Low-Energy Ionization Cooling of Muons for Reverse Emittance Exchange

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Abstract. At low muon momenta, ionization energy losses cool the beam transversely while heating the beam longitudinally. Transverse emittances can be minimized at the cost of enlarged longitudinal emittance. The specific cooling rate (dP/ds)/P scales as $1/\beta^4$, which compresses cooling lengths to impractically small systems. However use of gas absorbers decompresses these lengths and may enable practical beam manipulation. The combination of stronger focusing, H_2 gas absorbers and low-momentum transverse cooling may enable cooling to $\varepsilon_{N,eq}$ as low as 0.000001m, much smaller than the previous collider baseline values.

Introduction

High luminosity at high energy in a μ^+-μ^- collider requires cooling of muons to minmal transverse emittances.[1] The cooling method used is ionization cooling, which is limited by the beam focusing and multiple scattering in the cooling absorbers. The basic cooling equation for transverse muon cooling within energy loss absorbers is, approximately:

$$\frac{d\varepsilon_N}{ds} = -\frac{g_t}{P_\mu}\frac{dP_\mu}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^2 m_\mu c^3 L_R P_\mu}, \qquad [1]$$

where ε_N is the transverse normalized emittance, g_t is the transverse partition number (g_t=1 without emittance exchange), β = v/c, P_μ is the muon momentum, dP_μ/ds is the momentum loss in the absorber, E_s is the characteristic scattering energy (~14 MeV), L_R is the characteristic radiation length in the material, and $\beta_\perp$ is the transverse focusing function at the absorber.[1] Cooling is limited to the equilibrium transverse emittance, which is:

$$\varepsilon_{N,eq} = \frac{\beta_\perp E_s^2}{2 g_t \beta\, m_\mu c^2 L_R \frac{dE}{ds}} \ , \qquad [2]$$

where $dE/ds = \beta c\, dP_\mu/ds$. dE/ds is given by the Bethe-Bloch formula :

$$\frac{dE}{ds} \cong 4\pi N_A \rho\, r_e^{\,2} m_e c^2 \frac{Z}{A}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right) - 1 - \frac{\delta}{2\beta^2}\right] \qquad [3]$$

where N_A is Avogadro's number, Z, A are the material atomic number and weight, ρ is the material density and $\delta \approx 0$ is the density effect parameter (δ = ~0). The material with largest $L_R dE/ds$ is hydrogen, and therefore the best cooling material. I(Z) is the material ionization energy, and is approximately 16 $Z^{0.9}$ eV.

With relativistic cooling, a value of $\varepsilon_{N,eq}$ of ~ 0.0001m is typically obtained and values similar to that are used in initial muon collider scenarios. However, if the particle momentum becomes relatively small, the transverse cooling can, in principle, become somewhat stronger. For small β, dE/ds scales as $1/\beta^2$. Solenoidal focusing becomes stronger at non-relativistic momenta, and the focusing parameter $\beta_\perp = 2\,P_\mu/0.3\,B$ is proportional to β. ($\beta_\perp = 0.00133$m at P_μ = 10MeV/c, B=50T). The combined effect is that $\varepsilon_{N,eq}$ is proportional to β^2, and transverse emittances could be reduced by a factor of ~100 by cooling to $\beta = 0.1$.

With the combination of strong solenoidal fields (50T), hydrogen absorbers, and cooling to β=0.1, we can obtain $\varepsilon_{N,eq}$ = 0.000001m, a factor of 100 smaller than the collider baseline value.

Low-Energy "Cooling" Challenges

The major difficulty in lower energy cooling is that the increase of energy loss with reduced energy heats the beam longitudinally, increasing the longitudinal emittance. The equation for rms energy spread change is:

$$\frac{d\sigma_E^2}{ds} = -2\frac{\partial \frac{dE}{ds}}{\partial E}\sigma_E^2 + \frac{d\left\langle \Delta E_{rms}{}^2\right\rangle}{ds} \quad , \qquad [4]$$

where the second is the heating term caused by random fluctuations in the particle energy loss. In the long-pathlength Gaussian-distribution, limit, the second term in Eq. 2 is given by:

$$\frac{d\left\langle \Delta E_{rms}{}^2\right\rangle}{ds} = 4\pi\left(r_e m_e c^2\right)^2 n_e\gamma^2\left(1-\frac{\beta^2}{2}\right) \qquad [5]$$

The first term is strongly antidamping at low energies. The antidamping can be reduced by "emittance-exchange" cooling, in which the particle path through absorber is made energy-dependent and the energy spread can be relatively decreased. Such emittance-exchange increases transverse emittance by the same degree that longitudinal emittance decreases; 3-D emittance is not decreased.

The second term can be rewritten, approximately, as:

$$\frac{d\left\langle \Delta E_{rms}{}^2\right\rangle}{ds} \cong 0.157\rho\frac{Z}{A}\gamma^2\left(1-\frac{\beta^2}{2}\right) \qquad (\mathrm{MeV})^2/\mathrm{cm}, \qquad [6]$$

with ρ in gm/cm^3.

The heating or cooling effect of energy-loss can be expressed as a partition number where the partition number is the relative rate of cooling or heating compared to the fractional momentum change:

$$g_L = \frac{\frac{d\varepsilon_L/ds}{\varepsilon_L}}{\frac{dp/ds}{p}}$$

From the Bethe-Bloch expression (with δ=0) this becomes:

$$g_L \cong -\frac{2}{\gamma^2} + \frac{2(1-\frac{\beta^2}{\gamma^2})}{\left(\ln\left[\frac{2m_e c^2\beta^2\gamma^2}{I(Z)}\right] - \beta^2\right)}$$

This is ~ -1.6 at p_μ = 10 MeV/c. The transverse (x and y) partition numbers are both 1 (without emittance exchange). The sum of the x, y, and z partition numbers remains positive but is ~0.4 at low energies; it is ~2 at relativistic energies. Emittance exchange cooling transfers cooling decrements between transverse and longitudinal emittances; the sum of partition numbers is constant.

In the present case we choose to reduce transverse emittance while allowing longitudinal emittance to increase. Thus the low-energy cooling is used effectively as a "reverse emittance exchange" procedure, which allows a minimization of transverse emittance while longitudinal emittance is enlarged. This process is acceptable for a high-energy muon collider (within limits), since acceleration to high energy reduces the relative energy spread (δE/E) and makes large δE more tolerable. The research program will explore the low energy cooling to determine the limits of this transfer.

The second key difficulty is that the relative energy loss (1/E × dE/ds) becomes quite large and the characteristic cooling distance $L_C = (1/p \times dp/ds)^{-1}$ becomes very short in solids or liquids. For 10 MeV/c muons, this distance becomes L_C = ~0.062cm for liquid hydrogen and that is very short for a practical cooling system. A 50T magnet focusing system would be at least 10's of cm long. However, if the absorber is pressurized H_2 gas, the length can be extended to fit the dimensions of the cooling system. The density of H_2 gas is 0.0000838P gm/cm^3 at 295°K, with P the gas pressure in atmospheres, so L_C = 49/P cm for 10 MeV/c muons, and the gas pressure can be adjusted to obtain (almost) any desirable cooling length. At these low momenta, gas absorbers (gas jet absorbers) should be used and the densities tailored to optimize the final phase-energy distributions. Dispersion may be introduced to correlate energy/energy loss in that optimization. These options should be explored.

We note that L_C scales as $1/\beta^4$ for small β. For P_μ= 20 MeV/c, L_C=1.27cm in liquid H_2. For higher muon momenta, liquid or solid absorbers may be preferable to gas.

For momenta less than ~5 MeV/c, the Bethe-Bloch equation becomes inaccurate, and the muon-atom interactions are frictional with high probability of muon capture (for μ^-) or muonium (μ^+-e^-) formation. In this initial research, we will avoid this very low energy region, but note that it could be very useful for some applications.

Low Energy Cooling with gas absorbers

We first rewrite the rms cooling equations, with an emphasis on parameter values corresponding to low energy. At low energies, momentum (P_μ) is a more appropriate variable than energy for the discussion.

$$\frac{1}{P}\frac{dP}{ds} \cong 4\pi N_A \rho\, r_e^{\,2} m_e c^2 \frac{Z}{A}\frac{1}{\beta^2 \gamma m_\mu c^2}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right)-1-\frac{\delta}{2\beta^2}\right]$$

$$\frac{d\sigma_P^2}{ds} = -2g_L \frac{dP}{Pds}\sigma_P^2 + 0.157\rho \frac{Z}{A}\frac{\gamma^2}{\beta^2}\left(1-\frac{\beta^2}{2}\right)$$

Divide both terms on the right in the above equation by 2 if longitudinal oscillations are occurring. (The longitudinal heating and cooling rates decrease when the longitudinal coordinates δp-δz are mixed.)

Transverse cooling equations:

$$\frac{d\varepsilon_N}{ds} = -\frac{g_t}{P_\mu}\frac{dP_\mu}{ds}\varepsilon_N + \frac{\beta_\perp E_s^2}{2\beta^2 m_\mu c^3 L_R P_\mu}$$

$$\frac{d\left\langle \theta_{rms}{}^2\right\rangle}{ds} = -\frac{2g_t}{P_\mu}\frac{dP_\mu}{ds}\left\langle \theta_{rms}{}^2\right\rangle + \frac{E_s^2}{\beta^2 c^2 L_R P_\mu{}^2}$$

This rms equation assumes P_μ is continually restored by longitudinal reacceleration. A more general equation (which also applies when P_μ is not continually restored) is:

$$\frac{d\left\langle P_t^2\right\rangle}{ds} = -\frac{2g_t}{P_\mu}\frac{dP_\mu}{ds}\left\langle P_t^2\right\rangle + \frac{E_s^2}{\beta^2 c^2 L_R}$$

We will consider hydrogen (H_2) as the baseline absorber material. At liquid density, ρ=0.0708 gm/cm^3 and L_R=866cm. At standard temperature (T=20ºC), ρ = 0.0000838P gm/cm^3, where P is the pressure in atmospheres and L_R = 731400/P cm. We will consider typical values for some of these parameters.

ICOOL simulation studies

To test some of these concepts, we have run ICOOL[2] simulations of low energy muon beams traveling through absorbers, with some evaluation of cooling effects, without reacceleration, and we observed the cooling and heating effects. In a typical simulation, we passed the beam through enough hydrogen such that the beam would lose about a third of its momentum.

Table 1 summarizes results of 4 simulations, starting at momenta of 50, 30, 22, and 15 MeV/c. In each of these cases we introduce enough material to obtain a momentum loss of ~30% of the initial beam. In such a simulation, the x and y transverse emittances would decrease by ~25%, while longitudinal emittance increases by more than ~50%. The net effect is emittance exchange, in which the transverse cooling is balanced by longitudinal heating, while the 6-D emittance remains nearly constant.

These simulations demonstrated that effective emittance exchange occurred for a large variety of conditions, and demonstrate that low-energy absorber-driven energy loss can be used to reduce the beam to very small transverse emittance, at the cost of corresponding longitudinal emittance increase, with relatively constant 6-D emittance.

The last case in Table 1 was chosen as a limit case approaching minimum cooling. It shows that in a final stage one can cool from $\varepsilon_{\perp,N}$ = ~1.4×10^{-6} to ~1×10^{-6} using liquid hydrogen absorbers. (This is much less than usually considered in μ^+-μ^- collider scenarios.) This requires focusing to a beam size of ~0.1mm and is at a cost of longitudinal heating. That beam size would correspond to a $\beta_\perp$ = 0.001m. (In practice, one would probably not cool to that limit because of the accumulated longitudinal heating effects, and the difficulty of focusing to that small a spot size.) With a more modest beam size of 0.5mm (and $\beta_\perp$ = 0.005m) we would obtain cooling in $\varepsilon_{\perp,N}$ = ~7.18×10^{-6} to ~5.51×10^{-6}, with the same increase in longitudinal emittance.

Figure 3 shows the change in longitudinal phase space that accompanies the transverse cooling in a particular case of "cooling" from P_μ from ~15 to 10MeV/c, with $\delta P_{\mu,z}$ initially at 0.3 MeV/c. The momentum spread increases by nearly a factor of 3; however, the bunch length δz decreases by ~30% from adiabatic damping.

The ICOOL simulations showed significantly less multiple scattering than that predicted by a simple application of the rms cooling and scattering equations, particularly at the lowest momentum.[3] The ICOOL simulations have been performed using both a Bethe-Moliere model[4] and a Fano-based model[5] for the multiple scattering. The multiple scattering in hydrogen is 40% smaller than that indicated by the rms equation when the Bethe model is used and ~60% less when the Fano model is used. The Fano model is expected to be more accurate than the Bethe model and both models are expected to be more accurate than the simplified rms scattering equation.[3] The deviation is greater than that observed when cooling at the semirelativistic momenta (~200MeV/c) that are optimal for 6-D cooling; the multiple scattering at low energies is much less than that given by the rms formulae.

To obtain further cooling the beam would need longitudinal phase-energy rotation after each step, which requires a combination of drift + rf to reduce the momentum spread while increasing the mean beam energy and the bunch length. Because the beam energy is so low the drift distance required is relatively small. For instance, if the initial beam had a bunch length in $c\tau$ of 1cm at P_μ = 10 MeV/c, the physical bunch length (in $\delta z=\beta c\tau$) is only ~1mm. We would like that bunch length to stretch by a factor of ~3, so that the momentum spread (and energy spread) may be reduced by a factor of 3 in reacceleration. If the rms $\delta p/p$ is 10%, we would need a drift of only 3cm ($c\tau$=30cm) to obtain the required bunch length growth. A bunch length of $c\tau$ = 1m would require a 3m drift to be properly expanded.

If the beam transport is a bend-less and rf-less drift then the bunch length changes following, approximately:

$$\delta z(s) = \sqrt{\left(\delta z_0\right)^2 + \left(\frac{1}{\gamma^2}\frac{\delta P}{P}s\right)^2} ,$$

where $\delta P/P$ is the rms relative momentum spread. The distance required to expand the beam by a factor of 3 is then $\Delta s \cong 3\ \delta z_0\ \gamma^2 P/\delta P$. For non relativistic motion $\gamma \cong 1$, and we have typically used $\delta P/P \cong 0.1$, so $\Delta s \cong 30\ \delta z_0$, which can be small for small δz_0. This length could be extended significantly if a more isochronous transport is used.

To reduce the energy spread and develop multistep cooling, reacceleration would then be required. The total energy of reacceleration is relatively small ($\Delta E = \beta\Delta P$); only ~0.6MV is needed for the 15 to 10 MeV/c case, or ~2.1MV for the 30 to 21MeV/c example. A low gradient rf cavity or even an induction linac module could be adequate.

If the transport is a field-free drift, we can combine the energy regain with the drift, obtaining an estimate of required gradient, which depends on δz_0: $\Delta E/\Delta s \cong \beta c \delta P \Delta P / 3\delta z_0 P\gamma^2 \cong \beta c\ \Delta P/30\delta z_0$. At low energies and relatively long bunches, this can become fairly small. (At δz_0=10cm, ΔP=5MeV/c, β=0.1, this is 0.17MV/m.) There is, however, a serious practical problem in the fact that bunch lengthening must be applied in each step, since the longitudinal emittance is increasing and eventually the bunch becomes impractically lengthened. Splitting the beam longitudinally into multiple bunches may help one to extend this by another factor.

Liquid-density hydrogen was used as a reference material since it represents the maximum density available in that material. For most cooling sections we may consider at low energies, a much lower density could be used. Note that for the cooling sections described in Table l, the lengths of the absorbers at liquid hydrogen densities are mm to cm lengths, much smaller than a general transport length, so much lower densities (0.1 to 0.01 ρ_{LH}) could be used and may be more practically implemented – with such devices as gas-jet absorbers and gas-filled rf cavities. Spaced "foil" absorbers could also be used.

Comments

In the initial calculations of Table 1, we placed the beam within a drift space at a low β^*. If directly implemented, this would mean the beam size would blow up quickly exiting the focus, following ($\beta(s) = \beta^* + s^2/\beta^*$) and chromatic and geometric aberrations would distort the cooled emittance. If the beam focus is maintained by solenoidal fields, this distortion is greatly reduced and the cooled emittance can be maintained.

We have shown that the technique of emittance exchange by energy loss within absorbers can be extended to very low momentum with some potential to reduce the muon beam transverse emittance to small values. The process is limited by the fact that longitudinal emittance is increased by low-momentum-energy loss, with 6-D emittance approximately constant.

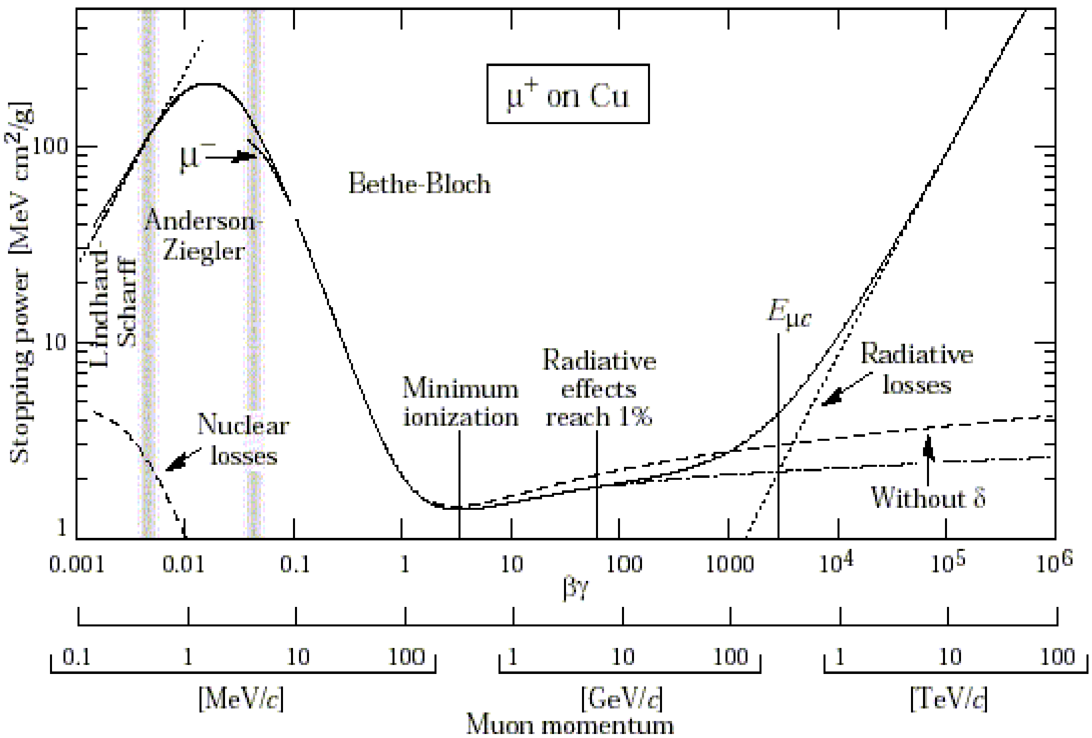

Figure 1: This displays the energy loss of charged particles as a function of particle momentum mβγ. In the present paper we are considering muon momenta of ~10 to 100 MeV/c, where the change of energy loss with respect to particle momentum is steeply negative (anti-damping).

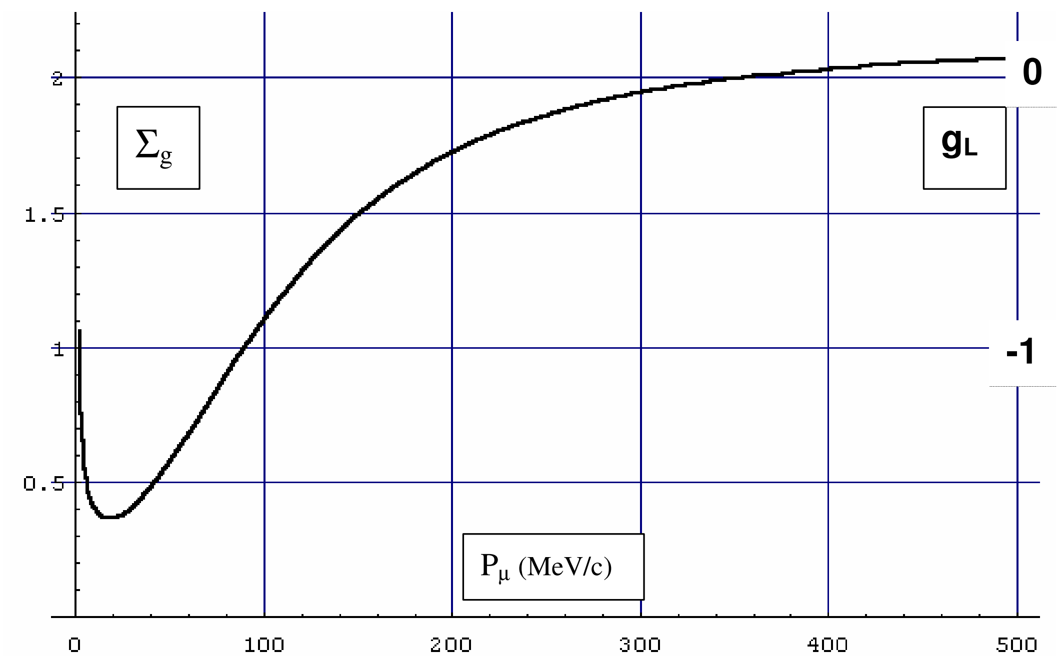

Figure 2: The longitudinal partition number, g_L, and the sum of x, y, z partition numbers, $\Sigma_g = 2 + g_L$ as a function of muon momentum P_μ . Σ_g varies from ~0.3 at P_μ = 20 MeV/c to slightly more than 2 for P_μ > 350 MeV/c. This particular graph is for hydrogen absorbers (with δ=0); there is only a weak dependence on absorber material. The parameters depend only on β=v/c of the incident particle. (Protons have the same Σ_g as muons with $P_\mu = (m_\mu/m_p) P_p$.)

Figure 3:
Longitudinal projections of the beam in simulations of low-energy cooling of muons in hydrogen absorbers. The initial beam is cooled transversely by ~30% in x and y emittances while the longitudinal emittance ~doubles. Note that in z-δp, z shortens (δt remains the same) while δp triples (δE doubles).

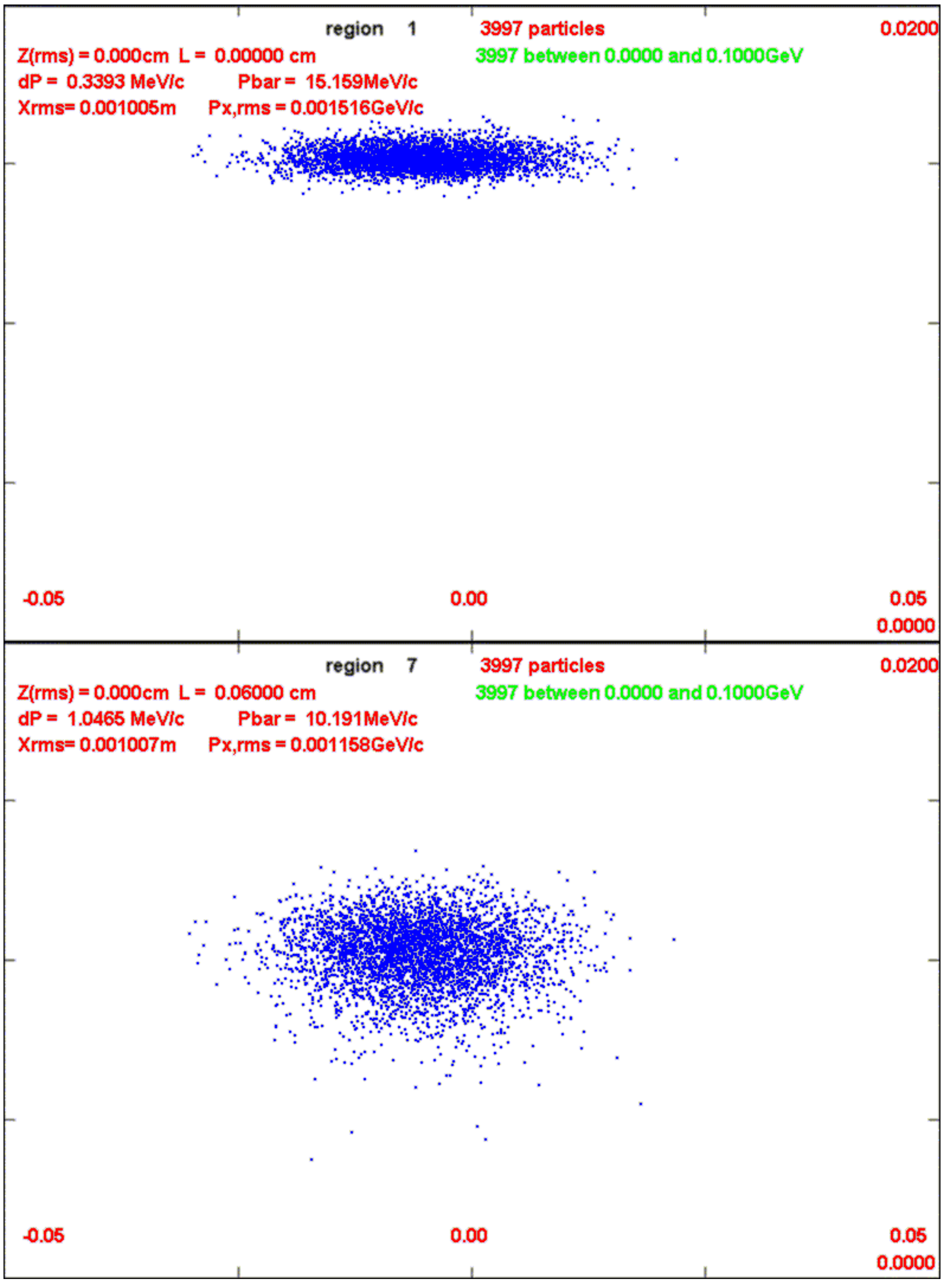

Figure 4.
Addition of low-E emittance exchange can reduce transverse emittance at the cost of increased longitudinal emittance. This process could fit reasonably well within a complete cooling scenario. This figure shows a baseline cooling scenario where a minimal 6-D emittance is obtained after a sequence of cooling steps at the end of the “New Ring”. (Original version is from a cooling scenario discussion of R. Palmer.[] Horizontal scale is transverse normalized rms emittance in mm-mrad; vertical scale is longitudinal rms emittance in mm.) We then add a low-E cooling line showing that a lower transverse emittance at enlarged longitudinal emittance that could be obtained by low-E steps. The 6-D cooling allows a low-E sequence that can reach a transverse emittance of ~5 mm-mmrad. (The method does parallel the 50-60T lens steps in the R. Palmer discussion; this section is similar to ours; the major difference is that we are extending the concept to even lower momenta where we can obtain similar emittance exchanges in shorter distances and lower magnetic fields.)

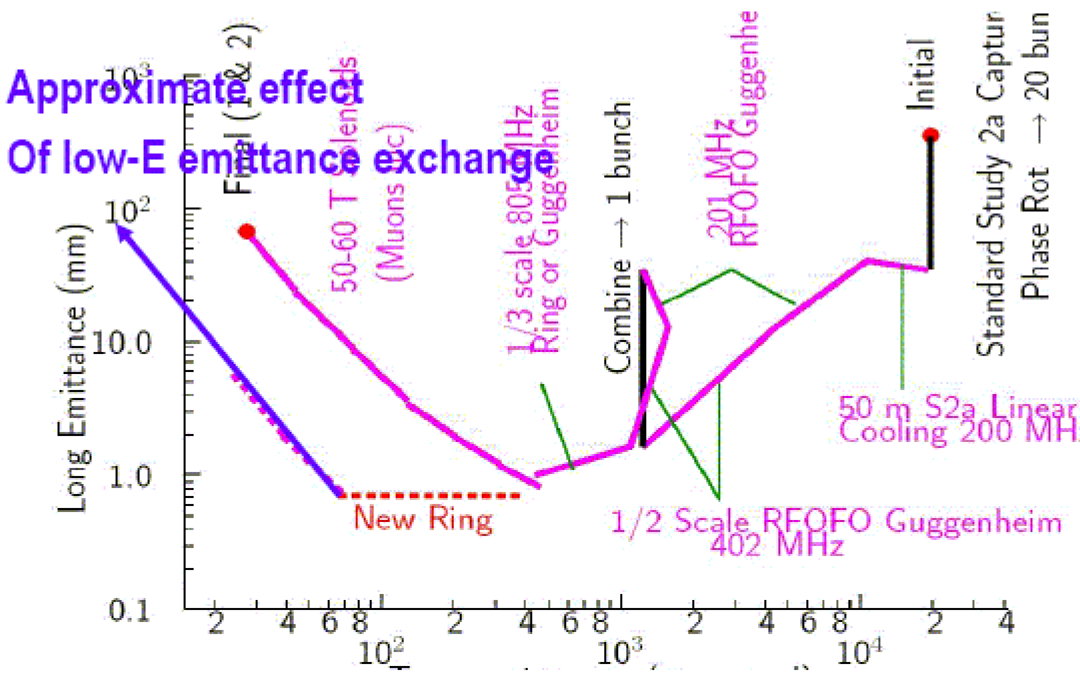

Table 1: Low-Energy Cooling Simulation Examples

Parameter	**Symbol**	**50MeV/c**	**30 MeV/c**	**22 MeV/c**	**15 MeV/c**
Initial momentum	P_i	50.7	30.35	22.3	15.16 MeV/c
Final momentum	P_f	41.4	21.4	15.3	10.19 MeV/c
LH absorber thickness		3cm	0.7cm	0.3cm	0.06cm
Focusing parameter	β*	0.083m	0.018m	0.0045m	0.001m
Initial transverse emittance	$\varepsilon_{t,I,N}$	0.057cm	0.00624cm	0.00112	0.000143cm
Final transverse emittance	$\varepsilon_{t,I,N}$	0.0485	0.00485	0.00091	0.000109cm
Initial δP	$\delta P_{rms,i}$	1.94	0.70	0.57	0.51 MeV/c
Final δP	$\delta P_{rms,f}$	3.16	1.85	1.7	1.69MeV/c
Initial kinetic energy	$E_{k,i}$	11.5	4.27	2.33	1.08 MeV
Final kinetic energy	$E_{k,f}$	7.82	2.145	1.10	0.49 MeV
6-D heating factor	F_{6D}	0.98	1.14	1.14	1.17
Bunch length	L	1cm	2mm	5mm	10cm

References

[1] D. Neuffer, Nucl., Inst. and Meth. A532, p.26 (2004).

[2] R. Fernow, "ICOOL, a Simulation Code for Ionization Cooling of Muon Beams", Proc. 1999 Particle Accelerator Conference, New York, p. 3020 (1999), see http://pubweb.bnl.gov/people/fernow/.

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[6] R. Palmer, unpublished (2006).