NFMCC Technical Note - 508

One-coil-per-cell solenoid lattices with the minimum beta function under the coil

R.C. Fernow
Brookhaven National Laboratory

20 February 2007

As part of a general survey of solenoidal ionization cooling lattices [1] we have discovered a set of lattice solutions that may be useful for transverse cooling of muon beams at neutrino factories or muon colliders. These solutions have one coil per cell and operate in the second momentum pass band.

1. Introduction

Periodic solenoid focusing has been widely used for ionization cooling lattices. The simplest lattice of this type has one solenoid coil per cell. So far Study 2a is the only example of this type that has been seriously considered [2]. In Study 2a the solution used the high energy (1^{st}) momentum pass band and the minimum of the beta function occurs between the coils. We examine here alternative schemes that use the 2^{nd} momentum pass band and have the minimum of the beta function directly beneath the coils.

2. One-coil-per-cell lattices

We confine our considerations here to periodic solenoidal ionization cooling lattices with one coil per cell. The geometry of the lattice is shown in Fig. 1.

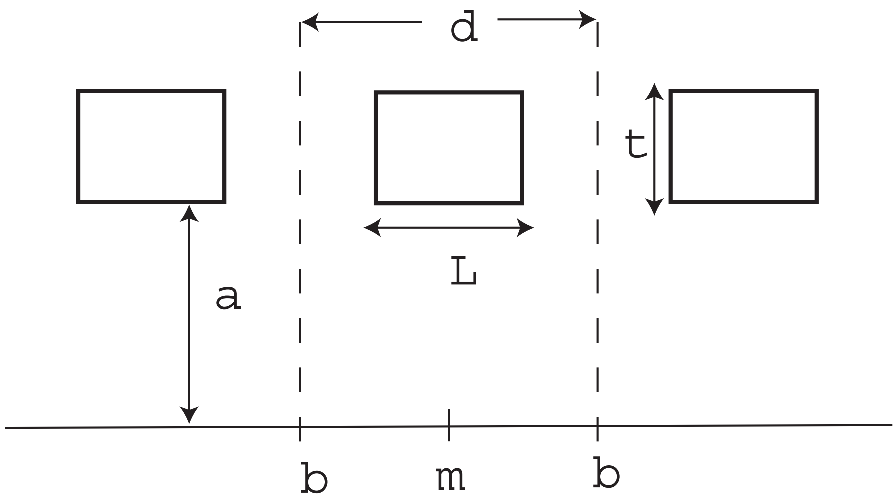

Figure 1. Schematic diagram of three cells of the solenoid lattice.

We define the cell length **d**, the coil length **L**, the coil inner radius **a**, and the coil radial thickness **t**. We define the cell boundary **b** to be the axial location midway between the coils and the cell midplane **m** to be the axial location of the center of the coil.

Periodic solenoid lattices have a series of pass bands and stop bands as a function of momentum, as shown in Fig. 2.

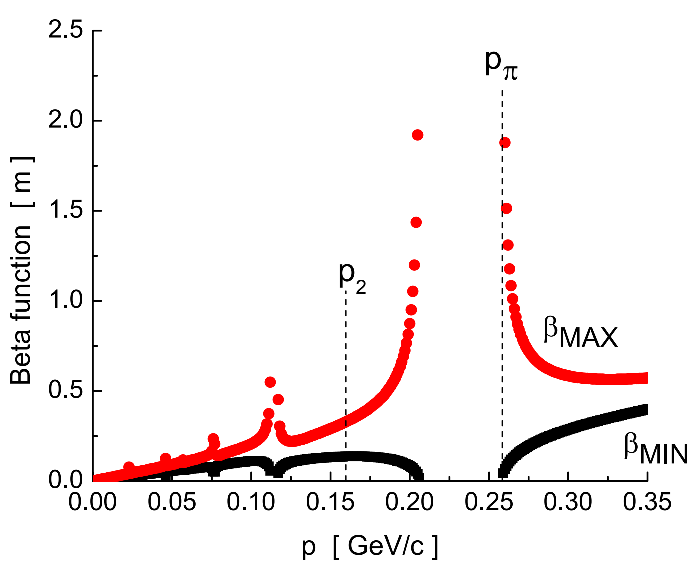

Figure 2. Pass bands as a function of momentum.

The highest energy pass band extends from infinite momentum down to the edge of the first stop band. We define the momentum of the upper edge of this stop band as $\mathbf{p_\pi}$ and the center of the next lower pass band as $\mathbf{p_2}$.

In general the beta function in these lattices can have its minimum value at the cell boundary or at the cell midplane. We will restrict ourselves here to solutions with the beta minimum at the midplane. Then there are the two symmetry classes shown in Fig. 3, depending on whether the polarity of the current alternates in adjacent cells.

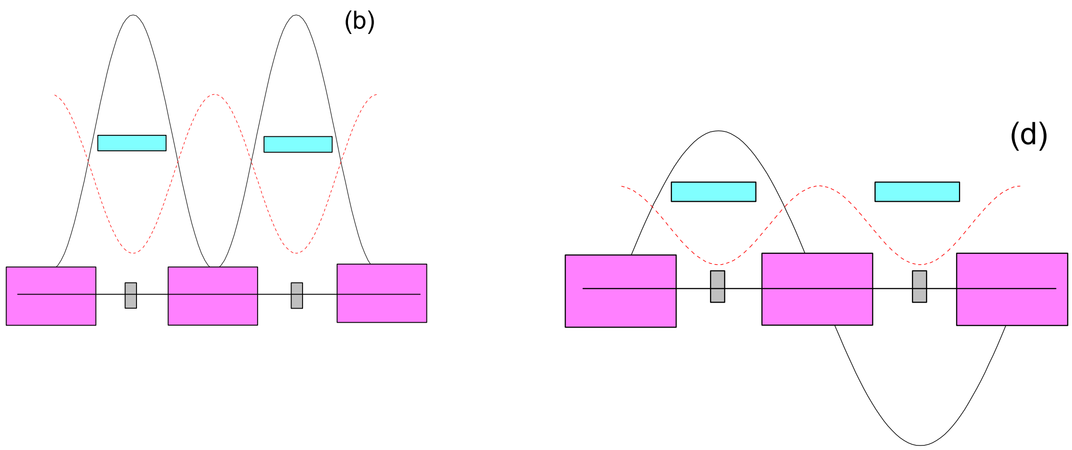

Figure 3. Lattice symmetry classes with the beta minimum at the cell midplane. Magenta box: rf cavity, cyan box: coil, gray box: absorber, black curve: magnetic field, dashed red curve: beta function.

The dependence of the peak field (B_p) and the maximum field on-axis (B_o) on the cell length and coil dimensions are shown in Fig. 4.

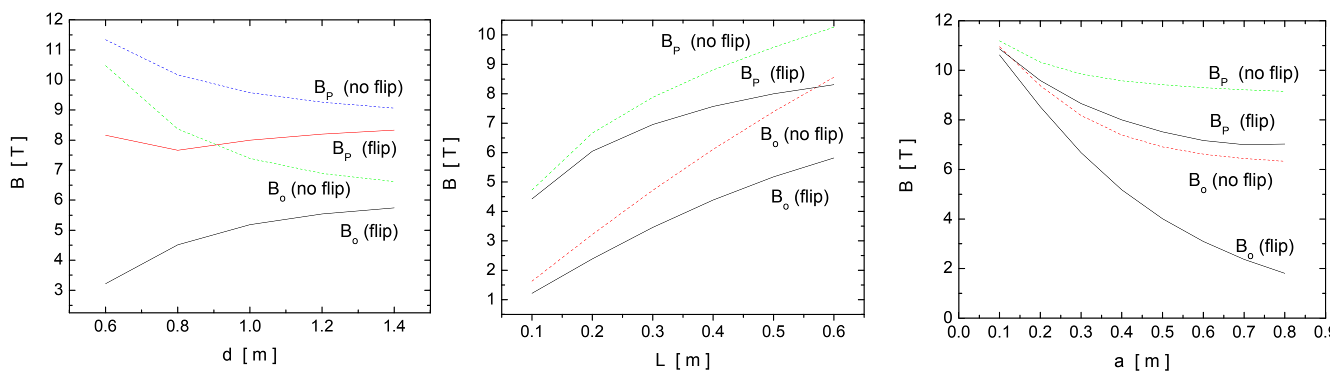

Figure 4. Dependence of the peak field on lattice parameters. The reference parameters are d = 100 cm, L = 50 cm, a = 40 cm, t = 10 cm and J= 100 A/mm^2.

For these plots all other lattice dimensions are kept constant. In general the peak field increases if the coil length is increased, while it decreases when the coil radius increases. The peak field increases linearly with increases in the current density and approximately linearly for increases in the radial thickness. The peak field is smaller for alternating polarity.

3. Properties of the second pass band solutions

We now consider solutions for the second momentum pass band. We look at solutions that have p_2 fixed at 200 MeV/c, which is a typical operating momentum for cooling lattices. We first look at the sensitivity of the solutions to changes in the cell geometry. Figure 5 shows the dependence on *d*. For these plots all other dimensions remain fixed.

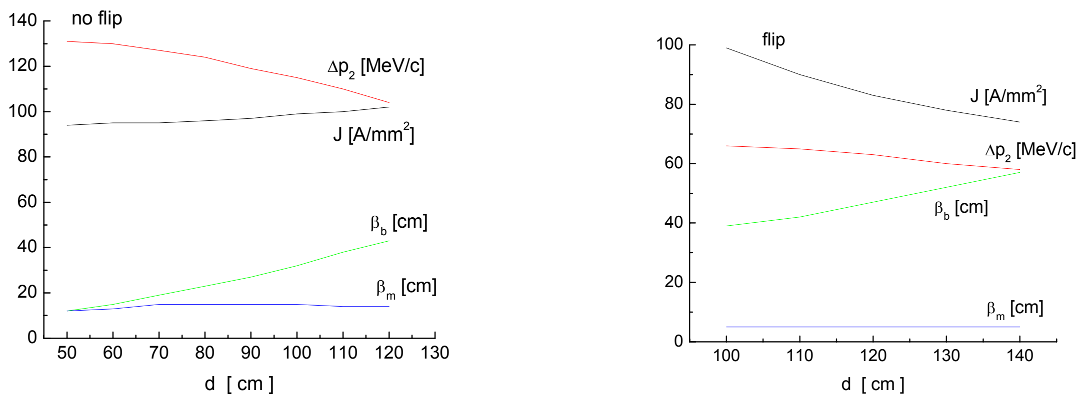

Figure 5. Dependence of the second pass band properties on the cell length. The other parameters are L = 20 cm, a = 40 cm, and t = 24 cm for the no-flip case, and L = 40 cm, a = 35 cm, and t = 24 cm for the flip case.

The current density is adjusted to keep p_2 fixed. Note that for the parameter range considered here the beta function at the midplane is insensitive to changes in the cell length, while the beta function at the boundary grows with increasing cell length.

Figure 6 shows the dependence on *L*.

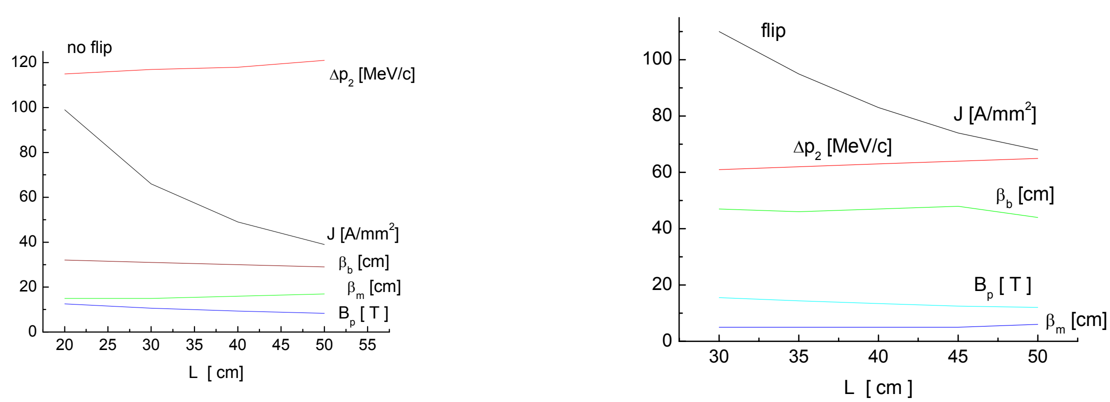

Figure 6. Dependence of the second pass band properties on the coil length. The other parameters are d = 100 cm, a = 40 cm, and t = 24 cm for the no-flip case, and d = 120 cm, a = 35 cm, and t = 24 cm for the flip case.

The beta function at the midplane is insensitive to changes in *L* for the parameter range considered here.

Figure 7 shows the dependence on *a*.

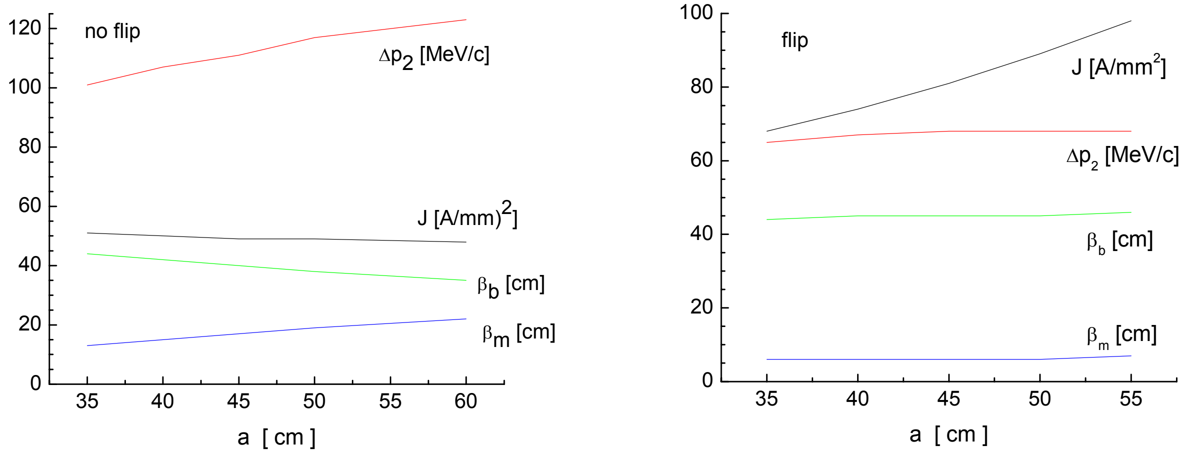

Figure 7. Dependence of the second pass band properties on the coil inner radius. The other parameters are d = 120 cm, L = 40 cm, and t = 24 cm for the no-flip case, and d = 120 cm, L = 50 cm, and t = 24 cm for the flip case.

The beta function at the midplane and the momentum acceptance both grow with *a* in the no-flip case.

Figure 8 shows the dependence on *t*.

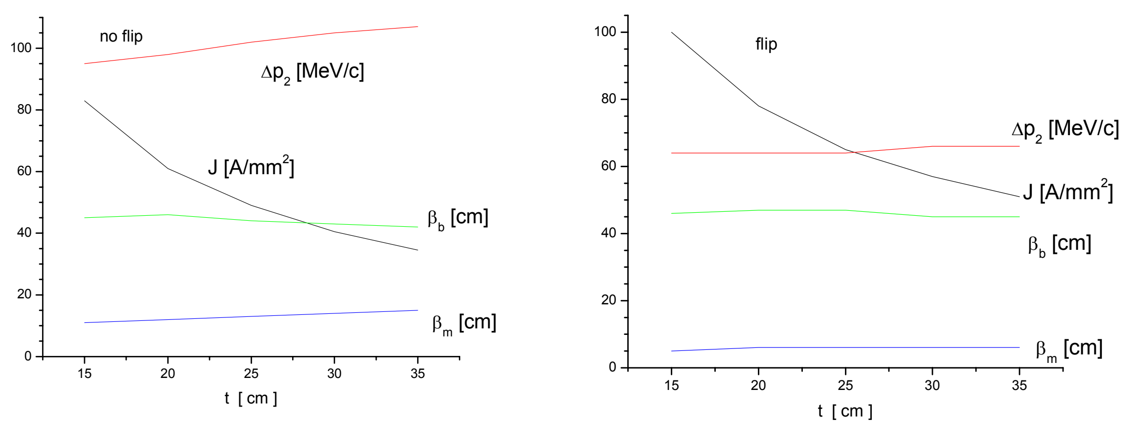

Figure 8. Dependence of the second pass band properties on the coil radial thickness. The other parameters are d = 120 cm, L = 40 cm, and a = 35 cm for the no-flip case, and d = 120 cm, L = 50 cm, and a = 35 cm for the flip case.

4. Geometric scaling

Lastly we consider the case when the cell length and all the coil dimensions are scaled in the same proportion. Figure 9 shows the dependence of the lattice properties on the scale factor for an alternating polarity lattice.

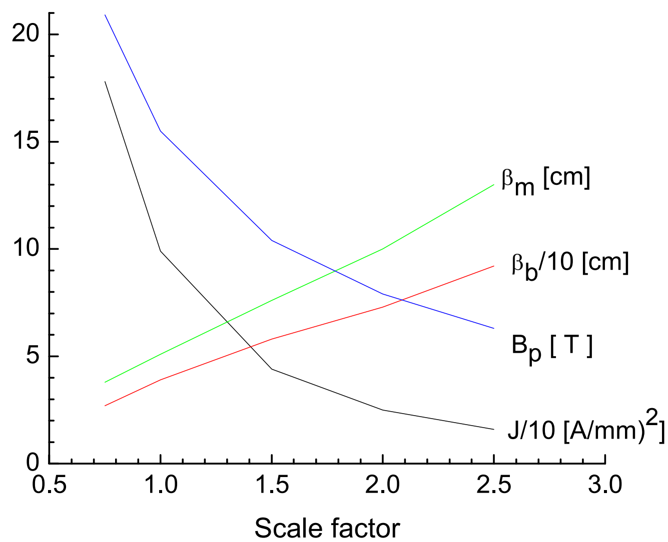

Figure 9. Dependence of the second pass band properties on scaled lattice dimensions for an alternating polarity lattice. A scale factor of 1 corresponds to d = 100 cm, L = 40 cm, a = 35 cm, and t = 24 cm.

J must be decreased quadratically to keep p_2 fixed at 200 MeV/c.[1] The peak field in the coil decreases for increasing scale factor. The maximum field on-axis tracks the peak field. The beta functions at the boundary and the midplane both grow linearly with increasing scale factor. The momentum acceptance (not shown) is independent of the scale factor.

Figure 9 shows that the beta function at the midplane remains small (3 – 13 cm) over a wide range of scale factors. With suitable adjustments of the coil dimensions and the geometric scale factor it is possible to design lattices with small beta function and an acceptable peak field in the coil. A muon collider design of this type for low emittances has been proposed recently [3].

Acknowledgements

We would like to thank Bob Palmer and Juan Gallardo for useful discussions.

[1] This dependence was pointed out by R. Palmer.

References

[1] R.C. Fernow & R.B. Palmer, Solenoidal ionization cooling lattices, to be published.

[2] J.S. Berg et al., Cost-effective design for a neutrino factory, PRSTAB 9,011001 (2006).

[3] R. Palmer, Lattices, Muon Collider Task Force Thursday Meeting presentation, 14 December 2006.