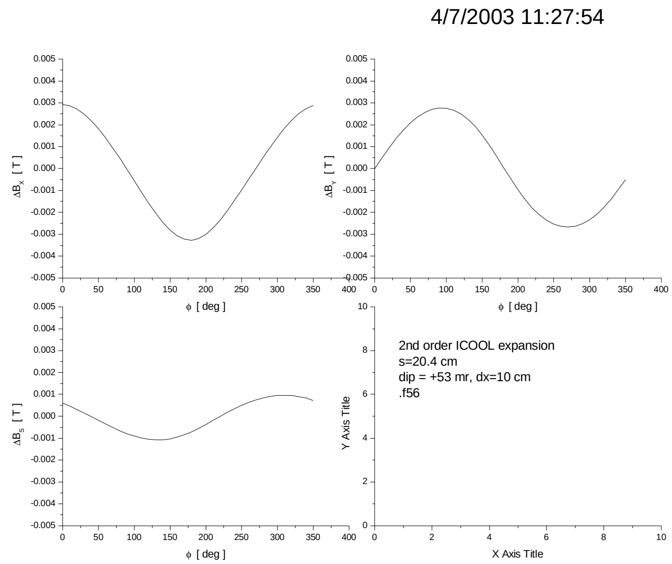


Figure 10. Errors in the field components for the 2^{nd} order expansions in ICOOL.

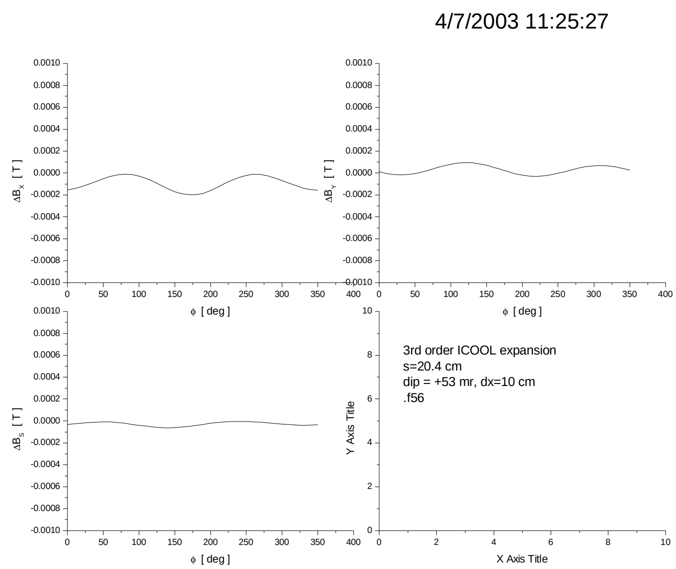


Figure 11. Errors in the field components for the 3rd order expansions in ICOOL.

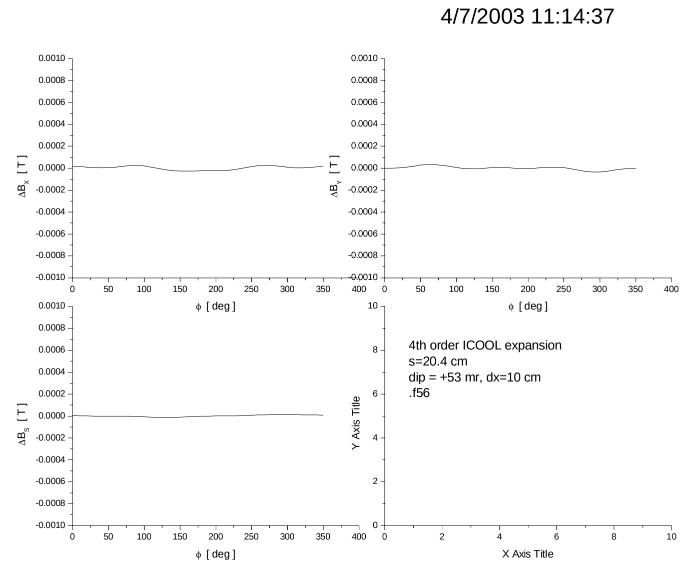


Figure 12. Errors in the field components for the 4th order expansions in ICOOL.

For the radial field component as the order is increased from 2 to 4 the maximum error drops from ~33 gauss to ~0.2 gauss for points on the test arc. The maximum error in the vertical component drops from ~28 gauss to ~0.2 gauss. The maximum error in the axial component drops from ~10 gauss to ~0.1 gauss.

4. Conclusions

For the complicated fields present in the RFOFO ring the 4th order off-axis expansions used in ICOOL reproduce the true field on a 10 cm test circle to ~0.2 gauss. This is probably sufficient for the simulations to reproduce any nonlinear effects introduced by the real field. The accuracy of the off-axis expansions increases steadily, up to 4^{th} order.

Acknowledgements

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References

[1] J.S. Berg, R.C. Fernow & R.B. Palmer, An alternating solenoid focused ionization cooling ring, Muon Collaboration technical note MUC-NOTE-COOL_THEORY-239, March 2002.
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