Guggenheim channel with realistic fields

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We examine the magnetic fields produced in the Guggenheim cooling channel. We describe the algorithms used to construct a cylindrical 3D field grid for use with ICOOL. The computed fields agreed well with the results from G4Beamline. We look at the characteristics of periodic orbits in the Guggenheim field. Simulations of the first 201 MHz channel for the muon collider showed good cooling performance, although the 52% overall transmission is still an issue.

1. Introduction

The Guggenheim channel for 6D cooling at the muon collider was first proposed by R. Palmer [1]. The design of the channel begins with the lattice of the RFOFO ring [2]. To form the Guggenheim channel the solenoids are elevated to follow the path of a helix. Until now simulations of the helical nature of the Guggenheim channel in ICOOL have been done by including an a_0 term in the Fourier expansion of the on-axis multipoles [3]. This approximation seemed to work for the RFOFO ring, but the accuracy of the off-axis expansions did not improve in a predictable way as the order of the expansion was increased. For this reason we are investigating other ways of simulating the channel in ICOOL. The method adopted here is to develop a new code GUGMAP to prepare a 3D Guggenheim field map for tracking.

2. GUGMAP algorithm

The GUGMAP procedure starts with the *reference circle* of the RFOFO ring. This circle has a circumference of 33 m and a radius of curvature *r* of 5.2521 m. It is the approximate location of the circulating muon beam. The centers of the solenoids start off located on a second *magnet circle*, which can be displaced from the reference circle by a distance *dr*. The parameter *dr* was used to zero out the net radial field per cell in the RFOFO ring design. The magnet circle has 12 identical cells, each with two oppositely directed solenoids. For the Guggenheim channel the centers of the solenoids are moved vertically so that they lie on a *reference helix*. The axes of the 24 solenoids start off tangent to the reference helix. Each solenoid can be rotated additionally thru a dip angle θ. The dip angle is used to control the net amount of vertical bending field per cell.

GUGMAP takes a superposition of the fields from all the solenoids in one or more turns of the helix. It calculates the solenoid fields using elliptic integrals [4]. As a check the field can also be calculated using the Biot-Savart formula. The calculated field components are checked by computing the Maxwell divergence and curl relations on the cylindrical grid. The program reproduces the RFOFO fields when the helical pitch is set to 0.

The internal field calculations in GUGMAP are done in Cartesian coordinates (B_X, B_Y, B_Z). However, for making the 3D map for ICOOL we use a cylindrical coordinate system where *r* is the radial coordinate, *s* is the azimuthal coordinate, and *y* is along the axis of the cylinder. These coordinate systems are related by

$$
\begin{aligned}
B_R &= B_x \sin\alpha + B_z \cos\alpha \\
B_Y &= B_y \\
B_S &= B_x \cos\alpha - B_z \sin\alpha
\end{aligned}
$$

where α is the azimuthal angle from the *z* axis in the reference plane perpendicular to *y*.

It is also useful to find the field components along the reference helix for the channel. It is convenient to use the Frenet-Serret coordinate system (**T**, **N**, **B**) for this purpose. The letters stand for the Tangential, Normal and Binormal directions. The Cartesian coordinates of a helix can be described parametrically by

$$
\begin{aligned}
x &= r \sin t \\
y &= h\,t \\
z &= r \cos t
\end{aligned}
$$

where the parameter *t* lies between 0 and 2π, the parameter *r* is the radius of the helix, and the parameter *h* is related to the helical pitch λ by

$$
h = \frac{\lambda}{2\pi}
$$

The Frenet unit vectors are given in terms of the Cartesian coordinates by

$$
\begin{aligned}
\widehat{T} &= \frac{1}{\sqrt{r^2 + h^2}} \left[r \cos t\ \widehat{x} + h\ \widehat{y} - r \sin t\ \widehat{z} \right] \\
\widehat{N} &= -\sin t\ \widehat{x} - \cos t\ \widehat{z} \\
\widehat{B} &= \frac{1}{\sqrt{r^2 + h^2}} \left[-h \cos t\ \widehat{x} + r\ \widehat{y} + h \sin t\ \widehat{z} \right]
\end{aligned}
$$

Taking the dot product with the Cartesian field components, we find the Frenet field components

$$B_T = \frac{1}{\sqrt{r^2 + h^2}}\left[r\cos t\ B_x + h\ B_y - r\sin t\ B_z\right]$$
$$B_N = -\sin t\ B_x - \cos t\ B_z$$
$$B_B = \frac{1}{\sqrt{r^2 + h^2}}\left[-h\cos t\ B_x + r\ B_y + h\sin t\ B_z\right]$$

Since one full turn of the helix corresponds to $t = 2\pi$ and it also corresponds to $\alpha = 2\pi$ in the reference plane, we find that $t = \alpha$. The axes of the solenoids are initially aligned along **T**. The ±dip angles are then applied to the two solenoids in each cell.

The arclength along one turn of the helix is given by

$$L = 2\pi r\sqrt{1 + \left(\frac{\lambda}{2\pi r}\right)^2}$$

For a pitch of 3 m the arclength on the helix is 1.004124 times longer than the arclength on the reference circle.

3. Guggenheim magnetic field

The Guggenheim magnetic fields were first rigorously computed by Amit Klier [5] using the G4Beamline code. The geometry of the channel is shown in Fig. 1.

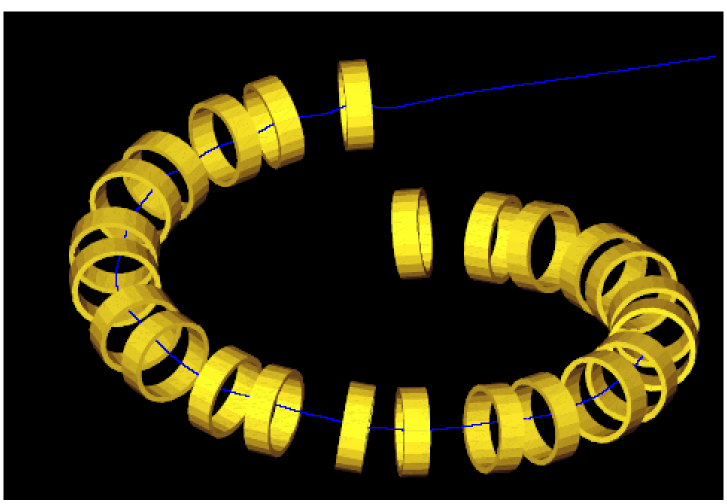

Figure 1. G4Beamline model of one turn of the Guggenheim channel (A. Klier)

The three Frenet field components along the reference helix that were computed by Klier are shown in Fig. 2.

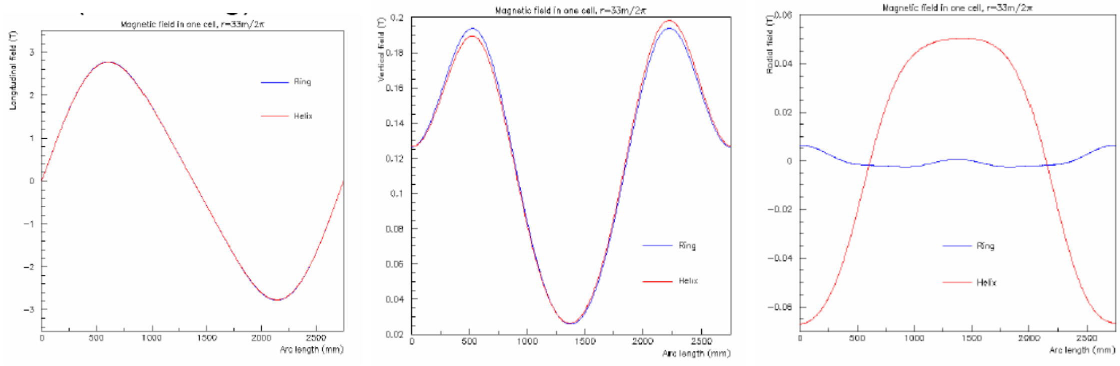

Figure 2. Magnetic field components for one cell of the Guggenheim channel computed with G4Beamline (A. Klier)

The corresponding Frenet field components from GUGMAP are shown in Fig. 3.

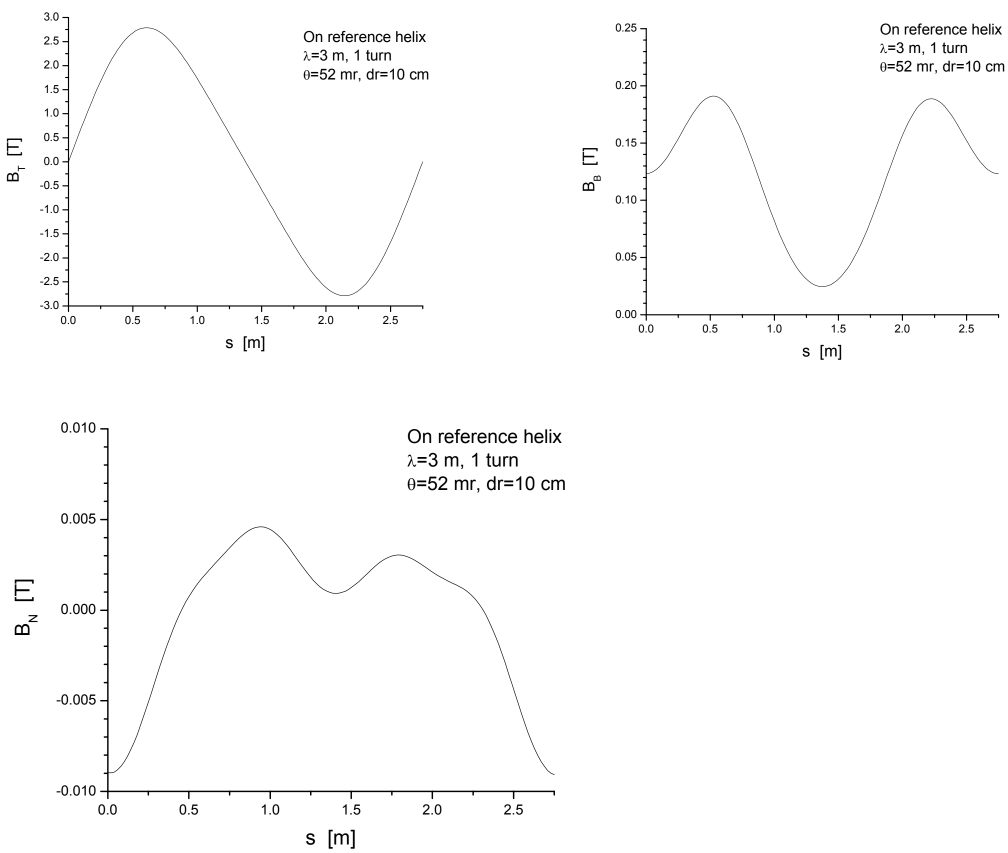

Figure 3. Frenet magnetic field components on the reference helix from GUGMAP for one turn of the Guggenheim channel

Comparing with the G4Beamline results we see that the longitudinal (T) and vertical (B) components agree very well. The radial components (N) have the same general shape, but they disagree in detail and in magnitude. Subsequent work[1] with the G4Beamline Guggenheim model found that the calculation of the radial field in Fig. 2 used an incorrect input parameter. The new results agree very well[2] with the radial field from GUGMAP [6]. In any case the average value of this component over the cell is small for both codes, so it shouldn't introduce a large uncertainty in the channel performance.

The field components on the reference helix in cylindrical coordinates for three turns of an unshielded Guggenheim channel are given in Fig. 4.

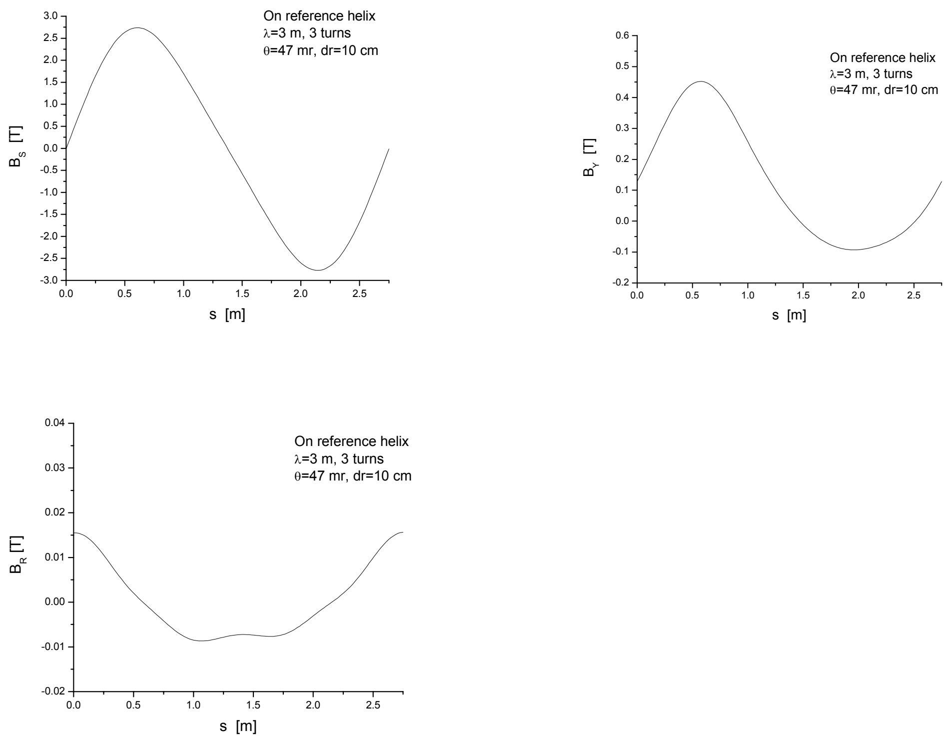

Figure 4. Cylindrical magnetic field components on the reference helix from GUGMAP for three turns of the Guggenheim channel

Note that in this figure we are looking at the net fields in a central cell for 3 turns in the helix. The additional turns increase the vertical field in the central cell, so we have slightly decreased the dip angle.

[1] Pavel Snopok, private communication.
[2] Apart from a different choice for the positive radial direction.

Finally we show in Fig. 5 the cylindrical field components on the reference circle. This is the center of the cylindrical 3D field grid used for ICOOL.

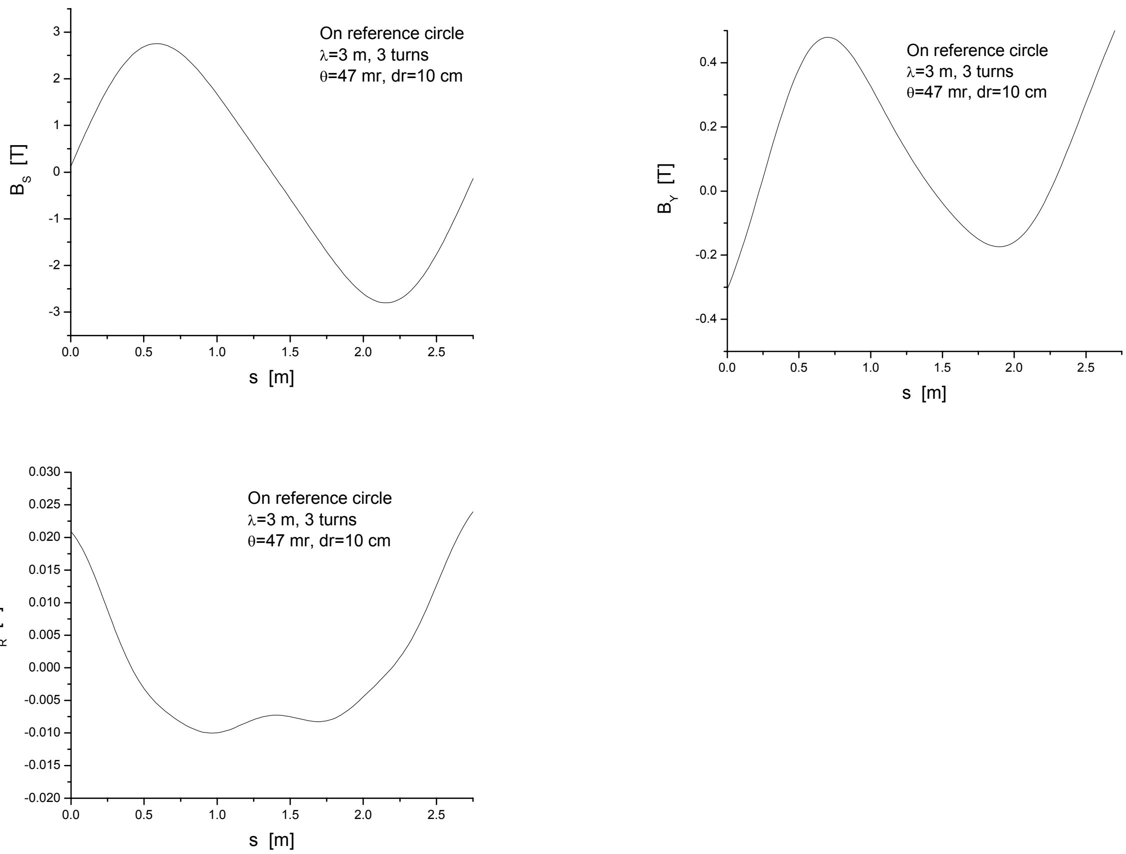

Figure 5. Cylindrical magnetic field components on the reference circle from GUGMAP for three turns of the Guggenheim channel

The average vertical field per cell is shown as a function of the dip angle θ in Fig. 6.

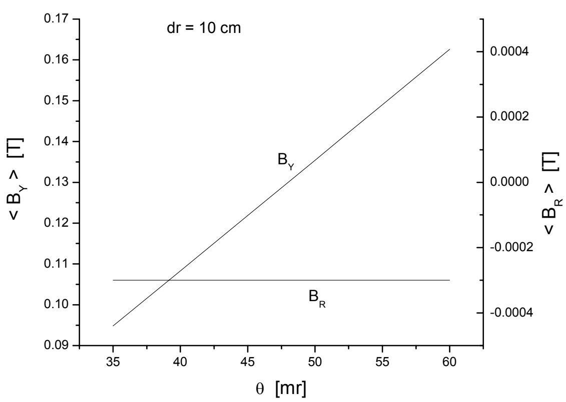

Figure 6. Average vertical and radial field per cell as a function of the dip angle θ

We see that the average vertical field per cell increases linearly with increasing dip angle. In order for a particle with momentum *p* to have a radius of curvature *r*, we need

$$\langle B_y \rangle = \frac{p}{e\,r}$$

For a 200 MeV/c particle this gives $\langle B_Y \rangle = 0.1270$ T. From Fig. 6 we see we can obtain this vertical field with a dip angle of 47 mr. We can also see from the figure that the dip angle does not affect $\langle B_R \rangle$.

The average radial field per cell is shown as a function of the radial offset *dr* in Fig. 7.

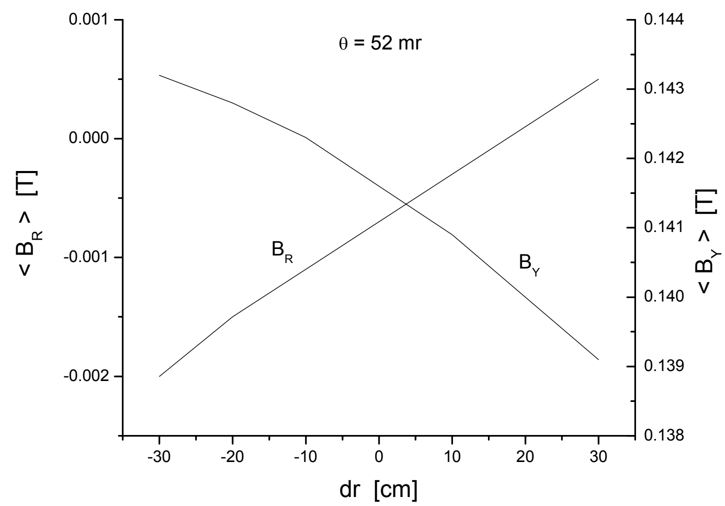

Figure 7. Average radial and vertical field per cell as a function of the radial offset *dr*

We see that the average radial field per cell increases linearly with increasing offset *dr*. We are limited to radial offsets of about ±30 cm or else the straight solenoid coils will interfere with the ±25 cm free aperture required around the curved beam path. We can also see from Fig. 7 that the radial offset also produces a small change in $\langle B_Y \rangle$.

4. Periodic orbits

Periodic orbits were determined by using ICOOL with the new field map to track thru one cell of the Guggenheim lattice. The output from ICOOL was used in a root-finding routine to determine the periodic solutions. Note that for a 3 m pitch the vertical change of the reference helix per cell is 25 cm. Thus the root finder required that the vertical position at the end of the cell was exactly 25 cm higher than the initial vertical position. Fig. 8 shows the transverse position offsets required at the start of the cell as a function of the total particle momentum.

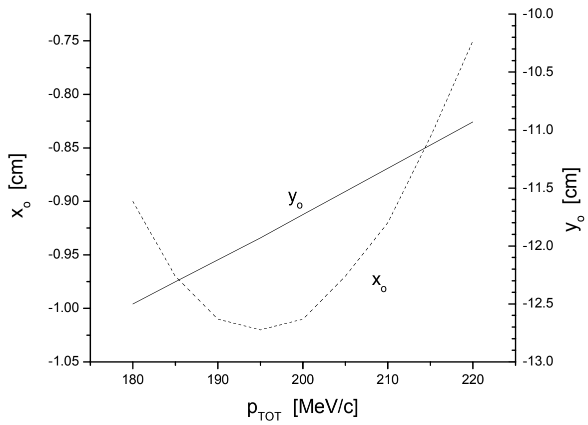

Figure 8. Initial transverse position offsets to produce a periodic orbit as a function of the total particle momentum.

For 200 MeV/c the required initial radial offset is -1.01 cm and the initial vertical offset is -11.73 cm. Fig. 9 shows the required transverse momentum offsets at the start of the cell as a function of the total particle momentum.

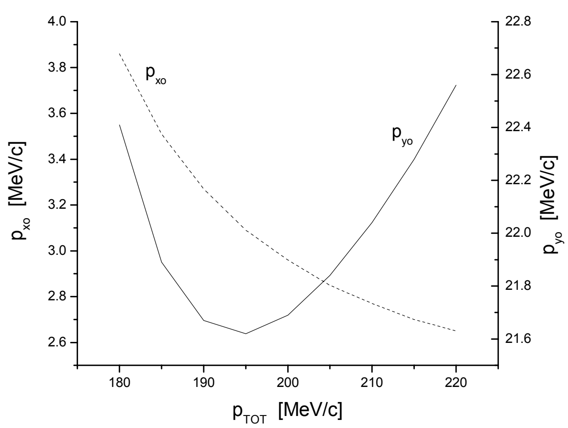

Figure 9. Initial transverse momentum offsets to produce a periodic orbit as a function of the total particle momentum.

For 200 MeV/c the required initial radial momentum offset is 2.96 MeV/c and the initial vertical momentum offset is 21.69 MeV/c. The corresponding β function at 200 MeV/c is 38 cm.

5. Cooling performance

We have used the new 3D map to study the cooling performance of the first 201 MHz Guggenheim channel in Bob Palmer's scenario for the muon collider [7]. The new field map can be used with ICOOL to accurately study the Guggenheim channel performance.

To do this the code tracks all the particles thru a cell using the map. Next the vertical positions of all particles are decreased by the 25 cm vertical pitch per cell of the reference helix. Then all particles are tracked thru the next cell using the same field map. This procedure is repeated for all the cells in the channel.

The cooling performance found using ICOOL is shown in Fig. 10.

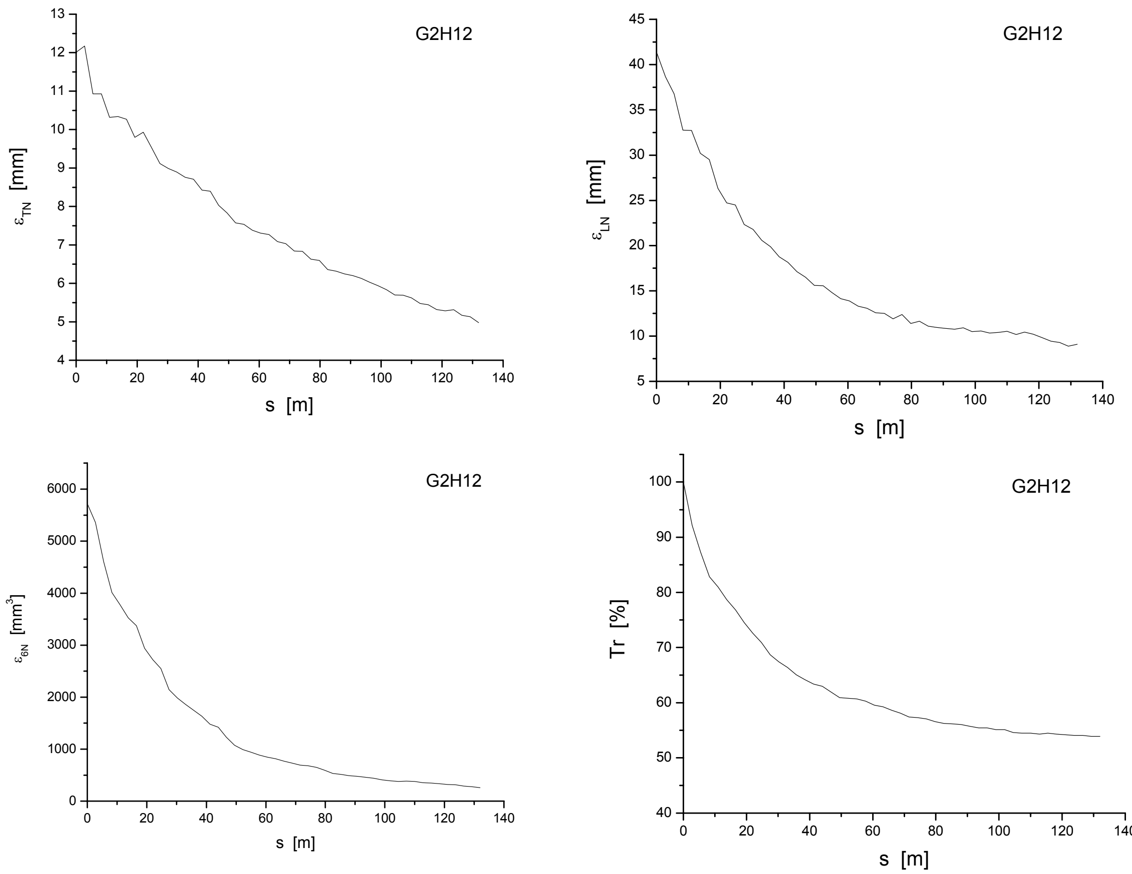

Figure 10. Cooling performance of the 201 MHz Guggenheim channel

The channel successfully reduces the transverse normalized emittance to 5 mm and the longitudinal emittance to 9 mm in four turns. The overall transmission is 52% including decays. The 6D emittance falls by a factor of 25, showing that real cooling is taking place in the channel.

6. Conclusions

We have computed a 3D magnetic field map that should allow accurate calculations of the cooling performance of the Guggenheim channel using ICOOL. The computed Frenet field components on the reference helix agree well with results from G4Beamline. We were able to find reasonable periodic orbit solutions using the new field map. The 201 MHz channel for the muon collider showed good cooling performance, although the low overall transmission is still an issue.

Acknowledgements

I would like to thank Juan Gallardo and Pavel Snopok for useful discussions.

References

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