

# Varying Gradients and Magnetic Fields and Muon Capture for a Neutrino Factory

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September 2009*

**Abstract.** Scenarios for capture, bunching and phase-energy rotation of  $\mu$ 's from a proton source have been developed. The goal is capture of a maximal number of muons in a string of rf bunches, with applications in neutrino factories and  $\mu^+\mu^-$  colliders. In this note we continue variation studies of the Front End. We begin with the  $n_B = 10$  example described in MuCOOL 520 and consider some variations motivated by recent observations on possible limitations of rf gradients within magnetic fields. In this paper we explore the effects of reducing rf gradients and magnetic fields from the previous baseline.

## Introduction

For a neutrino factory or a  $\mu^+\mu^-$  collider, short, intense bunches of protons are focused onto a target to produce pions, that decay into muons, which are then cooled and accelerated into a high-energy storage ring, where  $\mu$  decays can provide beams of high-energy neutrinos for a  $\nu$ -factory [1, 2, 3]. If the  $\mu^+$  and  $\mu^-$  bunches are counter-rotating and focused to collide in an interaction region, high-luminosity  $\mu^+\mu^-$  collisions are possible. [4]

The pions (and resulting muons) are initially produced within a short bunch length and a broad energy spread. In the “front end”, the  $\pi$ 's drift from the production target, lengthening into a long bunch with a high-energy “head” and a low-energy “tail”, while decaying into  $\mu$ 's. The beam is then transported through a “buncher” that forms the beam into a string of bunches, and an “rf rotator” section that aligns the bunches to (nearly) equal central energies, and then cooled in a “cooler” with rf cavities and absorbers. (see fig. 1 and 5) Table I shows baseline parameters for front end solutions that were developed in ref. [5]. The example displayed in fig. 1 and 5 is labeled  $N_B=10$ . That example obtains  $\sim 0.08 \mu^+$  and  $\mu^-$  per initial 8 GeV proton within the nominal acceptance of a  $\nu$ -factory.

**Table 1:** Parameters of some buncher/rotator scenarios.

Simulation cases		Study 2B		
Parameter		$N_B=18$	$N_B=10$	$N_B=7$
Bunch spacing number	$N_B$	18	10	7
Drift Length	$L_D$	110.7m	56.4m	37.75m
Buncher Length	$L_B$	51m	31.5	21
Buncher rf Gradient	$V_{rf}$	0 to 12	0 to 15	0 to 15
Buncher rf frequencies	$f_{rf,B}$	360→235MHz	360→240	350→240MHz
Rotator Length	$L_R$	54m	36m	27m
Rotator Bunch spacing	$N_B+\delta N_B$	18.05	10.08	7.08
Rotator gradient	$V_{rf}$	12	15	15
Rotator rf frequencies	$f_{rf,R}$	232 to 202	240 to 201.5	240 to 201.5
$\mu/24$ GeV p ( $A_T < 0.03$ , $A_L < 0.2$ m) after rotator.	$\mu/p_{24}$	0.126	0.124	0.10
$\mu/p$ ( $A_T < 0.03$ , $A_L < 0.2$ ) after LiH cooler	$\mu/p_{24}$	0.265	0.263	0.21
Final transverse emittance	$\epsilon_{T,rms}$	0.0076	0.0078	0.008
Final Longitudinal emittance	$\epsilon_{L,rms}$	0.071	0.076	0.091

In the baseline configuration, the rf cavities in the Buncher and the Rotator are closed-cell pillbox cavities placed within focusing solenoidal fields, with a nominal field of  $\sim 2$ T. (see fig. 2) Recent theoretical models and experimental studies suggest that this configuration enhances the possibility of rf breakdown. The model is that the solenoidal and rf fields would guide and accelerate emitted electrons across the cavity, causing large secondary emissions at the opposing surface, and an avalanche effect in multiple electron passages. It is considered likely that rf gradient will be limited by the magnetic field with the allowable gradient reduced with increasing magnetic field.[6] A model for that dependence is displayed in figure 3. The model is guided by breakdown observations in some magnet-rf configurations and extrapolated frequency dependences, although the front-end rf-magnet configuration has not been accurately tested.

If the model be accurate, the  $\sim 200$  MHz cavities of the baseline front end would be limited to  $\sim 6$ MV/m maximum gradient at  $B = 2$ T, somewhat less than the gradient in the baseline design, and it is uncertain that adequate  $\mu$  capture could then be obtained. The present baseline examples have up to 15MV/m gradient in the cavities, and would violate that limit. Note that even if the limitation model is accurate, the gradient limit could be anywhere from  $\sim 5$ MV/m to  $\sim 9$ MV/m at 200MHz and  $B=2$ T. The measurements and model are not yet established at a level that sets clear limitations.

There are several potential strategies to circumvent this difficulty (if it exists) which are under consideration:

1. The baseline rf cavities are pillboxes, with Be windows. However, experiments indicate that open-cell rf cavities may not be as limited. Open-cell cavities would require  $\sim$ twice as much rf power for equal gradient. This would be an undesired cost, but probably acceptable if needed. However, we are not yet certain that open cell rf can obtain the 10 to 15MV/m gradients of the baseline, and more direct experiments are needed.

2. Experiments in 800MHz button cavities indicate that gas-filled rf cavities are not limited in gradient by magnetic fields, and may enable higher gradients. The energy loss in the gas provides cooling, although additional gradient is needed to compensate that energy loss. Gas-filled rf could be used in the Cooler and would provide better muon cooling. We have previously simulated using gas-filled rf (with an alternating-solenoid focusing lattice) in the Rotator section, with a density large enough to provide cooling that replaced much of the Cooler. Performance was adequate. However, there is presently a concern that the electrons produced in the gas may drain rf energy from the cavities.[7, 8] Gas-filled cavities should be tested with beam to determine whether this is a genuine limitation at our parameters. We also need experimental verification that a gas-filled (~200 MHz) cavity can provide the needed gradient for our parameters.

3. Using “magnetically-insulated” rf cavities.[6] If emitted electrons are trapped along magnetic field lines, then reconfiguring the magnet and cavities so that the cavity surfaces are parallel to field lines prevents the cross-cavity accelerated electron paths that cause the breakdown. However such cavities would appear to have non-optimum shapes, and would also likely be open-cell cavities. (If open-cell cavities enable adequate gradient within the solenoidal fields, the added complication of magnetic insulation could be avoided.)

4. Avoiding a constant field solenoid and using an alternating-solenoid focusing lattice. In the baseline example, the Cooler has an alternating solenoid lattice with a period of 1.5m, obtained by solenoid coils placed between successive rf cavities (see fig. 4). In this configuration the magnetic field lines do not connect directly across the cavity and rf breakdown may be relatively suppressed. The on-axis field here is similar to the magnetically insulated field, but without the added complication of molding the cavity shape to the magnetic field lines. This case has been explored in other simulations, in which the buncher and/or rotator section are both placed within alternating solenoid lattices.[9]

5. Reconfiguring the front end so that the gradient does not exceed  $V'_{\max}(B)$ . This is discussed below. This may be implemented and could be adequate if  $V'_{\max}(B)$  is not too small, but we do not yet have an accurate expression for  $V'_{\max}(B)$ . The expression for  $V'_{\max}(B)$  may be dependent on cavity material, temperature and geometry. Palmer has suggested that the lower energy loss and high conductivity of Be may enable much higher gradients than the Cu cavities of the initial designs.[10]

While there are a variety of potential mitigation strategies, it is not yet known which of these strategies is necessary or sufficient. We will need experiments verifying any particular method. In the present note, we will explore some of these variations. In particular we will explore varying the rf gradients and the magnetic fields.

## VARIATION OF BUNCHER STRENGTH

In the baseline example, the buncher was  $L_B = 31.5\text{m}$  in length, and the rf gradient in rf cavities along the buncher increases quasi-adiabatically:

$$V'_{\text{rf}}(z) = 6\left(\frac{z}{L_B}\right) + 9\left(\frac{z}{L_B}\right)^2 \text{ MV/m.}$$

The rf frequency decreases from 360 to 240 MHz along the buncher. In the baseline configuration, the rf cavities have nominal lengths of 0.5m with pillbox fields, with 0.25m spacing between cavities, and a constant 2T solenoidal field is maintained throughout the buncher. The gradual increase of the rf field within the buncher is believed to enable a somewhat adiabatic capture of the muons into strings of bunches at different energies, preparing the bunches for lower-loss acceleration or deceleration in the Rotator.

The maximum rf gradient allowed within 2T fields may be somewhat less than the 15MV/m used in the baseline example. Extrapolation from the initial 200 MHz tests as reported in ref.[6] indicate that (at least) ~6MV/m should be acceptable. Therefore we explored the implications of reducing that maximum by simulation studies using ICOOL. In the simulations we used a buncher with a linear gradient increase, and varied the maximum gradient  $V_{\text{rf,max}}$ :

$$V'_{\text{rf}}(z) = V'_{\text{max}} \left( \frac{z}{L_B} \right) \text{ MV/m}$$

$V_{\text{rf,max}}$  was varied from 0 to 15 MV/m, and the Rotator and Cooler were left unchanged with the baseline configuration.

The front end capture was simulated using the ICOOL program[11], and results are displayed in table 3. In these simulations an initial population of  $\pi$ 's and  $\mu$ 's, as would be produced by 8GeV protons incident on a mercury target at the start of the front end transport, are tracked through the drift buncher, rotator and cooler. In these simulations muons are considered accepted if they meet the ECALC9 criteria of longitudinal amplitudes less than 0.2m and total transverse amplitudes less than 0.03m.[12] The number of muons within the reference acceptance at the beginning and end of the Cooler are displayed in Table 2.

**Table 2:**  $\mu$  Capture with varying Buncher rf strengths

$V_{\text{max}}$ (MV/m)	$\mu/p$ at z=128m	$\mu/p$ at z=210m	Total rf voltage
0	0.032	0.058	0 MV
3 (z/L)	0.036	0.075	31.5 MV
6 (z/L)	0.038	0.081	63 MV
9 (z/L)	0.040	0.084	94.5 MV
12 (z/L)	0.041	0.087	126 MV
15 (z/L)	0.041	0.087	157.5 MV
6 (z/L) + 9 (z/L) <sup>2</sup> - baseline parameters	0.039	0.082	126 MV

From these results we can obtain some conclusions on Buncher parameters.

1. The baseline configuration with a linear plus quadratic increase of rf field was not optimal. A simple linear ramp in gradient from 0 to 12 MV/m captured ~6% more  $\mu$ 's than the baseline.
2. The performance was not greatly sensitive to the final gradient within the buncher. Choosing any value from 9 to 15MV/m, obtained approximately the same acceptance. Reduction to 6MV/m only reduced acceptance by ~5% from the best values. Reduction to 3MV/m leads to less than 10% additional loss. The results show that if relatively modest gradients are achievable within solenoid fields, the Buncher can function adequately.

3. Even with no gradient within the Buncher (Buncher replaced by rf-less drift), the performance is not disastrously reduced. This corresponds to an initially non-adiabatic capture in the Rotator. The loss of ~30% of the potentially capturable beam is comparable to results that are obtained in simulations where non-adiabatic rf capture occurs.

The large variation in Buncher strength that can be explored with little performance loss indicates that the Buncher could be significantly shorter and/or weaker (and probably more affordable) than the baseline example with little loss in performance. Alternatively, its functions could be more gradually integrated with the Rotator functions. These variations should be explored in future studies.

### **Variation: Lower Gradients in Buncher and Rotator**

In another set of studies, we consider the possibility that the front end will not be able to use rf gradients within magnetic fields that are as large as those in the baseline designs. The baseline  $N_B=10$  example has rf gradients up to ~15MV/m with solenoidal fields of  $B=2T$  in the buncher and rotator. In the present studies we consider reducing the rf voltage in order to study the potential effects of restrictions on maximum rf gradient. From the previous study we choose a linear ramp in gradient from 0 to 9MV/m as a reasonable starting point and consider reducing rf gradients down to ~9MV/m in the Rotator and Cooler.

With the Cooler fixed at ~15MV/m, we first considered lowering the gradient in the rf rotator section. In the initial simulation the rf gradients in the Rotator were set to 15MV/m for all cavities, and in the present study we considered changing this to values as low as 9MV/m. In this initial study, we kept all parameters fixed except for the rf gradients in the Rotator and Cooler and changed these to new reference values. (rf phases and frequencies and other parameters were not reoptimized to match the new conditions.) Results from ICOOL simulations are displayed in table 3. In these simulations muons are also considered accepted if they meet the ECALC9 criteria of longitudinal amplitude less than 0.2m and total transverse amplitude less than 0.03m.[12] Under these criteria, the acceptance is somewhat reduced by the lower gradient in the rotator. At the exit of the rotator,  $V_B' = 15$  MV/m is reduced from ~0.04 to 0.0375 to 0.034 to 0.0315 as  $V_R'$  is reduced from 15 to 12 to 10 to 9 MV/m, respectively. However the cooling channel increases the accepted beam by a factor of ~2 for  $V_B' = 15$  MV/m and by a bit more for  $V_B' = \sim 10$  MV/m, so that  $\mu/p$  is reduced from ~0.081 at  $V_B' = 15$  MV/m to ~0.073 at 10MV/m.

We next considered changing gradients in both the Cooler and Rotator, with both gradients reduced toward 9MV/m. Since we expect that rf limitations are more stringent in the constant-B Rotator than in an alternating solenoid (ASOL) lattice, we usually run simulations with rf gradients in the Rotator less than or equal to rf gradients in the ASOL Cooler.

With lower gradients in the cooling channel, we find that the rf gradient becomes insufficient to recover the energy loss in the absorbers and maintain stable longitudinal motion and the acceptance drops dramatically. However one can maintain stable cooling by reducing the absorber length per cell and the acceptance remains large. In the present examples we reduced absorber lengths from 1.0cm to 0.8 to 0.65 as  $V_C'$  is reduced from 15 to 12 to 10 MV/m, respectively. One then finds that equivalent cooling requires a longer cooling channel. With these adjustments,  $\mu/p$  remains more stable, reducing from ~0.081 to 0.063 as  $V'$  is reduced from 15 to 10MV/m. The extra-long cooling channels are not so efficient at producing muons within the acceptance. Truncating the cooling channels at  $z=205m$  (~75m of cooling) reduces the  $\mu/p$  by ~5% for the 12 MV/m cases and ~10% for the 10MV/m cases.

We continued the study by changing gradients in both cooler and rotator in order to map out some of the potential changes in performance. We also included some cases with higher Cooler gradients; some potential for greater acceptance is possible there. The overall impression obtained in these studies is that, while acceptance decreases with reduced gradients, the decrease is not precipitous. Adequate performance for a neutrino factory would still be possible even if the acceptable gradient is reduced to  $\sim 10\text{MV/m}$  or less.

The  $\sim 10\text{MV/m}$  gradient is in a model with  $2/3$  rf occupancy. If more rf cavities are incorporated, and the drift spaces are filled with rf cavities, the gradient would be reduced to  $\sim 6.7\text{MV/m}$ , which could be more certain to be practical. We thus may also consider scenarios in which rf gradients are reduced by more completely filling the longitudinal space with cavities.

**Table 3:**  $\mu$  Capture with varying Rotator and Cooler rf strengths ( $B_0 = 2\text{T}$ )

V' (Rotator)	V' (Cooler)	$\mu/p$ at $z=129\text{m}$	$\mu/p$ at "end"	Final $z_{\text{opt}}$
15 MV/m	17 MV/m	0.040	0.087	207
15	16	0.040	0.085	205
15	15	0.040	0.081	205
14	15	0.039	0.080	205
12	15	0.0375	0.077	205
10	15	0.034	0.073	205
9	15	0.031	0.068	205
14 (z/L)	14	0.041	0.0776	205
12	14	0.0375	0.0754	205
10	14	0.034	0.068	205m
12	12	0.0375	0.072	215, 0.8cm
10	12	0.034	0.067	215, 0.8cm
10	10	0.0338	0.063	240, 0.65cm
9	10	0.0325	0.060	240, 0.65cm
9	9	0.0325	0.057	260, 0.6cm

### Variation: Lower Magnetic Field in Buncher/Rotator

We next consider lowering the magnetic field in the buncher and rotator. The baseline case used 2T. We reduce that to 1.25T throughout the drift, buncher, and rotator, matching into the study 2A ASOL channel for the cooling section. The 1.25T fields were matched into the cooling channel using a set of solenoidal coils generated by R. Palmer in a previous study. With this new lattice, we then proceeded to vary the bunching and rotator rf gradient strengths, similar to the pattern used in the above  $B=2\text{T}$  cases. Results are presented in Table 4, in much the same format presented in Table 3.

In these simulations we obtain the same pattern presented in Table 3, but with slightly reduced acceptances. Typical reductions are  $< \sim 5\%$ . This was a relatively small effect from a greatly reduced focusing field, and the  $B=1.25$  case may even be a better cost/performance option. It is likely that the 2T value is not optimal, and some intermediate field might have better performance and be a preferable option. Further study should be done to find a cost/performance optimum.

**Table 4:**  $\mu$  Capture with varying Rotator and Cooler rf strengths ( $B_0 = 1.25T$ )

$V'$ (Rotator)	$V$ (Cooler)	$\mu/p$ at $z=132m$	$\mu/p$ at $z_{opt}$	$z_{opt}, L_{abs}$
15	17	0.039	0.081	210
15	16	0.039	0.080	205
15	15	0.039	0.076	210
14	15	0.038	0.078	210
12	15	0.036	0.074	214
10	15	0.034	0.069	210
9	15	0.031	0.068	210
14 (z/L)	14	0.0396	0.072	210
10	14	0.0337	0.066	210
12	12	0.0373	0.0693	231, 0.8
10	12	0.035	0.0665	220, 0.8
9	12	0.0335	0.0621	220, 0.8
10	10	0.035	0.061	245, 0.65
9	10	0.0335	0.0587	245, 0.65

## Comments and Future Studies

From these studies, we see that adequate performance in the neutrino factory front end can be obtained with substantially reduced magnetic and rf gradient fields, although performance is somewhat reduced from higher-field parameters. When experiments and further studies specify acceptable or safe magnetic field and/or rf gradients, the front end design can be modified to operate within those “safe” levels. Another inferred result is that a front end that is unable to reach design gradients/fields can still operate adequately at somewhat reduced fields.

## Acknowledgments

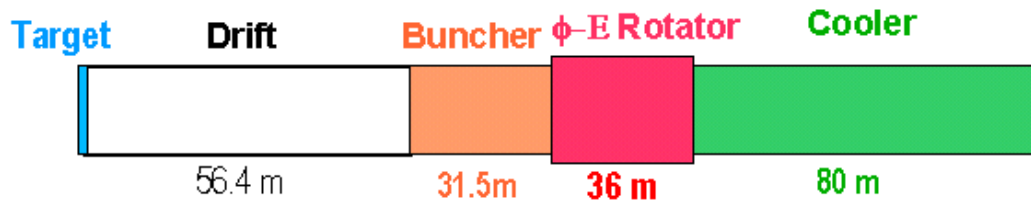
I thank C. Yoshikawa, R. Palmer, M. Zisman and R. Fernow for important contributions and discussions. Research supported by Department of Energy under contract no. DEAC02-07CH11359.

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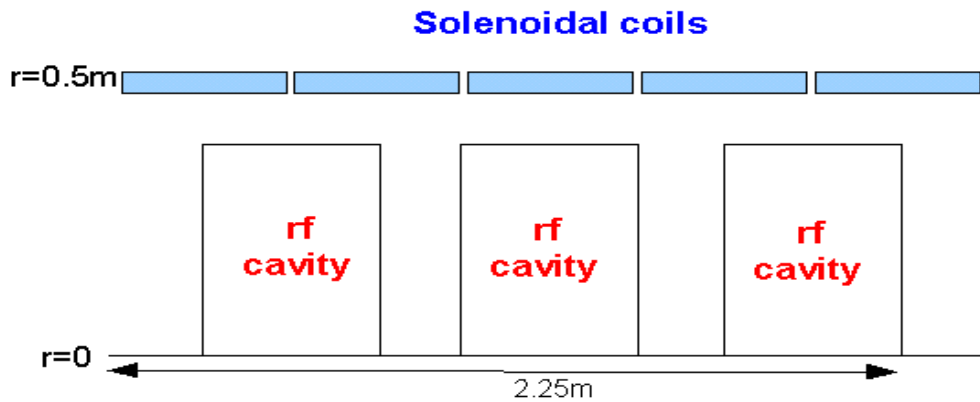
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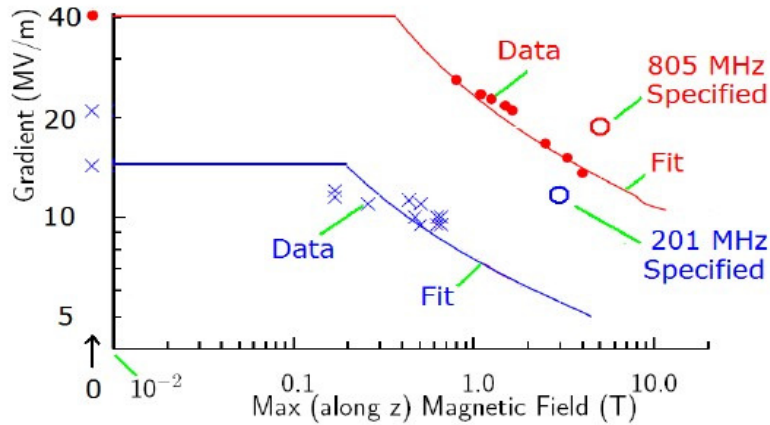




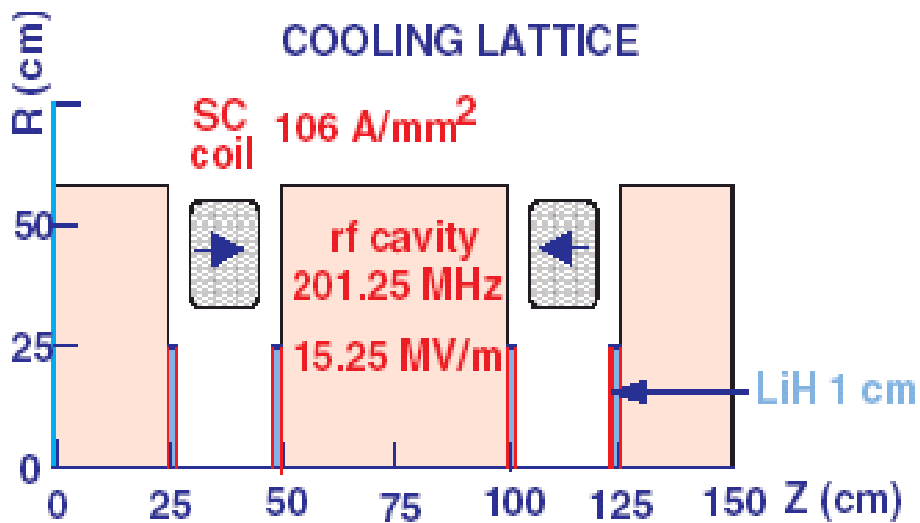
**FIGURE 1.** Schematic view of the components of the  $n_B=10$  front-end system, showing an initial drift (56.4m), the buncher (31.5m), and the phase-energy ( $\phi$ - $\delta E$ ) rotator (36m) leading into a cooling section of up to 80m.  $\pi$ 's would be produced by protons on a target at the beginning of the drift, decay to  $\mu$ 's in the drift, while lengthening in phase. The buncher and  $\phi$ - $\delta E$  rotator form the  $\mu$ 's into a string of bunches matched into the cooler.



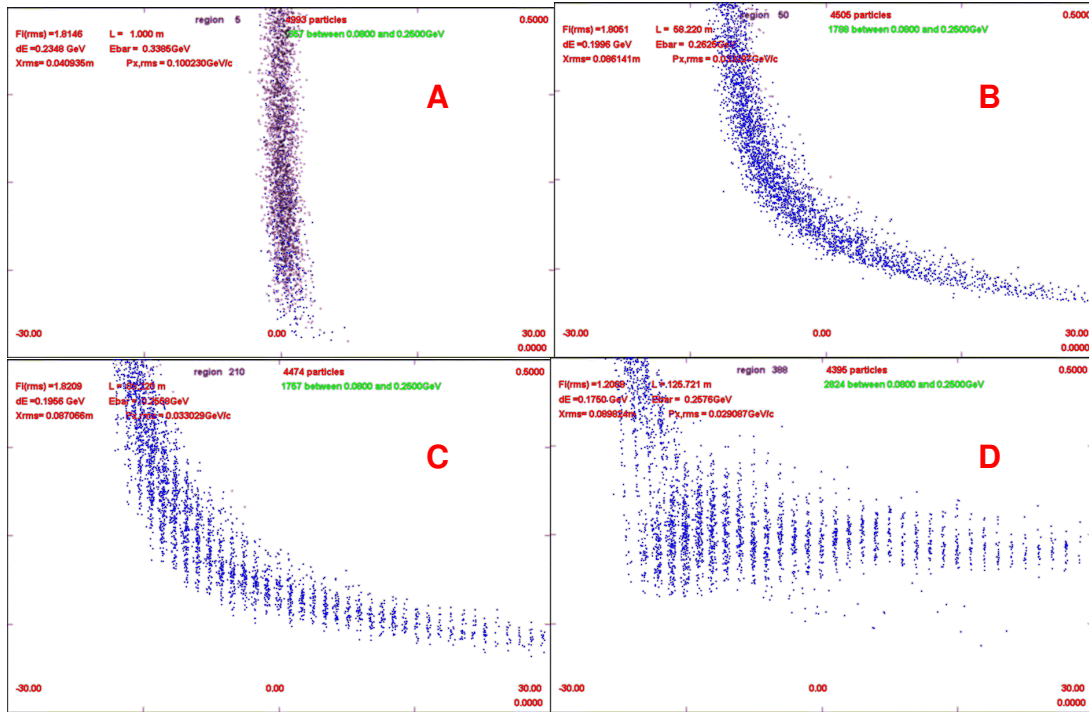
**Figure 2:** Baseline configuration for the buncher and phase-energy rotation sections of the v-factory front end. The sections consist of rf cavities (cylindrical “pill-boxes”) that are spaced at 0.75m intervals with a nominal length of 0.5m (0.25m between cavities). The cavities are placed within a constant-field solenoid, with  $B=2\text{T}$ .



**Figure 3.** Maximum Rf gradient versus magnetic field for 201 and 805 MHz rf cavities. The “data” points are based on experimental observations in a MuCOOL experimental run. The “Fit” lines are based on the data values and extrapolation within the model of ref. 6. The specified 805 and 201 MHz values correspond to a particular muon collider cooling scenario.



**Figure 4:** Cooling cell from Study 2B.[3] The cooling cell includes 2 rf cavities, 4 LiH absorbers and two superconducting coils. An alternating



**Figure 4:** ICOOL simulation results of the buncher and phase rotation, at the parameters of the  $N_B=10$  example described in the text. Each figure shows the A:  $\pi$ 's and  $\mu$ 's as produced at the end of a 1.0m long target. B:  $\mu$ 's at  $z=58\text{m}$  after a drift. C:  $\mu$ 's at  $z=93\text{m}$ , the end of the buncher. The beam has been formed into a string of  $\sim 200\text{MHz}$  bunches at different energies. D: At  $z=126\text{m}$  after  $\phi$ - $\delta E$  rotation; the bunches are aligned into nearly equal energies. In each plot the vertical axis is momentum (0 to 0.5 GeV/c) and the horizontal axis is longitudinal position with respect to a reference particle (-30 to 30m).